

Cambridge University Press

0521801605 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

Frontmatter

[More information](#)

Although topology was recognized by Gauss and Maxwell to play a pivotal role in the formulation of electromagnetic boundary value problems, it is a largely unexploited tool for field computation. The development of algebraic topology since Maxwell provides a framework for linking data structures, algorithms, and computation to topological aspects of three-dimensional electromagnetic boundary value problems. This book attempts to expose the link between Maxwell and a modern approach to algorithms.

The first chapters lay out the relevant facts about homology and cohomology, stressing their interpretations in electromagnetism. These topological structures are subsequently tied to variational formulations in electromagnetics, the finite element method, algorithms, and certain aspects of numerical linear algebra. A recurring theme is the formulation of algorithms for the problem of making branch cuts for computing magnetic scalar potentials and eddy currents. An appendix bridges the gap between the material presented and standard expositions of differential forms, Hodge decompositions, and tools for realizing representatives of homology classes as embedded manifolds.

Cambridge University Press

0521801605 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

Frontmatter

[More information](#)

**Mathematical Sciences Research Institute
Publications**

48

**Electromagnetic Theory and Computation
A Topological Approach**

Cambridge University Press

0521801605 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

Frontmatter

[More information](#)

Mathematical Sciences Research Institute Publications

- 1 Freed/Uhlenbeck: *Instantons and Four-Manifolds*, second edition
- 2 Chern (ed.): *Seminar on Nonlinear Partial Differential Equations*
- 3 Lepowsky/Mandelstam/Singer (eds.): *Vertex Operators in Mathematics and Physics*
- 4 Kac (ed.): *Infinite Dimensional Groups with Applications*
- 5 Blackadar: *K-Theory for Operator Algebras*, second edition
- 6 Moore (ed.): *Group Representations, Ergodic Theory, Operator Algebras, and Mathematical Physics*
- 7 Chorin/Majda (eds.): *Wave Motion: Theory, Modelling, and Computation*
- 8 Gersten (ed.): *Essays in Group Theory*
- 9 Moore/Schochet: *Global Analysis on Foliated Spaces*
- 10–11 Drasin/Earle/Gehring/Kra/Marden (eds.): *Holomorphic Functions and Moduli*
- 12–13 Ni/Peletier/Serrin (eds.): *Nonlinear Diffusion Equations and Their Equilibrium States*
- 14 Goodman/de la Harpe/Jones: *Coxeter Graphs and Towers of Algebras*
- 15 Hochster/Huneke/Sally (eds.): *Commutative Algebra*
- 16 Ihara/Ribet/Serre (eds.): *Galois Groups over \mathbb{Q}*
- 17 Concus/Finn/Hoffman (eds.): *Geometric Analysis and Computer Graphics*
- 18 Bryant/Chern/Gardner/Goldschmidt/Griffiths: *Exterior Differential Systems*
- 19 Alperin (ed.): *Arboreal Group Theory*
- 20 Dazord/Weinstein (eds.): *Symplectic Geometry, Groupoids, and Integrable Systems*
- 21 Moschovakis (ed.): *Logic from Computer Science*
- 22 Ratiu (ed.): *The Geometry of Hamiltonian Systems*
- 23 Baumslag/Miller (eds.): *Algorithms and Classification in Combinatorial Group Theory*
- 24 Montgomery/Small (eds.): *Noncommutative Rings*
- 25 Akbulut/King: *Topology of Real Algebraic Sets*
- 26 Judah/Just/Woodin (eds.): *Set Theory of the Continuum*
- 27 Carlsson/Cohen/Hsiang/Jones (eds.): *Algebraic Topology and Its Applications*
- 28 Clemens/Kollár (eds.): *Current Topics in Complex Algebraic Geometry*
- 29 Nowakowski (ed.): *Games of No Chance*
- 30 Grove/Petersen (eds.): *Comparison Geometry*
- 31 Levy (ed.): *Flavors of Geometry*
- 32 Cecil/Chern (eds.): *Tight and Taut Submanifolds*
- 33 Axler/McCarthy/Sarason (eds.): *Holomorphic Spaces*
- 34 Ball/Milman (eds.): *Convex Geometric Analysis*
- 35 Levy (ed.): *The Eightfold Way*
- 36 Gavosto/Krantz/McCallum (eds.): *Contemporary Issues in Mathematics Education*
- 37 Schneider/Siu (eds.): *Several Complex Variables*
- 38 Billera/Björner/Green/Simion/Stanley (eds.): *New Perspectives in Geometric Combinatorics*
- 39 Haskell/Pillay/Steinhorn (eds.): *Model Theory, Algebra, and Geometry*
- 40 Bleher/Its (eds.): *Random Matrix Models and Their Applications*
- 41 Schneps (ed.): *Galois Groups and Fundamental Groups*
- 42 Nowakowski (ed.): *More Games of No Chance*
- 43 Montgomery/Schneider (eds.): *New Directions in Hopf Algebras*
- 44 Buhler/Stevenhagen (eds.): *Algorithmic Number Theory*
- 45 Jensen/Ledet/Yui: *Generic Polynomials: Constructive Aspects of the Inverse Galois Problem*
- 46 Rockmore/Healy (eds.): *Modern Signal Processing*
- 47 Uhlmann (ed.): *Inside Out: Inverse Problems and Applications*
- 48 Gross/Kotiuga: *Electromagnetic Theory and Computation: A Topological Approach*

