From Spinors to Supersymmetry

Supersymmetry is an extension of the successful Standard Model of particle physics; it relies on the principle that fermions and bosons are related by a symmetry, leading to an elegant predictive structure for quantum field theory. This textbook provides a comprehensive and pedagogical introduction to supersymmetry and spinor techniques in quantum field theory. By using the two-component spinor formalism for fermions, the authors provide many examples of practical calculations relevant for collider physics signatures, anomalies, and radiative corrections. They present in detail the component-field and superspace formulations of supersymmetry and explore related concepts, including the theory of extended Higgs sectors, models of grand unification, and the origin of neutrino masses. Numerous exercises are provided at the end of each chapter. Aimed at graduate students and researchers, this volume provides a clear and unified treatment of theoretical concepts that are at the frontiers of high-energy particle physics.

Herbi K. Dreiner is Professor of Physics at the University of Bonn. He received his PhD from the University of Wisconsin, and also worked at the Deutsches Elektronen-Synchrotron (DESY), the University of Oxford, ETH Zürich, and the Rutherford Appleton Laboratory. He is the author of over 100 scientific papers on supersymmetry and has received several teaching prizes. He is a founder of the Bonn Physics Show, for which he received the European Physical Society High Energy Physics (EPS-HEP) Outreach Prize.

Howard E. Haber is Distinguished Professor of Physics at the University of California, Santa Cruz. He received his PhD from the University of Michigan and is a Fellow of the American Physical Society. He is a co-author of *The Higgs Hunter's Guide*, published by Perseus Books in 1990. He won an Alexander von Humboldt Research Award in 2009 and was a co-recipient of the 2017 American Physical Society J. J. Sakurai Prize for Theoretical Particle Physics.

Stephen P. Martin is Professor of Physics at Northern Illinois University, where he received the university's highest award for teaching excellence. He gained his PhD from the University of California, Santa Barbara and is a Fellow of the American Physical Society. He has authored over 100 papers in high-energy physics.

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HERBI K. DREINER
University of Bonn

HOWARD E. HABER
University of California, Santa Cruz

STEPHEN P. MARTIN
Northern Illinois University

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Preface

The discovery of the Higgs scalar boson in 2012 has put the Standard Model of particle physics on a firm footing. We now have a technically complete theoretical description of all collider phenomena that are known as of this writing. However, there are compelling reasons to believe that the Standard Model of particle physics must be extended. In our view, the foremost of these is that the squared-mass parameter of the Higgs field is quadratically sensitive, through radiative corrections, to every other larger mass scale to which it couples, directly or indirectly. This quadratic sensitivity is an indication of the so-called hierarchy problem.

The classical gravitational interaction lies outside the Standard Model. Using the fundamental constants $\hbar$, c, and Newton's gravitational constant G_N, one can construct a quantity with the units of energy called the Planck scale,

$$M_P c^2 \equiv \left(\frac{\hbar c^5}{G_N}\right)^{1/2} \simeq 1.221 \times 10^{19}\ \text{GeV}\,.$$

At the Planck energy scale, the quantum mechanical aspects of gravity can no longer be neglected. In particular, the gravitational potential energy Φ of a particle of mass m, evaluated at its Compton wavelength, $r_c = \hbar/(mc)$, must satisfy

$$|\Phi| \sim \frac{G_N m^2}{r_c} = \frac{G_N m^3 c}{\hbar} \lesssim 2mc^2$$

to avoid particle–antiparticle pair creation by the gravitational field. Since the creation of particle–antiparticle pairs is an inherently quantum mechanical phenomenon, quantum gravitational effects cannot be ignored if $m \gtrsim M_P$ [up to $\mathcal{O}(1)$ constants].[1] Thus, the Standard Model cannot be a theory of fundamental particles and their interactions at energy scales of order the Planck scale and above.

Consequently, the Standard Model must break down at some energy scale Λ that is bounded from above by the Planck scale. However, there are strong hints that Λ may in fact lie below $M_P c^2$. The quantization of weak hypercharge, the way that the fermion representations of the Standard Model fit into SU(5) and SO(10) multiplets, and the convergence of running gauge couplings (via renormalization group evolution) all hint at some sort of full or partial unification of forces, the scale of which (if it exists) must be very high to evade proton decay and other bounds. Of course, this might be just a coincidence, and the hierarchy problem should not be viewed as hinging on the existence of unification. Other affirmative,

[1] Note that for $m = M_P$, the Schwarzschild radius $r_s \equiv 2G_N m/c^2 \sim r_c$, which also suggests that the quantum mechanical nature of gravity cannot be neglected at mass scales above M_P.

and perhaps stronger, hints of the presence of mass scales far above the electroweak scale but below M_P include: (i) the presence of nonzero neutrino masses, which are most naturally explained with the seesaw mechanism; (ii) the puzzle of the origin of baryogenesis, which cannot be explained in the Standard Model alone because of the lack of sufficient CP violation; (iii) the solution of the strong CP problem, which can be explained by axions with a very high Peccei–Quinn breaking scale; and, (iv) the fact that many independent observations in astrophysics and cosmology point to the existence of dark matter.

