

Analytic Tomography

This book is about tomography, which is a way to see what is inside an object without opening it up. The unifying idea of tomography is the Radon transform, which is introduced in an informal and graphic way in Chapter 1. The remaining chapters deal with the basic and advanced properties of the Radon transform and related operators.

The book was written to appeal to the broadest possible group of readers. The first chapter, which introduces computerized tomography, x-ray imaging and the Radon transform, requires almost no mathematical background. The second chapter, which is devoted to a rigorous and detailed study of the basic properties of the Radon transform should be accessible to readers with a good undergraduate background in mathematics. The last three chapters are devoted to the more advanced areas of mathematical tomography and the Radon transform. These chapters require a more sophisticated background in mathematics. There are numerous figures and more than 600 references to literature in the field.

Andrew Markoe is Professor of Mathematics at Rider University. He is the author of 19 publications in the areas of several complex variables, Radon transforms, and mathematical tomography.

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 Andrew Markoe
 Frontmatter
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