# Introduction

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Although Computational Fluid Dynamics (CFD) has developed to a point where it is a routine tool in many applications, several difficulties remain. Numerical issues, such as grid generation, are often difficult and costly, in the sense that much time and effort has to be devoted to the task, but they are manageable. The other main problem concerns the realistic physical modelling of turbulent and transitional flow, and is much less tractable.

The aim of this volume is to provide a reasonably comprehensive, up-to-date and readable account of where the numerical computation of industrially important, single-phase turbulent flows has reached. Turbulent flow appears in such a diversity of guises that no single model used for engineering calculations can expect to mimic all the observed phenomena to the level of approximation sought. Thus, different levels and types of modelling are adopted according to the nature of the physical situation under study, the type of information to be extracted, and the accuracy required.

The book has been organized within three main sections. In Part A the focus is on techniques (with applications serving to illustrate the appropriateness of the technique adopted) while Part B examines particular types of flow, usually adopting a single preferred modelling strategy. Finally, in Part C, some current research approaches are introduced. Throughout, references to other articles in the book are given by their chapter number in square brackets. The individual articles themselves are sequenced broadly in terms of increasing complexity, at least within Parts A and B. The nomenclature undergoes some variation across the chapters, reflecting the differences habitually adopted in the journal literature over the different themes covered in the volume. Nomenclature for core variables is defined at the start of Chapter [1] and this is essentially common for Part A. Additional definitions or variants are provided in the individual chapters as needed.

## Part A: Physical and Numerical Techniques

Chapters [1]–[6] present a sequence of articles on single point closure. These represent the core of what is usually understood by 'turbulence modelling'. Chapter [1] by Gatski and Rumsey considers linear and non-linear models of eddy-viscosity type. It begins with algebraic variants of the mixing-length hypothesis and considers in turn various elaborations up to conventional two-equation models and the $k$-$\varepsilon$-$\overline{v^2}$ extension. The chapter closes with an extensive discussion of non-linear eddy-viscosity models, a closure level which apprecia-bly enlarges the range of flows that may successfully be modelled, usually for little additional cost. However, when transport or force-field effects on the turbulent fluctuations are large, a formal second-moment closure is usually to be preferred. Thus, one solves transport equations for all the 'second moments' i.e. the non-zero turbulent stresses and, in non-isothermal flows, the heat fluxes too. In Chapter [2] Hanjalić and Jakirlić provide an overview of the important modelling issues at this level and some of the modelling strategies adopted over the last 25 years. The chapter concludes by presenting an impressive range of test flows that have been computed with the form of closure adopted by the authors' group.

Chapters [3] and [4] which follow, by Craft and Launder (CL) and Durbin and Petterson-Reif (DP) provide, in greater detail, particular modelling strategies in second-moment closure especially for the crucially important pressure-strain terms. Both are motivated by the aim of replacing the widely adopted, though limited, algebraic 'wall-reflection' scheme that attempts to account for modifications to the pressure fluctuations brought about by a wall. The DP chapter reviews the current form of the 'elliptic-relaxation' method which replaces the algebraic scheme by a set of relatively simple partial differential equations. That by CL reviews the 'two-component-limit' strategy; their aim is partly to remove the need for wall reflection and partly to achieve a wider applicability of the model in free flows by adopting a more elaborate treatment for the case where walls are absent.

If one is going to adopt a model at second-moment closure level, one's output comprises point values of the stresses rather than the value of the eddy viscosity. The recommended strategies for incorporating such models into the computer code in order to achieve rapid convergence of the numerical solver are the subject of Chapter [5] by Leschziner and Lien.

Finally, from among this examination of single-point closures, Chapter [6] by Nagano considers the problem of turbulent heat (or mass) diffusion. The discussion covers both second-moment and eddy viscosity approaches with particular focus being placed on an equation for the dissipation rate of mean square temperature fluctuations. A major requirement for heat transport modelling is that, besides gaseous flow where the molecular diffusivities of heat and momentum are of a similar magnitude, one also needs to cope with Prandtl numbers both much less than (liquid metals) and much greater than (oils) unity.

