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978-0-521-79211-0 - Spectral Methods for Time-Dependent Problems

Jan S. Hesthaven, Sigal Gottlieb and David Gottlieb

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