

CLOSURE STRATEGIES FOR  
TURBULENT AND TRANSITIONAL FLOWS

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This book, which has grown out of a two-week instructional conference at the Newton Institute in Cambridge, is designed to serve as a graduate-level textbook and, equally, as a reference book for research workers in industry or academia.

CLOSURE STRATEGIES FOR  
TURBULENT AND TRANSITIONAL FLOWS

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## PREFACE

Material for this volume first began to be assembled in 1999 when a six-month Programme on Turbulence was held at the Isaac Newton Institute for Mathematical Sciences in Cambridge. The Programme had its own origins in an Initiative on Turbulence by the UK Royal Academy of Engineering, which had identified the prediction of turbulent flow as a key technology across a range of industrial sectors. Researchers from different disciplines gathered together at the Newton Institute building in Cambridge to work on aspects of the turbulence problem, one of the most important of which was closure of the averaged (or filtered) turbulent flow equations. An Instructional Workshop on *Closure Strategies for Modelling Turbulent and Transitional Flows* was held from April 6 to 16th. The aim of the workshop was for the experts on closure to present material leading up to the current state-of-the-art in a form suitable for research students and others requiring a broad overview of the field, together with an appreciation of current issues. A sequence of 38 lectures by 19 different lecturers provided background to techniques, examples of current applications, and reflections on possible future developments. Recognising that a gathering of so many experts from a variety of backgrounds was somewhat unusual, it was felt that a more polished version of the lecture material would be of interest to a wider audience, as a reflection upon the current state of prediction methods for turbulent flows. The result has taken rather longer to appear than originally intended, but the opportunity has been taken for contributions to be updated wherever possible, to take account of the most recent developments.

The Editors would like to thank the contributors for their efforts to improve the papers, with a view to making them more accessible to the intended audience, which remains that of the original workshop. Especial thanks are due to Dr T. Gatski who shouldered the not inconsiderable task of shaping three separate contributions into a single chapter on compressible flow. Thanks are also due to the Newton Institute for hosting the workshop, and the main sponsors of the Turbulence Programme, including the then British Aerospace (now BAE Systems), Rolls-Royce, The Meteorological Office, British Gas Technology, British Energy, and the UK Defence Evaluation Research Agency. Finally we record our appreciation to Mrs C. King who provided invaluable secretarial support throughout the Workshop and in the editing of the present volume.

## ACRONYMS

ABL	Atmospheric Boundary Layer
APG	Adverse Pressure Gradient
ASM	Algebraic Stress Model [1]
BL	Binomial Lagrangian [21]
C/D	Coalescence/Dispersion [21]
CDF	Cumulative Distribution Function [20]
CERT	Centre d'Études et de Recherches, Toulouse
CE	Constrained Equilibrium [21]
CFD	Computational Fluid Dynamics
CMC	Conditional Moment Closure [10]
CPU	Central Processing Unit
DES	Detached Eddy Simulation [8]
DIA	Direct Interaction Approximation
DNS	Direct Numerical Simulation [7]
DSM	Differential Second-moment-closure Model (also SMC) [2]
EASM	Explicit Algebraic Stress Model [1]
ECL	Ecole Centrale de Lyon
EDF	Electricité de France
EDQNM	Eddy-Damped Quasi-Normal Markovian [9]
EMST	Euclidean Minimum Spanning Tree [20]
ER	Elliptic Relaxation [4]
ERCOTAC	European Research Community On Flow, Turbulence and Combustion
ESRA	Extended SRA [19]
EVM	Eddy Viscosity Model [1]
FFT	Fast Fourier Transform
GGD/GGDH	Generalized Gradient Diffusion (Hypothesis) [2]
GLM	Generalized Langevin Model [21]
IEM	Interaction-by-exchange-with-the-mean [21] (also known as LMSE)
ILDm	Intrinsic Low-Dimensional Manifold [21]
IP/IPM	Isotropization of Production (Model) [2]
ISAT	<i>In Situ</i> Adaptive Tabulation [20,21]
KH	Kelvin–Helmholtz
LDA	Laser Doppler Anemometer
LDV	Laser Doppler Velocimetry
LES	Large Eddy Simulation [8]
LHS	Left Hand Side
LIA	Linear Interaction Approximation [19]
LIPM	Lagrangian IP Model [21]
LMSE	Linear Mean Square Estimation [20] (also known as IEM)
LRR	Launder, Reece and Rodi [2]
LSES	Large-Scale Eddy Structures
MCM	Mapping Closure Model [21]
MDF	Mass Density Fraction [21]

*Acronyms*

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MPP	Massively Parallel Processing
MUSCL	Monotone Upwind Schemes for Scalar Conservation Laws
NASA	National Aeronautics and Space Administration
NLEVM	Nonlinear Eddy Viscosity Model [1]
NSE	Navier–Stokes Equations
ONERA	Office National d'Études et de Recherches Aérospatiales
PBL	Planetary Boundary Layer
PDF	Probability Density Function [10]
PIV	Particle Image Velocimetry
POD	Proper Orthogonal Decomposition
PTM	Production-Transition Modification [18]
QI	Quasi-Isotropic
QUICK	Quadratic Upstream-Interpolation for Convection Kinematics [5]
RANS	Reynolds Averaged Navier–Stokes [1]
RDT	Rapid Distortion Theory [9]
RHS	Right Hand Side
RLA	Ristorcelli, Lumley and Abid [4]
RMS	Root Mean Square
RNG	Renormalization Group Theory [2]
RSM/RSTM	Reynolds Stress (Transport) Model [2]
SDM	Semi-deterministic Method [22]
SGD	Simple Gradient Diffusion [2]
SGS	Sub-Grid Scale [8]
SIG	Special Interest Group
SIMPLE	Semi-Implicit Method for Pressure-Linked Equation [5]
SLM	Simplified Langevin Model [21]
SLY	Savill, Launder and Younis (a second-moment transition model due to Savill [17])
SM	Smagorinsky Model [8]
SMC	Second Moment Closure [2]
SNECMA	Société Nationale d'Études et de Conception de Moteurs d'Avions
SRA	Strong Reynolds Analogy [19]
SSG	Sarkar, Speziale and Gatski [1]
SSM	Scale Similarity Model [8]
SST	Shear Stress Transport [1]
TCL	Two-Component Limit [3]
TFM	Test Field Model [9]
TKE	Turbulence Kinetic Energy
TPC	Two-Point Closure [26]
T-RANS	Time-dependent RANS [22]
T-S	Tollmien–Schlichting
T3A, T3B ...	A series of transition test cases [17]
TVD	Total Variation Diminishing
UMIST	University of Manchester Institute of Science and Technology
UMIST	Upstream Monotonic Interpolation for Scalar Transport [5]
UTS	University of Technology, Sydney
VLES	Very Large Eddy Simulation