Sandham (Chapter [7]) and Fröhlich and Rodi (Chapter [8]) introduce simulation-based approaches, dealing respectively with Direct Numerical Simulation (DNS), where all scales of turbulence are resolved, and Large-Eddy Simulation (LES), where large scales are resolved and small scales modelled. These approaches are becoming increasingly realistic as computer performance continues to improve. DNS provides reference solutions for simple canonical flows, against which turbulence closure assumptions can be checked, whilst LES is developing towards a practical method of prediction. Limitations on

Reynolds number, due to the range of turbulence scales that need to be resolved, are emphasised in these contributions.

An alternative perspective on turbulence closure is provided by Cambon in Chapter [9]. Here the single-point approaches discussed in [1]–[5] are placed into the context of multi-point and higher-order closures. Though mathematically more demanding, such approaches contain more of the physics of turbulent flow and provide useful insight into fundamental phenomena, such as nonlinearity and non-locality. Emphasis is placed on two-point closures, with practical examples of rotation and stratification used to illustrate the insight that can be obtained with this approach.

Part A concludes with Chapter [10], in which Roekaerts gives an introduction to the modelling of reacting flows. In this application mass-weighted averaging is introduced for the first time (this form of averaging is also used in Chapter [19], when compressible, high-speed flows are discussed). To account for chemistry effects, methods based on probability density functions (PDFs) are introduced. Applications of one-point scalar PDF methods and joint velocity-scalar PDFs appear later, in Chapters [20] and [21].

## Part B: Flow Types and Processes

Part B of the volume begins with a consideration of the capability of single-point closures and LES in tackling flows with separated flow regions and strong streamline curvature. Craft in [11] examines the strengths and, all too often, the weaknesses of single-point closure when applied to separated and impinging flows. This article, read in conjunction with the applications reported in [3]–[5], provides an overview of the performance achieved by the different modelling levels. In Chapters [12] and [13], Rodi and Laurence discuss the capabilities of LES, and we have our first glimpse of a current debate concerning the extent to which LES will replace single-point closure approaches for practical problems. The topic is revisited in [25] of Part C, but we see already in Chapter [12] the potential of LES, compared to single-point methods, for simulation of a laboratory experiment of flow around a bluff-body, dominated by separation and strong vortex shedding. Differences between techniques require further investigation, but the chapter ends on an optimistic note that LES will 'soon become affordable and ready for practical applications'. Laurence in [13], however, damps some of the high expectations, by suggesting that in many industrial problems the increase in computer power will simply result in more complete single-point predictions, and we may be waiting many years to see LES widely used.

The application of second- and third-moment closure to problems of horizontal shear flows affected by buoyancy is the theme of Chapters [14] and [15] by Craft and Launder, and Ilyushin. Stably stratified horizontal flows turn out to be far more difficult to capture than vertical mixed convection,

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where even a linear eddy viscosity model does fairly well in reproducing the observed phenomena. The reason for this difference in ease of predictability is that, for vertical flow, buoyant effects in the mean momentum equation introduce additional shear which is usually the dominant feature of any change in turbulence structure. In the horizontal shear flow the only important effects of stratification arise through the impact of buoyancy on the turbulent field itself. Because it is the vertical velocity fluctuations that are mainly affected by the stratification, second-moment closure is usually seen as the best starting point for closure. Yet, as both sets of authors point out, situations arise where second-moment closure is inadequate, though agreement with observation may be restored if, instead, closure is effected at third-moment level. Looking ahead, an alternative route for dealing with this type of problem is developed in [22] by Hanjalić and Kenjereš where the large-scale structures in Rayleigh–Bénard convection are resolved by employing a time-dependent solution of the Reynolds equations using just a (highly) truncated second-moment closure.

