

Contents

<i>Preface</i>	<i>page</i>	xiii
<i>Acknowledgments</i>		xvii
<i>Acronyms and Abbreviations</i>		xix

Part I Spin-1/2 Fermions in Quantum Field Theory, the Standard Model, and Beyond

1 Two-Component Formalism for Spin-1/2 Fermions	3
1.1 The Lorentz Group and Its Lie Algebra	3
1.2 The Poincaré Group and Its Lie Algebra	6
1.3 Spin-1/2 Representation of the Lorentz Group	7
1.4 Bilinear Covariants of Two-Component Spinors	12
1.5 Lagrangians for Free Spin-1/2 Fermions	20
1.6 The Fermion Mass Matrix and Its Diagonalization	24
1.7 Discrete Spacetime and Internal Symmetries	30
1.8 Parity Transformation of Two-Component Spinors	33
1.9 Time-Reversal of Two-Component Spinors	36
1.10 Charge Conjugation of Two-Component Spinors	40
1.11 CP and CPT Conjugation of Two-Component Spinors	41
Exercises	43
2 Feynman Rules for Spin-1/2 Fermions	52
2.1 Fermion Creation and Annihilation Operators	52
2.2 Properties of Two-Component Spinor Wave Functions	53
2.3 Charged Two-Component Fermion Fields	59
2.4 Feynman Rules for External Fermion Lines	61
2.5 Feynman Rules for Propagators	62
2.6 Feynman Rules for Fermion Interactions	70
2.7 General Structure and Rules for Feynman Graphs	79
2.8 Examples of Feynman Diagrams and Amplitudes	81
2.9 Self-Energy of Scalar and Vector Bosons	95
2.10 Self-Energy of Two-Component Fermions	107
2.11 Feynman Rules for External Fermion Lines Revisited	124
Exercises	127

3 From Two-Component to Four-Component Spinors	136
3.1 Four-Component Spinors and Dirac Gamma Matrices	136
3.2 Four-Component Spinor Indices	141
3.3 Bilinear Covariants and Their P, T, and C Properties	145
3.4 Lagrangians for Free Four-Component Fermions	151
3.5 Properties of Four-Component Spinor Wave Functions	155
3.6 Feynman Rules for Four-Component Fermions	159
3.7 Feynman Rules for External Fermion Lines	165
3.8 Examples of Feynman Diagrams and Amplitudes	166
3.9 Self-Energy of Four-Component Fermions	172
3.10 Feynman Rules for External Fermion Lines Revisited	182
Exercises	184
4 Gauge Theories and the Standard Model	191
4.1 Abelian Gauge Field Theory	191
4.2 Nonabelian Gauge Groups and Their Lie Algebras	192
4.3 Nonabelian Gauge Field Theory	196
4.4 Feynman Rules for Gauge Theories	199
4.5 Spontaneously Broken Gauge Theories	202
4.6 Complex Representations of Scalar Fields	210
4.7 The Standard Model of Particle Physics	214
4.8 Parameter Count of the Standard Model	223
Exercises	224
5 Anomalies	229
5.1 Anomalous Chiral Symmetries	229
5.2 Gauge Anomalies and Their Cancellation	243
5.3 Discrete Gauge Anomalies	249
Exercises	256
6 Extending the Standard Model	259
6.1 The Seesaw-Extended Standard Model	259
6.2 The Two-Higgs Doublet Model (2HDM)	264
6.3 Grand Unification and Unification of Couplings	298
Exercises	318

Part II Constructing Supersymmetric Theories

7 Introduction to Supersymmetry	331
7.1 Motivation: The Hierarchy Problem	331
7.2 Enter Supersymmetry	335
7.3 Historical Analogies	343
Exercises	346

8 Supersymmetric Lagrangians	347
8.1 A Free Chiral Supermultiplet	347
8.2 Interactions of Chiral Supermultiplets	352
8.3 Supersymmetric Gauge Theories	355
8.4 Gauge Interactions for Chiral Supermultiplets	357
8.5 Summary: How to Build a Supersymmetric Model	359
8.6 Soft Supersymmetry-Breaking Interactions	363
Exercises	365
9 The Supersymmetric Algebra	367
9.1 Extension of the Poincaré Algebra	367
9.2 The $N = 1$ Supersymmetry Algebra	368
9.3 Representations of the $N = 1$ Supersymmetry Algebra	371
9.4 Consequences of Super-Poincaré Invariance	380
9.5 Extended Supersymmetry	385
Exercises	391
10 Superfields	398
10.1 Supercoordinates, Superspace and Superfields	398
10.2 Supersymmetry Transformations the Superspace Way	402
10.3 Chiral Covariant Derivatives	404
10.4 Chiral Superfields	406
10.5 Vector Superfields	408
10.6 How to Make a Lagrangian in Superspace	410
10.7 Superspace Lagrangians for Chiral Supermultiplets	411
10.8 Superspace Lagrangians for Abelian Gauge Theory	414
10.9 Superspace Lagrangians for Nonabelian Gauge Theories	417
10.10 Nonrenormalizable Supersymmetric Lagrangians	421
10.11 R -symmetries	425
Exercises	427
11 Radiative Corrections in Supersymmetry	434
11.1 Introduction	434
11.2 Seiberg's Proof of the Nonrenormalization Theorem	436
11.3 Renormalization of the Wess–Zumino Model	437
11.4 Regularization by Dimensional Reduction	445
11.5 Renormalization Group Equations	459
11.6 Effective Potentials	466
Exercises	473
12 Spontaneous Supersymmetry Breaking	479
12.1 General Considerations for Supersymmetry Breaking	479
12.2 D -term SUSY Breaking: The Fayet–Iliopoulos Mechanism	482
12.3 F -term SUSY Breaking: The O'Raifeartaigh Mechanism	484

