

# Index

- Abstract graph, 6
- Adjacent vertices, 2
  - in graph of 3-blocks, 104
- Algebraic duality, 206
- Alternating barrier, 168–169
  - accessible vertices of, 170
  - and alternating paths, 169
  - center of, 169
  - components of, 168
  - condition for, 169
  - inner vertices of, 168
  - irregular darts in, 169
  - outer graph of, 168
  - outer vertices of, 168
  - Theorem of Alternating Connection, 170
- Angle in map, 268
- Arborescence, 126
  - condition for, 138
  - construction of spanning, 134–135
  - converging, 127
  - diverging, 126–127
  - multiple diverging, 142
  - paths in, 131–132
  - residual, 135
- Arc, 3–4
  - adjunction of, 57
  - in bridge, 29–30
- characterization of, 17
  - condition for, by paths, 132
  - and connection, 16, 21–22
  - decomposition, by vertex, 66
  - directed, 131
  - edge-contraction in, 39
  - internally disjoint, 47
  - subdivision of, 77
  - as tree, 21
- Arm, 64–66
  - and blocks, 64–66
  - connection of, 64
  - and 1-separation, 65
- Assignment Problem, 52
- Automorphism, of graph, 6
  - of premap, 263
- Automorphism group, of graph, 8, 31
  - of map, 263
  - of premap, 263
- $f$ -Barrier, 171
  - deficiency of, 172
  - even and odd components of, 171–172, 173
  - existence theorem for maximal, 176–177
  - and  $f$ -factor, 172
  - the  $f$ -Factor Theorem, 174–176

- maximal, 176
- outer graph of, 171
- parity theorem for, 173
- Basic permutation of preamp, 254
- Beraha numbers, 250
- BEST Theorem, 137
- Bicursal unit, 167
  - constituent of, 167–168
  - edges of attachment of, 167–168
- Binding number of complementary pair of subgraphs, 14
  - and cutting pairs, 43–44
- Binding set of pair of subgraphs, 14
- Bipartite graph, 51
  - complete, 79
  - $f$ -factors of, 180
- Bipartition of graph, 51
- Block, 60–64
  - and circuits, 61
  - and components, 61
  - and 2-connected subgraphs, 60
  - and cut-vertices, 62–64
  - extremal, 64
  - and loop or isthmus, 61
  - of planar graph, 298–299
  - of primitive chain-group, 208–209
  - in Reconstruction Theory, 121–122
  - and separable graphs, 61
- 3-Block, 83–104
  - closed outflow from, 87
  - edge-disjointness of, 92
  - graph of 3-blocks, 93–95
  - leading, 87
  - outflow from, 87
  - outlet of, 87
- Block-graph of graph, 63–64
- Bond of graph, 200–201
  - end-graphs of, 200–201
  - Hamiltonian, 236
- Bond-vertex, 296
- Boundary of 1-chain, 197
- Bounding sequence of face, 265
- Bridge, 27–30
  - algorithm for, 118
  - avoidance of, 311
  - circuit with two bridges, 316
  - degenerate, 28
  - equivalent bridges of circuit, 311
  - and Jordan's Theorem, 312
  - nucleus of, 28
  - outer vertex of, 28
  - overlapping, 311
  - overlap theorem of 3-connection, 317
  - skew, 311
  - in subgraphs, 30
- Bridge-fragment, 314
  - 2-connection of, 314
  - and planarity, 315
- Bridge-graph, 313
- Brooks' Theorem, 233–235
- Cap-graph, 294
- Cap-map, 294
- Capacity, of cut in digraph, 155
  - of vertex in digraph, 154
- Cell in map theory, 253
- Cell-base of chain-group, 186
  - condition for, 192
  - and elementary chains, 187, 191
  - existence theorem for, 186
  - primitive, 189
- Chain, 185
  - 0-chains and 1-chains, 197
  - elementary, 186
  - enclosure for, 186
  - linear dependence of, 186
  - nowhere-zero, 228–229
  - primitive, 188–189
  - support of, 186
  - unit, 185
- Chain-base, 189
- Chain-group, 186
  - primitive, 191
- Characteristic polynomial, 249–250
- Chromatic number of graph, 228
- Chromatic polynomial of graph, 226–233
  - of circuit, 229
  - of clique, 230
  - coefficients in, 231–232
  - for positive integers, 228
  - recursion formulae, 226–227
  - sign, 233
  - of tree, 229
  - of union, 230–231
  - and vertex-joins, 230
- Circuit, as 3-block, 95–98
  - characterization of, 17–18
  - condition for, by paths, 132
  - connection of, 16
  - directed, 131
  - edge-contraction in, 39
  - Hamiltonian, 236, 243
- Circuit-map, 287–288, 296
- Cleavage, 95
  - and virtual edges, 101
  - relation to another cleavage, 102–103