The problem of 'by-pass' transition has been the subject of single-point turbulence modelling since the early 1970s. The rationale was originally provided by the fact that at least some low-Reynolds-number two-equation eddy-viscosity models reproduced the reversion of a turbulent boundary layer back to (or towards) laminar when subjected to a severe acceleration. In view of that, it was conjectured that forward transition (from laminar to turbulent flow) in the presence of a turbulent external stream could also be predicted by the same model ... and so it proved. Since those early days the appreciation of the detailed processes taking place in by-pass transition has come a long way, progress being greatly assisted by DNS/LES studies of the type provided by Durbin, Jacobs and Wu in Chapter [16]. Savill's survey of modelling approaches is divided into two parts, Chapter [17] dealing with the use of conventional closures that have been designed for fully turbulent flows while Chapter [18] considers special modelling features. A major aim of current research efforts is to drive down the level of external-stream turbulence at which accurate prediction can be made and it is this goal that has led to the use of intermittency parameters and other devices discussed in [18].

Compressible flows, which form the subject of Chapter [19], in fact contain several different phenomena requiring the modellers' attention. The first is the question of how one should perform the averaging process in a fluid where the density is itself varying in time. From there, issues concerning the effects of density fluctuations on the different processes provide a major challenge. Finally structural changes to turbulence passing through a shock wave need to be considered. All these topics are addressed by Barre, Bonnet, Gatski and Sandham. It is noted that questions of numerical solution, addressed in Chapter [5], have taken account of the requirements of compressible flow. Indeed that chapter shows an application to a supersonic three-dimensional flow with a bow shock present.

In [20] Jones presents a review of the one-point scalar PDF approach, applied to flows with chemical reaction. It is argued that in the exact equation for a scalar PDF it is the term representing molecular mixing which presents the chief difficulty, and various approaches are described. Applications to a jet diffusion flame illustrate the current state of the art. Extensions to a joint velocity-scalar PDF, solved by means of a Monte Carlo method, are described by Wouters, Peeters and Roekaerts in Chapter [21]. This approach has a unified treatment of all the terms in the averaged equations that must be closed, including conventional Reynolds stresses. The method is expensive, but results for a bluff-body stabilized diffusion flame are promising.

## Part C: Future Directions

In this final part of the volume, space is allocated to some of the strategies that have not yet found an established place in the hierarchy of modelling or, as in [25], to issues of what directions are ready for further exploitation. As has been signalled earlier, Hanjalić and Kenjereš [22] report that the problem of Rayleigh–Bénard convection (in which a horizontal layer of fluid, confined within the space between horizontal planes, is heated from below) is captured much better with a truncated second-moment closure if one adopts a time-dependent rather than a steady-state numerical solution. Essentially what results from the time-dependent simulation is a close replica of the large eddy-simulation of the same flow. Put another way, the TRANS simulation is effectively a coarse-grid LES that uses a higher level of sub-grid model and is grid independent. Clearly, for the problem chosen it would seem that this fusion of RANS and LES strategy is wholly satisfactory and superior (from the standpoint of accuracy or cost) to either a steady state RANS or a conventional LES. Ilyushin in Chapter [23] also develops inter-linkages between two approaches to turbulence that are usually viewed as discrete. In this case he shows, among other contributions, how a knowledge of just the second- and third-order moments enable the probability density functions to be approximated.

Coleman and Sandham review the latest direct simulations of separation bubbles in Chapter [24]. Turbulent separation bubbles are at the current limits of computer power, with severe Reynolds number restrictions. However, DNS of transitional separation bubbles, an important phenomenon that in some cases controls the performance of aerofoils, are already at the Reynolds numbers encountered in applications. Guerts and Leonard consider, in Chapter [25], recent developments in LES and the issues facing LES that need to be addressed for it to be developed into a reliable predictive tool. The guidelines for developing reliable LES listed in section 5 complement those of Chapter [7] for DNS, and should be borne in mind by anyone interested in using LES for complex flow problems. Closure methods will continue to require guidance from experiment and theory, and in Chapter [26] we conclude the volume with a review by Cambon of the potential for further insight coming from recent developments in two-point closures.