Volumes 1–4 and 6–27 are published by Springer-Verlag

Cambridge University Press

0521801605 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

Frontmatter

[More information](#)

Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross

MSRI and HP/Agilent

P. Robert Kotiuga

Boston University



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press

0521801605 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

Frontmatter

[More information](#)

Paul Gross
pwgross@alum.bu.edu

P. Robert Kotiuga
Department of Electrical
and Computer Engineering
Boston University
8 Saint Mary's Street
Boston, MA 02215
United States
prk@bu.edu

Series Editor

Silvio Levy
Mathematical Sciences
Research Institute
17 Gauss Way
Berkeley, CA 94720
United States

MSRI Editorial Committee

Hugo Rossi (chair)
Alexandre Chorin
Silvio Levy
Jill Mesirov
Robert Osserman
Peter Sarnak

The Mathematical Sciences Research Institute wishes to acknowledge support by the National Science Foundation. This material is based upon work supported by NSF Grant 9810361.

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

<http://www.cambridge.org>

© Mathematical Sciences Research Institute 2004

Printed in the United States of America

A catalogue record for this book is available from the British Library.

Library of Congress Cataloging in Publication data available

ISBN 0 521 801605 hardback

Cambridge University Press

0521801605 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

Frontmatter

[More information](#)

T

able of Contents

Preface	ix
Introduction	1
Chapter 1. From Vector Calculus to Algebraic Topology	7
1A Chains, Cochains and Integration	7
1B Integral Laws and Homology	10
1C Cohomology and Vector Analysis	15
1D Nineteenth-Century Problems Illustrating the First and Second Homology Groups	18
1E Homotopy Versus Homology and Linking Numbers	25
1F Chain and Cochain Complexes	28
1G Relative Homology Groups	32
1H The Long Exact Homology Sequence	37
1I Relative Cohomology and Vector Analysis	41
1J A Remark on the Association of Relative Cohomology Groups with Perfect Conductors	46
Chapter 2. Quasistatic Electromagnetic Fields	49
2A The Quasistatic Limit of Maxwell's Equations	49
2B Variational Principles for Electroquasistatics	63
2C Variational Principles for Magnetoquasistatics	70
2D Steady Current Flow	80
2E The Electromagnetic Lagrangian and Rayleigh Dissipation Functions	89
Chapter 3. Duality Theorems for Manifolds With Boundary	99
3A Duality Theorems	99
3B Examples of Duality Theorems in Electromagnetism	101
3C Linking Numbers, Solid Angle, and Cuts	112
3D Lack of Torsion for Three-Manifolds with Boundary	117
Chapter 4. The Finite Element Method and Data Structures	121
4A The Finite Element Method for Laplace's Equation	122
4B Finite Element Data Structures	127
4C The Euler Characteristic and the Long Exact Homology Sequence	138