Supersymmetry is the principle that fermions and bosons are related by a symmetry. Remarkably, this simple statement leads ineluctably to a beautiful and predictive structure for quantum field theory. Of course, beauty is a subjective criterion, but historically it has been acknowledged as a useful guide to progress, as attested to by Einstein, Dirac, and many others. From a practical point of view, supersymmetry provides a way of tremendously reducing the hierarchy problem associated with the Higgs field, and nontrivially it also provides avenues of approach to the other outstanding new physics puzzles noted above, especially the dark matter and unification ideas. We have written this book to convey the essential features of supersymmetry as an extension of the Standard Model, but it is another purpose of ours to also provide a toolbox that is useful not only within supersymmetry but in other aspects of particle physics at the high-energy frontier.

A fundamental observation about physics at the electroweak scale is that it is chiral; the left-handed and right-handed components of fermions are logically distinct objects that have different gauge transformation properties. This points to the use of the two-component spinor formalism, which by construction treats left-handed and right-handed fermions separately from the start. Furthermore, within the context of supersymmetric field theories, two-component spinors enter naturally, due to the spinorial nature of the symmetry generators themselves, and the holomorphic structure of the superpotential. Despite this, textbooks on quantum field theory usually present calculations, such as cross sections, decay rates, anomalies, and radiative corrections, in the four-component spinor language. Parity-conserving theories such as QED and QCD are well suited to the four-component spinor methods. There is also a certain perceived advantage to familiarity. However, as we progress to phenomena at and above the scale of electroweak symmetry breaking, it seems increasingly natural to employ two-component spinor notation, in harmony with the transformation properties dictated by the physics.

One often encounters the misconception that even though the two-component fermion language may be better for devising many theories, including supersymmetry, it is somehow inherently ill-suited or unwieldy for practical calculations of physical observables. One of the goals of this book is to demonstrate that two-component fermion notation is just as useful for analyzing quantum field theories as it is for formulating them. The two-component spinor formalism employed here applies equally well to Dirac fermions such as the Standard Model quarks and charged leptons and to Majorana fermions such as the neutrinos of the Standard Model or the neutralinos predicted by supersymmetry.

We have therefore organized the material of this book with two primary goals in mind. The first goal is to present a comprehensive treatment of the two-component spinor formalism as applied in quantum field theory in 3+1 spacetime dimensions, and its applications in the Standard Model and extensions thereof. This is the focus of Part I, which comprises the first six chapters. In Part I, we have also reserved one chapter for providing a detailed translation between the two-component and four-component fermion formalisms, so that the reader can make connections with the vast majority of textbooks that employ the latter in their development of quantum field theory and the Standard Model of particle physics. The second goal is to provide a pedagogical introduction to the construction and application of supersymmetric theories in establishing realistic theories of fundamental particles and interactions beyond the Standard Model. This is accomplished in Parts II and III of this book. Finally, we provide in Part IV sample calculations in the Standard Model and its supersymmetric extension in some detail using the two-component spinor formalism. These calculations, which encompass decay processes, scattering processes, and radiative corrections, illustrate the power of these methods in practical calculations of experimental observables. Additional details of our toolbox of techniques are collected in Part V, consisting of 11 separate appendices.

There is a huge literature on spinors and supersymmetry. Many references to the original literature that are not explicitly cited in this book can be found in reviews and summer school lecture notes by the authors that appeared in Refs. [1–3]. The mathematical treatment of spinors and their applications to physics can be found in numerous textbooks (e.g., see [B1–B15]). Textbook treatments of supersymmetry and supergravity are also abundant (e.g., see [B16–B45]) along with many reviews and summer school lecture notes (e.g., see Refs. [2–27]). Additional references that trace the origins of supersymmetry can be found in [B46].[2]

We hope that this book serves as a useful addition to the literature cited above. In particular, we have strived to provide a level of detail not typically found in other textbooks. We have also attempted to address many questions that students often pose when first encountering this material that are typically ignored in the standard treatments. Numerous exercises are provided at the end of each chapter (and at the end of most of the appendices). These exercises serve a number of different purposes. Some of the exercises require rather straightforward calculations that allow the student to gain a better understanding of the material. Other exercises represent the treatment of additional topics that would have been included in the main text if space permitted. In these cases, we have often quoted the final result (which the student is encouraged to verify), which may be employed elsewhere in the book. Finally, some of the exercises are more substantial and would constitute a good term project for a special topics course. In this way, we believe that this book can provide a good basis for an advanced class in particle theory (where the instructor focuses on some subset of the book's chapters for a one-semester course), or provide a useful resource for self-study.