# Part A.

## Physical and Numerical Techniques

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# 1
# Linear and Nonlinear Eddy Viscosity Models

*T.B. Gatski and C.L. Rumsey*

## 1 Introduction

Even with the advent of a new generation of vector and now parallel processors, the direct simulation of complex turbulent flows is not possible and will not be for the foreseeable future. The problem is simply the inability to resolve all the component scales within the turbulent flow.

In the context of scale modeling, the most direct approach is offered by the partitioning of the flow field into a mean and fluctuating part (Reynolds 1895). This process, known as a Reynolds decomposition, leads to a set of Reynolds-averaged Navier–Stokes (RANS) equations. Although this process eliminates the need to completely resolve the turbulent motion, its drawback is that unknown single-point, higher-order correlations appear in both the mean and turbulent equations. The need to model these correlations is the well-known 'closure problem.' Nevertheless, the RANS approach is the engineering tool of choice for solving turbulent flow problems. It is a robust, easy to use, and cost effective means of computing both the mean flow as well as the turbulent stresses and has been overall, a good flow-prediction technology.

From a physical standpoint, the task is to characterize the turbulence. One obvious characterization is to adequately describe the evolution of representative turbulent velocity and length scales, an idea that originated almost 60 years ago (Kolmogorov 1942). The physical cornerstone behind the development of turbulent closure models is this ability to correctly model the characteristic scales associated with the turbulent flow. This chapter describes incompressible, turbulent closure models which (can) couple with the RANS equations through a turbulent eddy viscosity (velocity $\times$ length scale). In this context both linear and nonlinear eddy viscosity models are discussed. The descriptors 'linear' and 'nonlinear' refer to the tensor representation used for the model. The linear models assume a Boussinesq relationship between the turbulent stresses or second-moments and the mean strain rate tensor through an isotropic eddy viscosity. The nonlinear models assume a higher-order tensor representation involving either powers of the mean velocity gradient tensor or combinations of the mean strain rate and rotation rate tensors.

Within the framework of linear eddy viscosity models (EVMs), a hierarchy of closure schemes exists, ranging from the zero-equation or algebraic models to the two-equation models. At the zero-equation level, the turbulent velocity and

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## Nomenclature

| | |
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| $b_{ij}$, $\mathbf{b}$ | Reynolds stress anisotropy tensor, $(\overline{u_i u_j}/2k) - \delta_{ij}/3$ |
| $C_\mu$, $C_\mu^*$ | eddy viscosity calibration coefficient |
| $D/Dt$ | material derivative $(= \partial/\partial t + U_j \partial/\partial x_j)$ |
| $\mathcal{D}$, $\mathcal{D}_{ij}$ | represents the combined effect of turbulent transport and viscous diffusion |
| $k$ | turbulent kinetic energy $(\equiv \tau_{ii}/2)$ |
| $L$ | characteristic length scale in wall proximity |
| $l$ | mixing length |
| $P$, $\overline{p}$ | mean pressure |
| $\mathcal{P}$ | turbulent kinetic energy production term |
| $\mathbf{R}$ | symmetric, traceless tensor in algebraic stress equation |
| $\mathcal{R}^2$ | flow parameter $(\equiv -\{\mathbf{W}^2\}/\{\mathbf{S}^2\})$ |
| $S_{ij}$, $\mathbf{S}$ | mean strain rate tensor $(\equiv (\partial U_i/\partial x_j + \partial U_j/\partial x_i)/2)$ |
| $T$ | characteristic time scale in wall proximity |
| $\mathbf{T}^{(n)}$ | tensor basis element |
| $U_e$ | edge velocity |
| $U_i$ | mean velocity component |
| $u_\tau$ | friction velocity |
| $W_{ij}$, $\mathbf{W}$ | mean rotation rate tensor in noninertial frame $(\equiv (\partial U_i/\partial x_j - \partial U_j/\partial x_i)/2)$ |
| $W_{ij}^*$, $\mathbf{W}^*$ | modified mean rotation rate tensor in inertial frame |
| $\overline{W}_{ij}$ | mean rotation rate tensor in transformed frame |
| $\underline{\mathbf{X}}$ | orthogonal transformation matrix |
| $x_i$ | coordinate direction in inertial (Cartesian) frame $(x, y, z)$ |
| $\alpha_n$ | tensorial expansion coefficients |
| $\varepsilon$ | isotropic turbulent energy dissipation rate |
| $\hat{\varepsilon}$ | near-wall modified dissipation rate |
| $\varepsilon_{ij}$ | dissipation rate tensor |
| $\delta$ | boundary layer thickness |
| $\delta^*$ | displacement thickness |
| $\eta$ | scalar invariant $(\equiv \sqrt{S_{ik}S_{ki}})$ |
| $\kappa$ | von Karman constant |
| $\rho$ | density |
| $\sigma_{ij}$ | viscous stress tensor |
| $\Pi_{ij}$ | pressure strain rate correlation |
| $\nu$ | kinematic viscosity |
| $\nu_t$, $\nu_t^*$ | turbulent eddy viscosity |
| $\nu_{\mathrm{ti}}$, $\nu_{\mathrm{to}}$ | inner and outer eddy viscosity |
| $\tau$ | turbulent time scale $(= k/\varepsilon)$ |
| $\tau_{ij}$, $\boldsymbol{\tau}$ | Reynolds stress tensor $(\equiv \overline{u_i u_j})$ |
| $\Omega_{ij}$ | arbitrary time-independent rotation rate of noninertial frame |
| $\Omega_r$ | rotation rate of noninertial frame |
| $\omega$ | dissipation rate per unit kinetic energy |

