

Notation Index

The symbol $:=$ denotes ‘equals by definition’. The end of a proof is marked by \square . The beginning of an argument by contradiction is flagged by ‘suppose [by way of contradiction]’ and the lightning symbol, $\not\equiv$, signals that the required contradiction has been reached. See for example the proof of Lemma ‘1.17’.

\leq	2, 25	$P \cup Q$	17
$\langle P; \leq \rangle$	2	$P \oplus Q$	17
$<$	2, 25	\mathbf{M}_n	17
$\not\equiv$	2	$P_1 \times \cdots \times P_n$	18
\parallel	2	P^n	18
\mathbf{n}	3	$\downarrow Q, \uparrow Q$	20
$\bar{\mathbf{n}}$	3	$\downarrow x, \uparrow x$	20
\bar{S}	3	$\mathcal{O}(P)$	20
\cong	3	\hookrightarrow	23
$\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{N}$	3–4	$\psi \circ \varphi$	23
$\langle \mathbb{N}_0; \preceq \rangle$	4	id_S	24
$\wp(X)$	4	$f \leq g$	24
$\text{Sub } G, \mathcal{N}\text{-Sub } G$	4	$Y^X, (X \rightarrow Y)$	24
$\text{Sub } V$	4	$Y^{(X)}, \langle X \rightarrow Y \rangle$	24
$\langle X; T \rangle$	4, 275	$P \oplus Q$	28
\Rightarrow	5	$w(P)$	32
Σ^*, Σ^{**}	7	S^u, S^ℓ	33
$(X \dashrightarrow Y), (X \dashrightarrow X)$	7	$\sup S, \inf S$	34
$\text{dom } \sigma$	7	$x \vee y, x \wedge y$	34
$\text{graph } f, \text{dom } f$	7	$\bigvee S, \bigwedge S, \bigvee_P S, \bigwedge_P S$	34
\sqsubseteq	8	$\bigvee_{i \in I} A_i$	34
\succ, \prec	11	HK	38
$P^\partial, \Phi^\partial$	14	$\langle L; \vee, \wedge \rangle$	39
\perp, \top	15	0, 1	41
P_\perp	15	$\text{Sub } L, \text{Sub}_0(L)$	41
\mathbb{N}_\perp	16	\mapsto	43
$\text{Max } Q, \max Q, \min Q$	16	$\mathcal{I}(L)$	45

		<i>Notation index</i>	287
$\ell(P)$	51	$\theta_{\{Q\}}$	144
$\mathcal{J}(L), \mathcal{M}(L)$	53	$\Gamma(X)$	145
$[X]$	60, 111	$\bar{A}, C(A)$	145, 146
$\langle A \rangle$	62	P_c	146
(G, M, I)	66	\mathcal{L}_C	147
gIm	67	$C_{\mathcal{L}}(A)$	147
A', B'	67	$\bigsqcup D$	149
$\mathfrak{B}(G, M, I)$	67	$F(L)$	152
$\mathfrak{B}_G, \mathfrak{B}_M$	68	$K(L)$	152
$\gamma(g), \mu(m)$	70	$p^{\triangleright}, q^{\triangleleft}$	155
(G, \bar{M}, \bar{I})	83	$(\triangleright, \triangleleft)$	155
$\mathbf{M}_3, \mathbf{N}_5$	87	$\varphi^{\sharp}, \varphi^b$	161, 162
$\mathbf{M}_{3,3}$	92	$\mathbf{DM}(P)$	166
a'	93	$\mathcal{I}(S)$	170
$\text{FC}(X)$	95	$P \oplus_{\perp} Q$	176
τ, \mathbf{F}	96	$P \oplus_{\vee} Q$	177
\rightarrow	96	$[P \rightarrow Q]$	178
\equiv	97	\perp	181
BT	98	fact	10, 181
F_p	98	$\text{fix}(F), \text{pre}(F), \text{post}(F)$	182
$[c, d]$	106	$\mu(F), \nu(F)$	182
$\mathcal{A}(L)$	113	F^n	183
$\mathbf{D}_F, \mathbf{P}_F$	120	$\mu_*(P)$	186
a^*	128, 262	$F(P)$	202
$a \equiv b \pmod{\theta}, a \theta b$	130	$\mathbf{A} = \langle A, \text{Con}, \vdash \rangle$	205
$(a, b) \in \theta$	130	Con	205
$[a]_{\theta}$	130	\vdash	205
$\text{Con } L$	132, 138	$Y \subseteq A$	205
$\ker f$	132	\perp_n	207
L/θ	133	$ \mathbf{A} $	207
$\langle a, b; c, d \rangle$	135	\bar{X}	207
$\mathbf{0}, \mathbf{1}$ (in $\text{Con } L$)	138	$\mathbf{IS}(\mathcal{L}), \mathbf{IS}(D)$	209
$\theta(a, b)$	138	\trianglelefteq	210
θ_J	141	\mathfrak{N}	211

