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978-0-521-79163-2 - Derivatives in Financial Markets with Stochastic Volatility

Jean-Pierre Fouque, George Papanicolaou and K. Ronnie Sircar

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