- Clique, 2–3
  - connection of, 16
  - 3-connection of, 76
- Coboundary of 0-chain, 200
  - unoriented, 219
- Coboundary group, 200
  - cell-base of, 203
  - as dual of cycle-group, 203
  - elementary chains of, 201–202
  - primitivity of, 202
  - rank of, 204
- Coboundary rank of graph, 204
  - and planar duality, 288–289
- Coloring of graph, 227
  - and chromatic polynomial, 228
  - condition for 2-coloring, 235
  - and Hamiltonian bond, 236
- Combinatorial sphere, 276
- Complementary subgraph, 13
- Component of graph, 14–18
  - algorithm for, 118
  - characterization by paths, 133
  - connection of, 16
- Component of premap, 256–257
- Component of map, 260
- Conductance of dart, 138
- Conforming chains, 194–195
- Conforming pair in map theory, 277
- Connection of graph, 14–18
  - by arcs, 21–22
  - of bridge, 29
  - list of graphs with, 22–27
- 2-Connection of graph, 54
  - and bridges, 56
  - and circuits, 56
  - constructions for, 56–60
  - and loops and isthmuses, 55
  - in planar maps, 68, 297
  - and recognizability, 117
  - and separation numbers, 56
  - and unions, 57
  - and vertical 2-connection, 74
- 3-Connection of graph, constructions for, 74–83
  - and edge-contraction, 105–111
  - and edge-number, 71
  - in planar maps, 113, 317–320
  - and strictness, 71
- $m$ -Connection in graph, 70
  - cyclic, 71
  - vertical, 70
- Connection-set of premap, 256
- Connectivity of graph, 72
  - and duality, 286
  - and edge-contraction or edge-deletion, 73–74
  - infinite, 72
- Connectivity of chain-group, 210–215
  - in planar maps, 296–306
- Contraction of chain-group, 187
- Contraction of graph, 32–36
  - and components, 35
  - repeated, 36
  - and subgraphs, 35
- Contraction of link in map, 272–273
  - use of, 276–277
- Cross, in map theory, 253, 283–284
- Cross-cap, 277
- Cross-cut of circuit in map, 306
  - skew cross-cuts, 310
- Cross-Cut Theorem, 306–307
- Cubic graph, 5
- Current in dart, 145
- Cut-vertex, 54
- Cutting pair of edge-sets, 43–44
- Cutting pair of vertex-sets, 50
- Cycle, 197
  - bounding, 283
  - elementary, 199
- Cycle-group, 197
  - cell-base of, 199
  - as dual of coboundary-group, 203
  - primitivity of, 199
  - rank of, 200
- Cycle-rank of graph, 288
- Cyclomatic number of graph, 19
  - of components, 40
  - of forest, 19
- Dart, 125
  - cursal, 161
- Deletion of edge in map, 273–275
- Dichromate of graph, 243–248
  - of 4-clique, 246
  - coefficients, 246
  - and duality, 289–290
  - examples, 245–246
  - recursion formulae, 244
  - and spanning trees, 247–248
  - and tree-number, 246–247
  - Unimodal Conjecture, 246
- Dichromatic polynomial, 225–226
- Digraph, 125
  - connected, 132
  - duality for, 283
  - equivalence with graph, 127