12.4 Effective Theory of Supersymmetry Breaking	488
Exercises	490

Part III Realistic Supersymmetric Models

13 The Minimal Supersymmetric Standard Model	493
13.1 A Warmup Exercise: SUSY-QED	493
13.2 MSSM Superpotential and Supersymmetric Interactions	494
13.3 <i>R</i> -Parity, Also Known As Matter Parity	499
13.4 Soft Supersymmetry Breaking in the MSSM	501
13.5 Parameter Count of the MSSM	502
13.6 Hints of an Organizing Principle	504
13.7 Renormalization Group Equations for the MSSM	511
13.8 Electroweak Symmetry Breaking and the Higgs Bosons	516
13.9 Neutralinos and Charginos	528
13.10 The Gluino	532
13.11 Squarks and Sleptons	533
13.12 Summary: The MSSM Sparticle Spectrum	538
Exercises	543
14 Realizations of Supersymmetry Breaking	548
14.1 Communication of Supersymmetry Breaking	548
14.2 The Goldstino and the Gravitino	550
14.3 Planck-Scale-Mediated SUSY Breaking	553
14.4 Gauge-Mediated SUSY Breaking	560
14.5 Extra-Dimensional and Anomaly-Mediated SUSY Breaking	566
14.6 Relating the μ -Term to the SUSY-Breaking Mechanism	570
Exercises	574
15 Supersymmetric Phenomenology	575
15.1 Superpartner Decays	575
15.2 Signals at Hadron Colliders	581
15.3 Signals at e^+e^- Colliders	588
15.4 The Lightest Supersymmetric Particle and Dark Matter	592
Exercises	597
16 Beyond the MSSM	599
16.1 The Next-to-Minimal Supersymmetric Standard Model	600
16.2 The Supersymmetric Seesaw	610
16.3 Supersymmetric Grand Unified Models	617
16.4 Discrete Gauge Symmetries in Supersymmetry	632
16.5 <i>R</i> -Parity Violation	637
Exercises	653

Part IV Sample Calculations in the Standard Model and Its Supersymmetric Extension

17 Practical Calculations Involving Two-Component Fermions	667
17.1 Conventions for Fermion and Antifermion Names and Fields	667
17.2 Z Vector Boson Decay: $Z \rightarrow f\bar{f}$	672
17.3 Bhabha Scattering: $e^+e^- \rightarrow e^+e^-$	674
17.4 Polarized Muon Decay	676
17.5 Top-Quark Condensation in a Nambu–Jona–Lasinio Model	679
Exercises	680
18 Tree-Level Supersymmetric Processes	682
18.1 Sneutrino Decay: $\tilde{\nu}_e \rightarrow \tilde{C}_i^+ e^-$	682
18.2 $\tilde{N}_i \rightarrow Z\tilde{N}_j$	683
18.3 $\tilde{N}_i \rightarrow \tilde{N}_j N_k \tilde{N}_l$	685
18.4 Three-Body Slepton Decays: $\tilde{\ell}_R^- \rightarrow \ell^-\tau^\pm\tilde{\tau}_1^\mp$	687
18.5 Neutralino Decay to Photon and Goldstino: $\tilde{N}_i \rightarrow \gamma\tilde{G}$	690
18.6 R -Parity-Violating Neutralino Decay: $\tilde{N}_i \rightarrow \mu^- u \bar{d}$	692
18.7 $e^-e^- \rightarrow \tilde{e}_L^-\tilde{e}_R^-$	694
18.8 $e^+e^- \rightarrow \tilde{N}_i \tilde{N}_j$	696
18.9 $\tilde{N}_1 \tilde{N}_1 \rightarrow f\bar{f}$	699
18.10 Gluino Pair Production from Gluon Fusion: $gg \rightarrow \tilde{g}\tilde{g}$	708
Exercises	712
19 One-Loop Calculations	719
19.1 Wave Function Renormalization in Softly Broken SUSY-QED	719
19.2 Electroweak Vector Boson Self-Energies from Fermion Loops	726
19.3 Self-Energy and Pole Mass of the Top Quark	730
19.4 Self-Energy and Pole Mass of the Gluino	739
19.5 The Anomalous Magnetic Moment of the Muon	742
19.6 Anapole Moment of the Muon	761
19.7 One-Loop MSSM Contributions to $g-2$ of the Muon	765
19.8 One-Loop Corrected MSSM Higgs Masses	768
19.9 The MSSM Wrong-Higgs Yukawa Couplings	776
Exercises	779

