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0521786754 - Finite Group Theory, Second Edition
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Finite group theory

During the last 40 years the theory of finite groups has developed dramatically. The finite simple groups have been classified and are becoming better understood. Tools exist to reduce many questions about arbitrary finite groups to similar questions about simple groups. Since the classification there have been numerous applications of this theory in other branches of mathematics.

Finite Group Theory develops the foundations of the theory of finite groups. It can serve as a text for a course on finite groups for students already exposed to a first course in algebra. For the reader with some mathematical sophistication but limited knowledge of finite group theory, the book supplies the basic background necessary to begin to read journal articles in the field. It also provides the specialist in finite group theory with a reference in the foundations of the subject.

The second edition of *Finite Group Theory* has been considerably improved, with a completely rewritten chapter 15 considering the 2-Signalizer Functor Theorem and the addition of an appendix containing solutions to exercises.

M. Aschbacher is Shaler Arthur Hanisch Professor of Mathematics at Caltech. His books include *Finite Group Theory* (Cambridge University Press, 1986), *Sporadic Groups* (Cambridge University Press, 1994), and *3-Transposition Groups* (Cambridge University Press, 1997).

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FINITE GROUP THEORY

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521781459

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First Edition Published 1986
Second Edition Published 2000

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Aschbacher, Michael, 1944–
Finite group theory / M. Aschbacher. – 2nd ed.
p. cm. – (Cambridge studies in advanced mathematics ; 10)
Includes bibliographical references and index.
ISBN 0-521-78145-0 (hb) – ISBN 0-521-78675-4 (pbk.)
1. Finite groups. I. Title. II. Series.
QA177 .A82 2000
512′.2 – dc21 99-055693

ISBN-13 978-0-521-78145-9 hardback
ISBN-10 0-521-78145-0 hardback

ISBN-13 978-0-521-78675-1 paperback
ISBN-10 0-521-78675-4 paperback

Transferred to digital printing 2006

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More information

Contents

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M. Aschbacher
Frontmatter
More information

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Preface

Finite Group Theory is intended to serve both as a text and as a basic reference on finite groups. In neither role do I wish the book to be encyclopedic, so I've included only the material I regard as most fundamental. While such judgments are subjective, I've been guided by a few basic principles which I feel are important and should be made explicit.

One unifying notion is that of a group representation. The term representation is used here in a much broader sense than usual. Namely in this book a representation of a group G in a category $\mathscr{C}$ is a homomorphism of G into the automorphism group of some object of $\mathscr{C}$. Among these representations, the permutation representations, the linear representations, and the representations of groups on groups seem to be the most fundamental. As a result much of the book is devoted to these three classes of representations.

The first step in investigating representations of finite groups or finite dimensional groups is to break up the representation into indecomposable or irreducible representations. This process focuses attention on two areas of study: first on the irreducible and indecomposable representations themselves, and second on the recovery of the general representation from its irreducible constituents. Both areas receive attention here.

The irreducible objects in the category of groups are the simple groups. I regard the finite simple groups and their irreducible linear and permutation representations as the center of interest in finite group theory. This point of view above all others has dictated the choice of material. In particular I feel many of the deeper questions about finite groups are best answered through the following process. First reduce the question to a question about some class of irreducible representations of simple groups or almost simple groups. Second appeal to the classification of the finite simple groups to conclude the group is an alternating group, a group of Lie type, or one of the 26 sporadic simple groups. Finally invoke the irreducible representation theory of these groups.

The book serves as a foundation for the proof of the Classification Theorem. Almost all material covered plays a role in the classification, but as it turns out almost all is of interest outside that framework too. The only major result treated here which has not found application outside of simple group theory is the Signalizer Functor Theorem. Signalizer functors are discussed near the end of the book. The last section of the book discusses the classification in general terms.

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Frontmatter
More information

The first edition of the book included a new proof of the Solvable Signalizer Functor Theorem, based on earlier work of Helmut Bender. Bender's proof was valid only for the prime 2, but it is very short and elegant. I've come to believe that my extension to arbitrary primes in the first edition is so complicated that it obscures the proof, so this edition includes only a proof of the Solvable 2-Signalizer Functor Theorem, which is closer to Bender's original proof. Because of this change, section 36 has also been truncated.

In some sense most of the finite simple groups are classical linear groups. Thus the classical groups serve as the best example of finite simple groups. They are also representative of the groups of Lie type, both classical and exceptional, finite or infinite. A significant fraction of the book is devoted to the classical groups. The discussion is not restricted to groups over finite fields. The classical groups are examined via their representation as the automorphism groups of spaces of forms and their representation as the automorphism groups of buildings. The Lie theoretic point of view enters into the latter representation and into a discussion of Coxeter groups and root systems.

I assume the reader has been exposed to a first course in algebra or its equivalent; Herstein's *Topics in Algebra* would be a representative text for such a course. Occasionally some deeper algebraic results are also needed; in such instances the result is quoted and a reference is given for its proof. Lang's *Algebra* is one reference for such results. The group theory I assume is listed explicitly in section 1. There isn't much; for example Sylow's Theorem is proved in chapter 2.

As indicated earlier, the book is intended to serve both as a text and as a basic reference. Often these objectives are compatible, but when compromise is necessary it is usually in favor of the role as a reference. Proofs are more terse than in most texts. Theorems are usually not motivated or illustrated with examples, but there are exercises. Many of the results in the exercises are interesting in their own right; often there is an appeal to the exercises in the book proper. In this second edition I've added an appendix containing solutions to some of the most difficult and/or important exercises.

If the book is used as a text the instructor will probably wish to expand many proofs in lecture and omit some of the more difficult sections. Here are some suggestions about which sections to skip or postpone.

A good basic course in finite group theory would consist of the first eight chapters, omitting sections 14, 16, and 17 and chapter 7, and adding sections 28, 31, 34, 35, and 37. Time permitting, sections 32, 33, 38, and 39 could be added.

The classical groups and some associated Lie theory are treated in chapter 7, sections 29 and 30, chapter 14, and the latter part of section 47. A different sort of course could be built around this material.

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Chapter 9 deals with various concepts in the theory of linear representations which are somewhat less basic than most of those in chapters 4 and 12. Much of the material in chapter 9 is of principal interest for representations over fields of prime characteristic. A course emphasizing representation theory would probably include chapter 9.

Chapter 15 is the most technical and specialized. It is probably only of interest to potential simple groups theorists.

Chapter 16 discusses the finite simple groups and the classification. The latter part of section 47 builds on chapter 14, but the rest of chapter 16 is pretty easy reading. Section 48 consists of a very brief outline of the proof of the finite simple groups makes use of results from earlier in the book and thus motivates those results by exhibiting applications of the results.

Each chapter begins with a short introduction describing the major results in the chapter. Most chapters close with a few remarks. Some remarks acknowledge sources for material covered in the chapter or suggest references for further reading. Similarly, some of the remarks place certain results in context and hence motivate those results. Still others warn that some section in the chapter is technical or specialized and suggests the casual reader skip or postpone the section.

In addition to the introduction and the remarks, there is another good way to decide which results in a chapter are of most interest: those results which bear some sort of descriptive label (e.g. Modular Property of Groups, Frattini Argument) are often of most importance.

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