288

Notation index

\mathcal{L}_\perp	212	X_α	238
$\mathcal{L} \boxtimes \mathcal{K}, \mathcal{L} \boxplus \mathcal{K}, \mathcal{L} \boxplus_\perp \mathcal{K}, \mathcal{L} \boxplus_\vee \mathcal{K},$ $(I \boxplus \mathcal{L})$	212	$\langle \mathcal{I}_p(B); T \rangle$	248
$A \dot{\cup}_\vee B$	212	$\wp^\tau(X)$	249
$Y \rightsquigarrow b$	213	\mathbb{N}_∞	250
$ r $	214	\models	253
$\mathcal{V}[[P]]$	217	$\sim_{\models} \sim_{\vdash}$	254
\mathbf{tt}, \mathbf{ff}	218	$\text{LA}_\vdash, \text{LA}_{\models}$	254
St	218	$\mathcal{O}^\tau(X)$	258
<i>cond</i>	219	$\langle X; \leq, T \rangle$	258
$\mathbf{IC}(Q)$	224	$\langle \mathcal{I}_p(L); \subseteq, T \rangle$	259
$\mathcal{I}(Q)$	233	\mathbf{D}, \mathbf{P}	262
$\mathcal{I}_p(L), \mathcal{F}_p(L)$	233	$\mathbf{D}(L, K), \mathbf{P}(Y, X)$	264
		θ_Y	266
(AC)	229	(Gal)	155
(ACC)	51	(Gal1)–(Gal3)	159
(AM1)–(AM2)	213	(IS1)–(IS5)	205
(BPI)	235	(JID)	240
(BUF)	237	(KL)	229
(CD)	239	(L1)–(L4), (L1) [∂] –(L4) [∂]	39
(clo1)–(clo3)	145	(M)	90
(DCC)	51	(MID)	240
(D), (D) [∂]	85	(P1)–(P5)	68
(DMI)	237	(T1)–(T3)	275
(DPI)	235	(ZL), (ZL)', (ZL)''	229
(FI1)–(FI3)	192		

Index

Page numbers given in boldface refer to definitions and those in *italic* to exercises, with the latter overriding the former where definitions are given in exercises.