Part V The Appendices

Appendix A Notations and Conventions	791
A.1 Matrix Notation and the Summation Convention	791
A.2 Conjugation and the Flavor Index	791
A.3 Conventional Units for Particle Physics	792
A.4 Spacetime Notation	793

A.5 The Pauli Matrices	794
A.6 How to Translate between Metric Signature Conventions	795
A.7 Two-Component Spinor Notation	797
A.8 Group Indices and the ϵ Symbol	799
A.9 Lorentz Transformations	799
A.10 Relating Higher-Rank Spinors and Lorentz Tensors	802
A.11 Four-Component Spinors and the Dirac Matrices	803
Appendix B Compendium of Sigma Matrix and Fierz Identities	807
B.1 Sigma Matrix Identities	807
B.2 Two-Component Spinor Product Identities	809
B.3 Fierz Identities	811
B.4 Sigma Matrix Identities in $d \neq 4$ Dimensions	816
Exercises	818
Appendix C Behavior of Fermion Bilinears under P, T, C	819
C.1 Two-Component Fermion Field Bilinear Covariants	819
C.2 Four-Component Fermion Field Bilinear Covariants	822
Appendix D Kinematics and Phase Space	825
D.1 Relativistic Kinematics	825
D.2 Lorentz-Invariant Phase Space	829
D.3 Dimensionally Regularized Phase Space	832
D.4 Decay Rate	834
D.5 Cross Section	836
Exercises	839
Appendix E The Spin-1/2 and Spin-1 Wave Functions	842
E.1 Fixed-Axis Spinor Wave Functions	842
E.2 Fixed-Axis Spinors in the Nonrelativistic Limit	847
E.3 Helicity Spinor Wave Functions	850
E.4 Covariant Spin Operators for a Spin-1/2 Fermion	852
E.5 Two-Component Bouchiat–Michel formulae	855
E.6 Four-Component Spinor Wave Functions	858
E.7 Four-Component Bouchiat–Michel Formulae	861
E.8 Polarization Vectors for Spin-1 Bosons	863
Exercises	869
Appendix F The Spinor Helicity Method	873
F.1 Massless Spinors: the Bracket Notation	874
F.2 Including Massless Vector Bosons	877
F.3 Simple Application of the Spinor Helicity Method	879
Exercises	881

Appendix G Matrix Decompositions for Fermion Mass Diagonalization	883
G.1 Singular Value Decomposition (SVD)	884
G.2 Takagi Diagonalization	886
G.3 Relating Takagi Diagonalization and the SVD	888
G.4 Real Normal Form of a Complex Antisymmetric Matrix	890
Exercises	893
Appendix H Lie Group and Algebra Techniques for Gauge Theories	901
H.1 Lie Groups, Lie Algebras, and their Representations	901
H.2 Matrix Exponentials	903
H.3 Dynkin Index and Casimir Operator	906
H.4 The Techniques of Cartan and Dynkin	909
H.5 Tables of Dimensions, Indices, and Branching Rules	926
Exercises	938
Appendix I Interaction Vertices of the SM and Its Seesaw Extension	947
I.1 Standard Model Fermion Interaction Vertices	947
I.2 Interaction Vertices of the Seesaw-Extended Standard Model	949
Appendix J MSSM and RPV Fermion Interaction Vertices	952
J.1 MSSM Higgs–Fermion Couplings	952
J.2 Gauge Boson Couplings to Neutralinos and Charginos	956
J.3 Higgs Couplings to Charginos and Neutralinos	958
J.4 Chargino and Neutralino Couplings to Fermions and Sfermions	960
J.5 SUSY-QCD Feynman Rules	965
J.6 Trilinear R -Parity-Violating Yukawa Couplings	967
Appendix K Integrals Arising in One-Loop Calculations	968
K.1 The Formulae of Dimensional Regularization	968
K.2 The Passarino–Veltman Loop Functions	969
Exercises	979
<i>Bibliography</i>	985
<i>References</i>	992
<i>Index</i>	999