[2] Books are listed in the Bibliography and are indicated by the prefix B. Review articles, lecture notes, and journal articles are independently numbered and appear separately in the References.

Prerequisites for this book include a basic knowledge of quantum field theory (e.g., see [B47–B74]), gauge theories (e.g., see [B75–B81]), and the Standard Model of particle physics and beyond (e.g., see [B82–B96]). Some review of this material is also provided in Part I of this book. Important mathematical methods employed in this book are treated in the appendices, including Lie groups and algebras (e.g., see [B97–B125]) and matrix algebra (e.g., see [B126–B129]).

Although we have endeavored to produce a book that is free of errors, it is very likely that typographical errors, misprints, and various inconsistencies have escaped our attention. We will therefore maintain updated information on any errors that we discover after the initial publication, which can be found on the home page of our book:

`www.cambridge.org/spinors-to-susy`

Of course, the authors would be most grateful to readers who detect any errors in our book and provide us with the relevant details. We will be happy to list the errata along with the appropriate citations on the home page of this book.

We conclude this preface by acknowledging that the Large Hadron Collider (LHC), operating at center-of-mass energies up to 13 TeV, has not seen any evidence of supersymmetry as of this writing, confounding the hopes of many. Several comments are in order here. First, the fact that the Higgs boson mass has turned out to be 125 GeV (rather than, say, 115 GeV, as seemed plausible when the LHC turned on) hints at considerably larger masses for the supersymmetric partners of the Standard Model particles than one might have guessed previously. This is because of the way that the superpartner masses feed logarithmically into the Higgs boson mass in radiative corrections, as explained in Sections 13.8 and 19.8. From this point of view, it is hardly surprising that direct evidence for supersymmetry has not been found within the presently explored mass ranges. While this seemingly leads to a little hierarchy problem associated with the ratio of the superpartner masses to the electroweak symmetry-breaking scale, that hierarchy remains minuscule compared to the problem of the large hierarchy between the electroweak and Planck scales in non-supersymmetric extensions of the Standard Model.

Furthermore, the existence of a 125 GeV Higgs boson is quite compatible with supersymmetric particles somewhat above the TeV scale, as had been noted long before the LHC turned on. Many other competitor theories either predicted no Higgs boson at all, or predicted a much heavier Higgs boson, or simply did not dare to make any firm prediction about it whatsoever. Supersymmetry compares favorably in this regard. Finally, the lack of any LHC evidence for other new physics means that many of the alternatives to supersymmetry that have been proposed to address the hierarchy problem are either eliminated or highly disfavored, and in our view none of these approaches can be said to be in better shape than supersymmetry.

Therefore, while we of course make no guarantees, for it would be foolish to do so, we remain optimistic that supersymmetry is an essential feature of the fundamental laws of physics, and we look forward to this being clarified as the great adventure of exploration continues. We hope that this book will be useful along the way.

Acknowledgments

This book is the culmination of a project that has lasted 20 years. The conception of this book project was strongly encouraged by Robert N. Cahn, who provided us with the initial connection with Cambridge University Press. We are especially grateful to Simon Capelin for guiding us for many years during this effort and for his patience and understanding over numerous missed deadlines. Although we had hoped to have a finished project in time for his retirement celebration, a few unforeseen events followed by the Covid-19 pandemic ultimately delayed the final delivery of the manuscript. We would like to thank Simon's successor, Vince Higgs, for his continued confidence and support through the latter stages of our project, Sarah Armstrong and Clare Dennison for guiding us through the final steps to publication, John King for his superb copy-editing skills, and Suresh Kumar and the team of LaTeX experts at Cambridge University Press for their assistance on a number of typographical issues.