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length scales are specified algebraically whereas, at the two-equation level, differential transport equations are used for both the velocity and length scales. Within the framework of nonlinear eddy viscosity models (NLEVMs), the characterizing feature is the (polynomial) tensor representation for the second-moments or Reynolds stresses. However, the method of determining the expansion coefficients differs among models. In some methods the expansion coefficients are determined through calibrations with experimental or numerical data and the imposition of dynamic constraints. In other methods, the expansion coefficients are related directly to the closure coefficients used in the full differential Reynolds stress equations. The models derived using these latter methods are sometimes referred to as explicit algebraic stress models.

Over the years, there has been a multitude of models at the EVM and NLEVM levels proposed for the RANS equations. No attempt is made (since we would surely fail) to be all inclusive with the choice of models for each level of closure discussed. Our goal, however, is to provide the reader with a broad perspective on the development of such models, so that, with this broader view, he or she will be better prepared to assess the viability of using a particular closure scheme.

## 2 Reynolds-averaged Navier–Stokes formulation

As a prelude to the discussion of the linear and nonlinear eddy viscosity models, it is desirable to describe the Reynolds averaging procedure and the resulting form of the mean momentum and continuity equations. In the Reynolds decomposition, the flow variables are decomposed into mean and fluctuating components as

$$f = \overline{f} + f'. \tag{2.1}$$

The average of a fluctuating quantity is zero $\overline{f'} = 0$, and the mean quantity $\overline{f}$ can be extracted if a statistically steady or a statistically homogeneous turbulence is assumed. For example, if the turbulence is stationary,

$$\overline{f(\mathbf{x})} = \lim_{\mathcal{T}\to\infty} \frac{1}{\mathcal{T}} \int_{t_0}^{t_0+\mathcal{T}} f(\mathbf{x}, t)\, dt, \tag{2.2}$$

and the average of the product of two quantities is $\overline{f\,g} = \overline{f}\overline{g} + \overline{f'g'}$.

The velocity $(u_i)$ and pressure $(p)$ fields can be decomposed into their mean $(U_i,\ P)$ and fluctuating parts $(u_i,\ p)$, and the resulting Reynolds-averaged Navier–Stokes (RANS) equations can be written as

$$\frac{DU_i}{Dt} = \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}. \tag{2.3}$$

For an incompressible flow, the mass conservation equation reduces to the mean continuity equation,

$$\frac{\partial U_j}{\partial x_j} = 0. \tag{2.4}$$

 