- Eulerian, 125, 133–134
  - graph-connected, 132
  - strongly connected, 132
- Disconnected graph, 121
- Drawing of graph, 2
- Drawing of map, 263–268, 286–287
- Duality, of chain-groups, 188
  - of cycles and coboundaries, 283
  - intimations of, 36, 41
  - of loop and isthmus, 290
  - of premaps and maps, 259–261
  - of primitive chain-groups, 193–194
  - of spanning trees, 289
- Edge of graph, 1
  - of attachment, 166
  - contraction of, 37–41, 67–68
  - cursal, bicursal, unicursal, 161–162
  - deletion of, 18–22, 66–68
- Edge of premap, 255
- Edge-reconstruction, 122–123
- Edmonds' Construction, 255
- End of arc, 5
- End of edge, 1
- End-graph of bond, 200–201
- End-graph of isthmus, 18
  - in arc, 21
  - in tree, 20–21
- Entrance of bicursal component, 163
- Entry-dart, entry edge, 163, 168
- Entry-path of bicursal component, 163
- Erdős–Gallai Theorem, 181–182
- Essential edge in 3-connected graph, 109–111
- Euler characteristic of premap, 260
  - additivity over components, 261
  - maximal value for map, 276
- Euler's Theorem, 137
- 1-Factor of graph, 51
  - partial, 51
  - theorem, 177
- $f$ -Factor, 170
  - algorithm for, 183
  - of bipartite graph, 180
  - and parity, 171
  - sufficient condition for, 171
  - theorem, 177
- Flow in digraph, 147
  - 5-Flow Conjecture, 240
  - $n$ -Flow, 240
  - Flow polynomial, 237–240
  - Forest, 19–20
  - Four-Color Theorem, 237
  - Frame of graph, 199
    - Frame, in theory of 3-connection, 106
    - $V$ -Function of graph, 221–226, 250
- Girth of graph, 83
- Graph, 1
  - dual, 281
  - of path, 132–133
  - planar, 285
  - of preamp, 255
  - reconstructible, 116
- Graph-function, 221
- Graphic partitions, 181–182
- Grinberg's Theorem, 236–237
- Hadwiger's Conjecture, 52
- Hall's Theorem, 50–52, 181
- Hinge of 2-separation, 83
- Hinge-graph of 2-separation, 95
- Half-edge in premap, 255
- Handle, in map theory, 277
- Hinge of 2-separation, 83
- Hub of wheel, 78
- Identification of vertices, 149–152
- Impedance, 143
- Incidence number, 197
- Incidence Matrix, 217–219
- Intake of graph, in 3-block theory, 87
- Interchange Theorem for cell-bases, 190
- Internal vertices of arc, 5
- Intersection of subgraphs, 10, 27, 63
- Invalency of vertex in digraph, 125
- Involution in map theory, 253
- Isolated vertex, 5
- Isomorphism of graphs, 5–9
- Isomorphism of premaps, 261–263
- Isomorphism class, 6, 261
- 2-Isomorphism, 114
- Isthmus, 18–19
- Jordan's Theorem, 290–296
- Kelly's Lemma, 119–122
- Kirchhoff matrix, 138, 141
- Kirchhoff's Laws, 143
- Klein bottle, 281
- Kruskal's Conjecture, 52
- Kuratowski graphs, 313
- Kuratowski's Theorem, 320–325
- Link, 1
  - addition of, 24
  - contraction of, 38–41
  - of premap, 255

- Link-adjunction, 74–75
  - and subdivision, 77
- Link-graph, 2
- Linkage, 26
  - as 3-block, 96–98
- Link-map, 262
  - planarity of, 276
- Loop, 1
  - contraction of, 37–38
  - of premap, 255
- Loop-graph, 2
- Loop-map, 262
  - planarity of, 276
  
- Map, alternating, 158–159
  - canonical, 280
  - canonical projective, 262, 266–267
  - on circuit, 287
  - dual, 260–261
  - genus, 280
  - on graph, 255, 298
  - orientability-character, 272
  - orientable, unorientable, 257–258
  - planar, 276
  - rooted, 263, 283
  - self-dual, 262
  - on tree, 287
  - unitary, 275–276
- Matrix-Tree Theorem, 140–142
- Matroid, 71, 219
- Max Flow Min Cut Theorem, 157–158
- Menger's Theorem for graphs, 46–50, 53
- Menger's Theorem for digraphs, 158
- Minor of graph, 36
- Minor of chain-group, 187
  - of cycle and coboundary groups, 204–205
  - and primitivity, 191
- Multiple join, 2
  
- Null graph, 2
  
- Orbit of permutation, 254
  - conjugate, 254
- Orientability of map, 257–259
- Orientation of graph, 128
  - acyclic, 229
- Orientation-class of map, 257
- Oriented map, 258
  - dual, 281
- Orthogonality of chains, 187
- Outflow, in 3-block theory, 87
- Outlet, in 3-block theory, 87
- Outlet of vertex-set in digraph, 155
- Outvalency of vertex in digraph, 125
  
- Path, alternating, 161
  - circular, 130
  - dart-simple, 130
  - degenerate, 129
  - in digraph, 129–133
  - edge-simple, 132
  - Eulerian, 133
  - factorization of, 130
  - in graph, 132
  - head-simple, 130
  - inverse of, 133
  - length of, 129
  - linear, 130–131
  - origin and terminus of, 129
  - product, 129–130
  - simple, 130
  - tail-simple, 130
- Path-bundle, 152
  - digraph of, 152
  - $f$ -limited, 154
  - rotational and irrotational, 152–154
- Peripheral circuit, 314
  - and 3-connection, 317–322
  - existence theorems, 318–320
  - intersection of, 323
- Petersen graph, 80
  - 3-connection of, 83
  - and Tait colorings, 242
- Petersen's Theorem, 177–178
- Pfaffian, 183
- Pinch-graph of circuit in planar map, 293
- Pinch-map of circuit in planar map, 293
  - planarity of, 294
- Planar mesh, 299–302
  - and circuit, 316
  - imposition of, on subgraph, 308
  - and peripheral circuit, 314
  - uniqueness of, for 3-connection, 317
- Planarity, 285–326
  - algorithm for, 314–316
  - condition for nonplanarity, 316
  - MacLane's Test for, 301
  - tests for, 299, 320
- Pole in digraph, 145
- Potential in digraph, 145
- Premap, 254
  - planar, 285
  - self-dual, 262
- Prism, 113
- Projective plane, 266–267