- absorption laws 39
- Adequacy Theorem 253, 255
- adjoint (lower, upper) **156**, 161–162, 172
- Alexander's Subbasis Lemma 257, 261, 274, 278
- algebra of propositions 96–98
- algebra of sets **95**
- algebraic closure operator **150**–152, 153, 154, 194, 204
- algebraic \cap -structure **150**, 151, 153–154, 176, 203, 208–209, 211, 214
- algebraic lattice **153**–155, 170, 203, 204, 223, 241, 242, 243, 246, 268, 284
- algebraic semilattice (= domain) **202**, 204
- antichain **3**, 21, 27, 32, 122, 149, 169
- antisymmetry **2**
- approximable mapping 213–215, 225
- Arrow's Theorem 5
- ascending chain condition, (ACC) 51–53, 55, 64, 149, 153, 155, 171
- associative laws 39
- atom **113**–115, 144, 246, 268
- atomic **113**, 240, 246
 - weakly **242**–243, 246
- attribute (of a context) **66**
- Axiom of Choice, (AC) 51, 52, 228–230, 231, 236–237
- Banach's Contraction Mapping Theorem 183
- Banach's Decomposition Theorem 63
- basis (for a topology) 248, 257, **276**
- bijjective 3
- Birkhoff's representation theorem 118, 119, 126, 174, 257
- block **130**–131, 132, 133, 134–137
- Boolean algebra(s) **94**–104, 109–111, 239, 240, 247–248, 283
 - atomless 246, 251
 - examples 95–96, 246, 255
 - finite 114–115, 233
 - representation of 240, 248, 256
- Boolean homomorphism **94**, 134, 255, 268
- Boolean lattice **93**, 108, 109, 113–114, 122, 128, 234, 235, 237
- Boolean ring 109–110
- Boolean space **249**–250, 256, 267, 268
- Boolean term (polynomial) **98**–104, 110–111
- bottom (element), \perp **16**, 17, 34, 41, 175, 176, 180
- bounded lattice **41**
 - (BPI) **235**–237, 255, 256
 - (BUF) **237**, 245, 253
- Cancellation Rule 159
- Cantor set, space 251, 261
- Cantor's Theorem 95, 221
- category 25
- chain(s) **3**, 27, 32, 36, 41, 58, 83, 87, 122, 129
 - and directed sets 149, 179, 184, 195, 231
 - conditions 50–52
 - maximal 229–230

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Index

[More information](#)

290

choice function **229**
 clopen down-set 256–259
 clopen set 95, 247, **276**
 closed set 4, 49, 147, 195, 275–276
 closure operator **145–147**, 150–152,
 169, 194, 204, 246
 algebraic (*see* algebraic closure oper-
 ator) **150**
 examples 147–148, 173
 and topped \cap -structures 68, 147,
 160, 246
 closure system (= topped \cap -structure)
48
 coalesced sum, \oplus_v **177**, 193, 212
 cofinite sets 62, 245
 commutative laws 39
 compact element **152–153**
 compact space **277–279**
 compactification, 1-point 251
 comparable 2
 compatible with join and meet **131**
 complement **93**, 108, 126
 complete distributivity law, (CD)
239–240
 complete lattice(s) **34**, 46–50, 52, 63,
 160–161, 272, 284
 and closure operators 146–147, 161,
 246
 and concept lattices 69–73
 examples 49, 147–148
 and fixpoints 50, 186, 189, 199
 of sets **36–37**, 47, 243
 and topped \cap -structures 48, 147,
 160, 246
 complete partially ordered set (= CPO)
175
 complete semilattice 201
 completely distributive lattice **239**,
 240, 243
 completely inductive 179, **231**
 completely join-irreducible element
242, 243, 246

Index

completely join-, meet-prime element
242, 246
 completely meet-irreducible element
242, 243, 246
 completion **165**, 285
 by cuts (= Dedekind–MacNeille
 completion) **166**
 Dedekind–MacNeille **166–169**, 174
 ideal 224
 normal (= Dedekind–MacNeille
 completion) **166**
 composite map 23, 183
 computer architecture 99–102
 concept 65–**67**,
 concept analysis, formal 6, 65–84, 283
 concept lattice(s) **67–84**, 160
 algorithm for drawing 76
 fundamental theorem of 70–72
 lattice theoretic examples of 73
 congruence(s) 130–144
 blocks of 134–137
 Boolean **134**, 142, 272
 duality for 119–123, 144, 266–267
 and ideals 142, 246
 join of 139
 lattice of 137–138, 139–140, 144
 principal **138–139**, 142, 143, 144
 Connecting Lemma 39
 consistent set
 in information system 205–206
 in ordered set 8, **201**
 consistently complete **202**
 context 65, **66**, 157
 complementary 83
 continuous
 on base sets **216**
 map(s) between CPOs and pre-CPOs
 177–179, 194–195, 199
 maps between domains 203, 213,
 215, 216, 221