Our book incorporates some material that was originally published in a chapter by Stephen P. Martin, entitled "A Supersymmetry Primer," which appeared in *Perspectives on Supersymmetry*, edited by Gordon L. Kane (World Scientific, Singapore, 1998), pp. 1–98, and subsequently updated in *Perspectives on Supersymmetry II*, edited by Gordon L. Kane (World Scientific, Singapore, 2010), pp. 1–153. In addition, some of the exercises in our book first appeared in lecture notes by Howard E. Haber and Laurel Stephenson Haskins, entitled "Supersymmetric Theory and Models," which was published in *Anticipating the Next Discoveries in Particle Physics*, Proceedings of the 2016 Theoretical Advanced Study Institute in Elementary Particle Physics (TASI-2016), edited by R. Essig and I. Low (World Scientific, Singapore, 2018), pp. 355–499. Lastly, we have made substantial use of the review article by Herbi K. Dreiner, Howard E. Haber, and Stephen P. Martin, "Two-component spinor techniques and Feynman rules for quantum field theory and supersymmetry," Physics Reports **494**, 1–196 (2010), which was published by Elsevier. We are grateful to World Scientific and Elsevier for granting permission for the reuse of these materials in this book.

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Finally, for their work on promoting and nurturing national and international scientific collaborations, we wish to acknowledge and celebrate the Alexander von Humboldt Foundation and the Aspen Center for Physics (the latter located just 10 miles down the road from the picturesque Maroon Lake that adorns the cover of this book). These two institutions were instrumental in the research lives of the three authors of this book, and both played a significant role in providing the collaborative opportunities that allowed the authors to carry out this work.

We dedicate this book to our respective spouses, Heike, Marjorie, and Jeanette, who are probably even more enthusiastic than we are that this book project has finally come to a conclusion.

Acronyms and Abbreviations

1PI	one-particle irreducible
2HDM	two-Higgs doublet model
AMSB	anomaly-mediated supersymmetry breaking
ATLAS	A Toroidal LHC Apparatus general-purpose detector
BCH	Baker–Campbell–Hausdorff
BPS	Bogomol'nyi–Prasad–Sommerfield
BR	branching ratio
BRST	Becchi–Rouet–Stora–Tyutin
$\mathbb{C}$	set of complex numbers
CKM	Cabibbo–Kobayashi–Maskawa
CM	center-of-momentum
CMS	Compact Muon Solenoid general-purpose detector
CMSSM	Constrained Minimal Supersymmetric Standard Model
CPT	charge conjugation/parity/time reversal
DGS	discrete gauge symmetry
DM	dark matter
$\overline{\mathrm{DR}}$	modified minimal subtraction scheme using DRED
DRED	dimensional reduction
DREG	dimensional regularization
EM	electromagnetic
FCNC	flavor-changing neutral current
FI	Fayet–Iliopoulos
FT	Fourier transform
GBP	generalized baryon parity
GF	gauge-fixed
GIM	Glashow–Iliopoulos–Maiani
GLP	generalized lepton parity
GMP	generalized matter parity
GMSB	gauge-mediated supersymmetry breaking
GUT	grand unified theory
GWP	Glashow–Weinberg–Paschos
HM	"HiggsMass" scheme
IR	infrared
irrep	irreducible representation
LEP	Large Electron-Positron Collider
LHC	Large Hadron Collider

Lips	Lorentz-invariant phase space
LSP	lightest supersymmetric particle
LSZ	Lehmann–Symanzik–Zimmermann
MHV	maximally helicity-violating
$\overline{\text{MS}}$	modified minimal subtraction scheme using DREG
MSSM	Minimal Supersymmetric Standard Model
mSUGRA	minimal supergravity
NLSP	next-to-lightest supersymmetric particle
NMSSM	Next-to-Minimal Supersymmetric Standard Model
NSVZ	Novikov–Shifman–Vainshtein–Zakharov
NUHM	nonuniversal Higgs mass model
OS	on-shell
PMNS	Pontecorvo–Maki–Nakagawa–Sakata
PMSB	Planck-scale-mediated supersymmetry breaking
pMSSM	phenomenological MSSM
PQ	Peccei–Quinn
PS	Pati–Salam
QCD	quantum chromodynamics
QED	quantum electrodynamics
Q4S	"quasi-4-dimensional" space
$\mathbb{R}$	set of real numbers
RG	renormalization group
RGE	renormalization group equation
RPV	R-parity-violating
SE-MSSM	seesaw-extended MSSM
SM	Standard Model
SQED	superpartner contribution of SUSY-QED
SUSY	supersymmetry (or supersymmetric)
SUSYGUT	supersymmetric grand unified theory
SUSY-QED	supersymmetric QED
SUSY-QCD	supersymmetric QCD
SVD	singular value decomposition
TASI	Theoretical Advanced Study Institute in Particle Physics
UV	ultraviolet
VEV	vacuum expectation value
WB	Wess–Bagger
XMSB	extra-dimensional-mediated supersymmetry breaking
YM	Yang–Mills
$\mathbb{Z}$	set of integers