Cambridge University Press

0521801605 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

Frontmatter

[More information](#)

viii

TABLE OF CONTENTS

Chapter 5. Computing Eddy Currents on Thin Conductors with Scalar Potentials	141
5A Introduction	141
5B Potentials as a Consequence of Ampère's Law	142
5C Governing Equations as a Consequence of Faraday's Law	147
5D Solution of Governing Equations by Projective Methods	147
5E Weak Form and Discretization	150
Chapter 6. An Algorithm to Make Cuts for Magnetic Scalar Potentials	159
6A Introduction and Outline	159
6B Topological and Variational Context	161
6C Variational Formulation of the Cuts Problem	168
6D The Connection Between Finite Elements and Cuts	169
6E Computation of 1-Cocycle Basis	172
6F Summary and Conclusions	180
Chapter 7. A Paradigm Problem	183
7A The Paradigm Problem	183
7B The Constitutive Relation and Variational Formulation	185
7C Gauge Transformations and Conservation Laws	191
7D Modified Variational Principles	197
7E Tonti Diagrams	207
Mathematical Appendix: Manifolds, Differential Forms, Cohomology, Riemannian Structures	215
MA-A Differentiable Manifolds	216
MA-B Tangent Vectors and the Dual Space of One-Forms	217
MA-C Higher-Order Differential Forms and Exterior Algebra	220
MA-D Behavior of Differential Forms Under Mappings	223
MA-E The Exterior Derivative	226
MA-F Cohomology with Differential Forms	229
MA-G Cochain Maps Induced by Mappings Between Manifolds	231
MA-H Stokes' Theorem, de Rham's Theorems and Duality Theorems	232
MA-I Existence of Cuts Via Eilenberg–MacLane Spaces	240
MA-J Riemannian Structures, the Hodge Star Operator and an Inner Product for Differential Forms	243
MA-K The Operator Adjoint to the Exterior Derivative	249
MA-L The Hodge Decomposition and Ellipticity	252
MA-M Orthogonal Decompositions of p -Forms and Duality Theorems	253
Bibliography	261
Summary of Notation	267
Examples and Tables	273
Index	275

Cambridge University Press

0521801605 - Electromagnetic Theory and Computation: A Topological Approach

Paul W. Gross and P. Robert Kotiuga

Frontmatter

[More information](#)

P

reface

The authors are long-time fans of MSRI programs and monographs, and are thrilled to be able to contribute to this series. Our relationship with MSRI started when Paul Gross was an MSRI/Hewlett-Packard postdoctoral fellow and had the good fortune of being encouraged by Silvio Levy to coauthor a monograph. Silvio was there when we needed him, and it is in no way an understatement to say that the project would never have been completed without his support.

The material of this monograph is easily traced back to our Ph.D. theses, papers we wrote, and courses taught at Boston University over the years. Our apologies to anyone who feels slighted by a minimally updated bibliography. Reflecting on how the material of this monograph evolved, we would like to thank colleagues who have played a supporting role over the decades. Among them are Alain Bossavit, Peter Caines, Roscoe Giles, Robert Hermann, Lauri Kettunen, Isaak Mayergoyz, Peter Silvester, and Gilbert Strang. The authors are also indebted to numerous people who read through all or part of the manuscript, produced numerous comments, and provided all sorts of support. In particular, Andre Nicolet, Jonathan Polimeni, and Saku Suuriniemi made an unusually thorough effort to review the draft.

Paul Gross would like to acknowledge Nick Tuffiaro at Hewlett-Packard and Agilent Technologies for mentoring him throughout his post-doc at MSRI. Tim Dere graciously provided his time and expertise for illustrations. This book could not have happened without help and encouragement from Tanya.

Robert Kotiuga is grateful to the students taking his courses at Boston University, to Nevine, Michele, Madeleine, Peter and Helen for their support, and to Boston University for granting him a leave while the book was in its final stages.

Paul Gross and Robert Kotiuga
August 2003