- separately 194
 with respect to Scott topology 195
 topologically 276
- convex 63, 135
- covering relation 11, 13, 26, 27, 53
- CPO(s) (and pre-CPO(s)) 149,
 175–181, 189, 193, 194, 195, 202,
 229–231
- fixpoints in 183–189, 197–198, 199,
 231
- free 224
- maps between 177–179, 194, 195,
 199
- of partial maps 180–182, 184, 193
- CPO Fixpoint Theorem I 183, 184,
 185, 188
- CPO Fixpoint Theorem II 187, 188,
 189, 199, 231
- CPO Fixpoint Theorem III 188, 198
- cross-table (of a context) 67
- CTOD space (= Priestley space) 258
- cube 12
- dcpo (=pre-CPO) 175
- dcppo (= CPO) 175
- decreasing set (= down-set) 20
- Dedekind–MacNeille completion 166
 –169, 174
- de Morgan’s laws 93
- denotational semantics 10, 217–221
- descending chain condition, (DCC)
 51, 52, 55, 64
- description problem 79
- determination problem 75
- deterministic (program) 6
- Diagonal Rule 200
- diagram 11–14, 18–19, 134–137
- diamond, M_3 87
- Dilworth’s Theorem 32, 127
- directed set 148–150, 179, 194, 193,
 195
- directed union 149
 closed under 150
- discrete order 2, 256, 258
- discrete topology 249, 256, 278
- disjoint union, $\dot{\cup}$ 17, 27
 of ordered sets 17
 topology 267, 269
- disjunctive normal form, (DNF) 103–
 104, 111
- distributive inequality 58
- distributive lattice(s) 86–93, 104–106,
 140, 233–237
 bounded 257–267
 characterizations of 89, 105, 106,
 107, 118, 142, 245
 congruences of 142
 duality for 119–123, 144, 266–267
 finite 116–129, 233
 as lattice of sets 118, 238–239, 243
 prime ideals of 233–234, 235–237,
 238–239, 272
 prime ideal space of 257
 representation of 118, 243, 259
- (DMI) 237, 246
- domain 9–10, 202–205, 224, 285
 constructors 215–217, 226
 equation 216, 221–223, 226–227
 maps between 203
 recursively defined 222
- domain of a map 7, 184
- down-set(s) 20–21, 27–28, 30
 clopen 256–259
- down-set lattice, $\mathcal{O}(P)$ 20–23, 166,
 225
 characterization of 243, 284
- (DPI) 235–237, 246, 257
- dual of an ordered set 14
- dual space (= prime ideal space) 248,
 257, 260–261
- dual statement 14, 111

292

duality (dual category equivalence)
 123, 264
 for Boolean algebras 251
 between \mathbf{D} and \mathbf{P} 262–267, 270
 –274, 284
 between \mathbf{D}_F and \mathbf{P}_F 120–123,
 128–129
 for congruences 144, 266–267, 272
 for ideals 263–264, 272

Duality Principle
 for Boolean algebras 111
 for lattices 39
 for ordered sets 15

element (in an information system)
 207–208

embedding
 lattice-, \mapsto 43, 107
 order-, \leftrightarrow 23, 43

empty set, join and meet of 34

endofunctor (= self-map) 182

entailment relation, \vdash 205
 \vdash -closed 207

equivalence class (= block) 130

equivalence relation 49, 130

equivalent (Boolean terms) 98

exact real arithmetic 8–9

Exchange Rule 191

extent (of a concept) 65, 67

extremely disconnected 267

extremely order-disconnected 272

factorial function 10, 181, 184

fence 28

Fibonacci sequence 28

field of sets (= algebra of sets) 95

filter 45, 107, 272
 distributive 245
 maximal (= ultrafilter) 233
 prime 233–234, 245

Index

proper 45, 232

finite distributive lattice(s) 116–129,
 233
 duality for 119–123
 prime ideals of 233
 representation of 116–118, 256

finite element 152–153, 202

finite length 51, 107

finite-cofinite algebra 95, 250–251

First Isomorphism Theorem (= Homomorphism Theorem) 130

fixpoint(s) 50, 182, 196–199, 285
 calculational rules for 186, 189–192,
 200
 equation 182
 greatest 50, 182
 least 182, 186, 197–200
 existence of 50, 183, 186, 187,
 188, 189, 198, 231
 induction 192
 minimal, existence of 188, 198
 operator 199