- Rank of primitive chain-group, 191
- Realization of sequence of subgraphs, 120
- Recognizability, 115–116
- Reconstruction Problem, 115–122
  - and characteristic polynomial, 250
  - and dichromate, 248–249
  - and Hamiltonian circuits, 249
  - Generalizations of, 123
- Reduction of chain-group, 187
- Reduction of graph, 16
- Regular chain-group, 194–196
  - representative matrix of, 196
  - residual chain-group of, 195–196
- Regular element of ring, 188
- Regular graph, 5
  - recognizability of, 116
  - reconstructibility of, 116
- Residual graph of circuit in planar map, 291
  - enclosure by, 291–292, 306
  - marginal enclosure by, 306
  - and partition of vertex, 292
- Residual graph for Tait colorings, 241–242
- Restriction of mapping, 11
- Rim of wheel, 78
- Rotor, 159, 250
  
- Scalar product of chains, 187
- Separable graph, 54
- 1-Separation of graph, 54–55
- 2-Separation of graph, 70
  - A*-major, 87
  - A*-maximal, 84–86
  - hinge of, 84
- n*-Separation of graph, 70
  - cyclic, 71
  - vertical, 70
- Separation number of edge-sets, 43–45
- Separation number of vertex-sets, 50
- Smith's Theorem, 243
- Snark, 242
- Splitting a vertex, in map, 270
- Spoke of wheel, 78
- Star, 23
- Strict graph, 2, 7
- String of blocks, 66
- String of 3-blocks, 105
- Structural property of graph, 11
- Subdigraph, 125–126
- Subdivision of edge, 77
- Subgraph, 9–14
  - bicursal, 163–167
  - and darts, 163–166
  - deficiency, 170
  - induced, 10
  - of planar graph, 297
  - primal, 115
  - proper, 9
  - spanning, 9
- Submap, 296
- Subpremap, 296
- Superficial transformation, 272
- Surface, 275–281
  - genus of, 281
  - orientable and unorientable, 281
  
- Tait colorings, 240–243
  - and bonds, 241
  - and 4-flows, 241
- Tait cycle, 242
- Theta-graph, 307
  - planarity of, 307
- Thomsen graph, 79–80, 113
- Tie between edge-sets, 47–50
- Torus, 267–268, 281
- Tour in digraph, 136
  - Eulerian, 136
- Trace of vertex or face in map, 291
- Trace-graph of vertex or face in map, 291
- Transpedance in digraph, 143
  - as determinant, 146–147
- Transpedance in graph, 148–149
- Transportation theory, 152–158
- Tree, 19–22
  - and arborescence, 128
  - recognizability of, 117
  - spanning, 20
  - as union of trees, 93
  - uniqueness of arcs in, 22
- Tree-number, 39–40
- Triad in graph, 105
- Triangle in graph, 104–105
- Turn in map, 268
- Twig, 22
- Type of 2-connected graph, 84
  
- Underlying graph, of digraph, 125
  - of path, 131
- Unimodal conjectures, 233, 246
- Union of subgraphs, 10, 63
  - and connection, 17
  - for *J*-detached subgraphs, 27
  
- Valency, in graph, 4, 20
  - in map, 288, 325

- Vertex of attachment, 11–14
  - and 3-blocks, 98
  - and contraction, 41–42
- Vertex of graph, 1
  - bicursal and unicursal, 162
  - and 3-connection, 71–72
  - expansion of, into circuit, 80–83
- Vertex of premap, 255
- Vertex-bond, 103
- Vertex-coboundary, 203
- Vertex-graph, 2
- Vertex-isomorphism, 33–34
- Vertex-join, 230
- Vertex-map, 276
- Vertex-splitting in graph, 75–76
- Virtual edge of graph, 91
  - and cleavage, 101
- Wheel, 78
  - 3-connection of, 78
  - Theorem, 111–113
- Y-graph, 308
  - arms of, 308
  - and circuit, 309–310
  - existence theorem for, 312