flat ordered set 16, 175, 178, 203

free pre-CPO 224

full \sqcap -structure 210

function space 24–25, 31, 128, 178,
 203, 214, 216, 222, 224

Fundamental Theorem of Arithmetic
 53, 55, 108

fundamental theorem of concept lattices
 70–72, 161

Fusion Rule (Transfer Rule) 191

Galois connection(s) 68, 155–165,
 171–174, 285
 calculational rules for 159, 171

gate 99
 diagram 99–102, 110

graph 7

greatest common divisor 37

greatest element 16

- greatest lower bound (*see also* meet) **33–34**, 39
- group (*see also* subgroups) 130–131, 140
- Hasse diagram (= diagram) **11**
- Hausdorff space **277**
- higher-order function 24
- highest common factor 37
- homeomorphism **276**
 order- **259**
- homomorphism **43–44**, 61, 88, 131–132, 133–134, 142
 of Boolean algebras (*see* Boolean homomorphism) **94**
 $\{0, 1\}$ -homomorphism **43**, 122–123, 128, 244, 264–266, 270
- Homomorphism Theorem 131
 for Boolean algebras 134
 for groups 131
 for lattices 133–134
- homset 265, 273–274
- ideal(s)
 completion 224
 duality for 263–264, 272
 in join semilattice 170
 in lattice **44–45**, 62, 64, 107, 141, 232, 244
 lattice of 49, 148, 155, 267
 maximal **233–235**, 237, 272
 order (= down-set) **20**
 in ordered set 223
 prime (*see* prime ideal) **233**
 principal **45**, 64, 272
 proper **45**, 232
 in ring 4, 49, 110, 231
- idempotency laws 39
- identities, lattice 39, 57
- identity map 24
- inclusion order 4
- increasing (map) **186**, 187, 188, 193
- increasing set (= up-set) **20**
- indecomposable (lattice) 126
- induced (inherited) order **3**
- induction, fixpoint 192
- Induction Rule 186
- inductive (ordered set) 231
- infimum (= greatest lower bound) **34**
- information ordering 6, 9
- information systems **205–217**, 223, 224–226, 285
 and algebraic \cap -structures 204, 208–209
 and domains 204, 209–210
- injective 271
- integers, \mathbb{Z} , chain of 3–4, 36, 62
- intent (of a concept) 65, **67**
- interior 50, 147
- invariant (F -invariant) 108, **187**
- intersection structure(s) (\cap -structure(s)) **48–49**, 204
 algebraic 150–152, 153–154, 204, 208–209
 combinations of 212
 topped **48–49**, 69, 138, 147–148, 160–161, 204
- interval 106, 108, 170
- interval order **5**
- interval topology 268
- irredundant join 108
- isomorphism
 of lattices **43**, 44
 of ordered sets **3**, 13–14, 23–24, 47
- join
 as binary operation 39–41
 in \cap -structure 48, 150
 irredundant 108
 properties of 34–36, 46
 in a subset 47

294

join-dense **53**, 55, 64, 70–72, 75, 112, 167, 168, 243

Join-Infinite Distributive Law, (JID) **106**, **240**, 241, 242, 243

join-irreducible element(s) **53–56**, 107, 112, 113, 116–120, 125–126, 128, 168

join-preserving map **43**, 44

join semilattice **170**, 223

kernel **132**, 142, 144

Knaster–Tarski Fixpoint Theorem **50**, 63, 188, 189

Kuratowski’s Lemma, (KL) **229**, 230

lattice(s)

- as algebraic structure **39–45**, 57–58
- algebraic (*see* algebraic lattice) **153**
- Boolean (*see* Boolean lattice) **93**
- bounded **41**
- chain conditions in **50–52**
- complete (*see* complete lattice) **34**
- concept (*see* concept lattice) **67**
- congruence **132**
- distributive (*see* distributive lattice) **86**
- examples **36–38**
- finite **45**, 46, 50
- homomorphism of (*see* homomorphism) **43**
- identities **39**, 57
- as ordered set **34–36**
- with no infinite chains **51**, 52, 55, 168
- modular (*see* modular lattice) **86**
- product of **42–43**, 119, 244
- quotient **133–134**
- of sets **36–37**, 47, 239
- of subgroups (*see* subgroups, lattice of)

least common multiple **37**

Index

least element **16**

least fixpoint **182**, 186, 197–200

- existence of **50**, 183, 186, 187, 188, 189, 198, 231

least pre-fixpoint **186**

least upper bound (*see also* join) **33–35**, 39–41

length **50–51**, 64, 127

lexicographic order **18**, 27

lifting **15–16**, 17, 176, 180, 216

Lindenbaum algebra (LINDA) **252–255**, 268

linear extension **32**, 244

linear sum, \oplus **17**, 62, 269

linearly ordered set (= chain) **3**

logical connectives **96**

logically equivalent, \equiv **97**

lower adjoint **156**, 161–162

lower bound(s) **33**, 156

M_n **17**, 37, 51, 73, 104, 143, 169

M_3 **18**, 87, 138, 143, 144

M_3 – N_5 Theorem **88–93**, 144

map

- composite **23**, 183
- continuous (*see* continuous map) **177**, **276**
- join-, meet-preserving **43**, 44
- order-preserving (*see* order-preserving map(s)) **23**
- partial (*see* partial map) **7**
- preserving existing joins, meets **46–47**, 161
- strict **177**, 184, 195
- total **7**

maximal element **16–17**, 52, 229, 231, 271

maximal filter (= ultrafilter) **233**

maximal ideal **233–234**, 239, 272

maximum (= greatest) element **16**

median inequality **58**, 85

- meet
 as binary operation 39–41
 properties of 34–36, 46
 in a subset 47
- meet-dense 53, 55, 70–72, 112, 167, 168, 242
- Meet-Infinite Distributive Law, (MID) 240
- meet-irreducible element 53, 112, 125–126
- meet-preserving map 43, 44
- minimal element 16, 271
- Mini-Max Theorem 58
- minimum (= least) element 16
- modular lattice 86–93
 characterizations of 86, 89, 105, 106, 107
- modular law 86
- monotone (= order-preserving) 23
- Monotonicity Rule 190
- \mathbb{N}_5 87, 106–107, 138, 143, 144
- natural numbers, \mathbb{N} , chain of 3, 36
- natural numbers under division, $(\mathbb{N}_0; \preccurlyeq)$
 4, 37, 41, 51, 54, 64, 87, 106, 108, 125, 127, 153, 155, 244
- neighbourhood filter 45
- non-comparable, \parallel 2
- non-principal ultrafilter 234, 245
- normal completion (= Dedekind–Mac-Neille completion) 166
- normal subgroups 4, 38, 49, 87, 131, 140
- number systems 3–4
- object (of a context) 66
- one 41
- open cover 277
- open set 4, 49, 275
- order (relation) 2
 coordinatewise (on product) 18
- inclusion 4
- induced (on subset) 3
- inherited from (subset) 3
- lexicographic 18, 27
- linear (= chain) 3
- linear extension of 32, 244
- pointwise 24
- strict 2, 25
- order-embedding 23, 43
- order filter (= up-set) 20
- order-homeomorphism 259
- order ideal (= down-set) 20
- order-isomorphism 3, 13–14, 23–24, 43, 44, 47
- order-preserving map(s) 23–24, 162, 178, 199, 200, 203
 fixpoints of 50, 187, 188, 189, 231
 function space of 24, 31, 128, 224
- ordered set(s) 2, 282
 chain conditions in 50–52
 connected 28
 examples 3–5
 flat 16, 175, 178, 203
 with no infinite chains 51, 52
 product of 18–19, 26, 27
 sums of (*see* sum) 17
- ordered Stone space (= Priestley space) 258
- ordered (topological) space 258
- ordinal 188, 198, 268
- partial order (= order) 2
- partially ordered set (= ordered set) 2
- partial map(s) 7–8, 149, 180–181, 184, 193, 202
- pentagon, \mathbb{N}_5 87
- pointwise order 24
- polar 67
- poset (= partially ordered set) 2
- postcondition 158

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Index

[More information](#)

296

post-fixpoint **182**
 powerset **4**, 234–235
 powerset algebra 95, 113, 114–115
 characterization of 115, 240, 284
 powerset lattice 19, 36, 45, 54, 73, 86,
 153
 precondition **158**
 pre-CPO (*see also* CPO) **175**, 193,
 224, 271
 predicate **4–5**, 158
 transformer 158, 164, 165
 pre-fixpoint **182**, 186
 pre-order (=quasi-order) **2**
 Priestley space **258–262**, 268–272
 Priestley's representation theorem 259
 prime filter **233–234**, 245
 prime ideal(s) **233–237**, 238–239, 272
 prime ideal space
 of Boolean algebra 247–248, 251
 of bounded distributive lattice **257**
 Prime Ideal Theorem 236
 principal congruence **138–139**, 142,
 143, 144
 principal down-set **20**, 161
 principal filter **45**
 principal ideal **45**, 64, 272
 product
 of chains 27, 127
 of complete lattices 63
 of CPOs 177, 194
 of domains 212, 216
 of downset-lattices 22, 119
 of \bigcap -structures 212
 of lattices **42–43**, 64, 88, 119, 144
 of ordered sets **18–19**, 64
 program 6, 157–158
 proper (filter, ideal) **45**, 232
 propositional calculus 96–98, 252–256
 p -space 272, 273

Index

pseudocomplement(ed) 128–129,
 262–263, 272
 quadrilateral **135**
 quadrilateral argument 135–137
 quadrilateral-closed **136**
 quasi-order **2**, 141, 173
 quotient lattice **133–134**

 rational numbers, \mathbb{Q} , chain of 4, 36,
 62, 168
 real numbers, \mathbb{R} , chain of 3, 36, 61,
 168
 recursion 10, 181, 200
 recursively defined domains 222–223
 refine(ment) 8, 163–165, 280
 reflexivity **2**
 representability (of ordered set) 261
 representation
 of Boolean algebras 114–115, 240,
 248, 256
 of distributive lattices 118, 243, 259
 of domains
 as information systems 209
 as \bigcap -structures 208
 as lattice of sets 118, 238–239, 243
 of lattices 174
 remarks on 112–113, 256
 ring 4, 49, 170, 231
 Boolean 109–110
 Rolling Rule 190

 scheduling 5–6
 Schröder–Bernstein Theorem 50, 64
 Scott topology 195
 self-map **182**
 semantic(s) 9–10
 denotational 10, 217–221
 for a deduction system 253
 domain 9–10
 Semi-inverse Rule 159

- semilattice
 algebraic (= domain) **202**, 204
 complete **201**
 join *170*
 separated sum, \oplus_{\perp} **176**, 177, 193, 212
 separately continuous map *194*
 series-parallel switching circuit 99
 simple lattice 140, *143*
 social choice function 5
 specification 163–165
 Square Rule *200*
 Stone's representation for Boolean algebras 248, 251, 256
 strict map **177**, 184, *195*
 strict order **2**, *25*
 strings, binary **7**, 9, 12, 15, 17, *26*, 176, 185, 202, 203, 208, 222
 subalgebra (of a Boolean algebra) **94**, 95, *111*
 subbasis (for a topology) 257, **276**, 278
 sub-CPO **176**
 subgroups of a group 4, 12
 as \cap -structure 49, 147, 155
 lattice of 38, 49, *56*, 63, *106*, *127*, 153, 156
 lattice of normal 38, 49, 87, 140
 unions of 148
 sublattice(s) **41–42**, 44, *58*, *60*, *61*, 88, 92, 282
 ordered set of 49, *148*
 sublattice-of-a-product technique 88, 92
 subspace(s) of a vector space 4, 49, 87, 148, 149, 153, 155
 substructure **210**, 211, 216, 222, *226*
 subsystem **210**
 sum 17–18, 176–177
 coalesced, \oplus_{\vee} **177**, 193, 212
 disjoint union, $\dot{\cup}$ 17, 27
 horizontal *84*
 of \cap -structures 212
 linear, \oplus **17**, *63*, *269*
 separated, \oplus_{\perp} **176**, 177, 193, 212
 vertical, $\overline{\oplus}$ *28*, *84*, *269*
 state 6, 157
 supremum (= least upper bound) **33**
 syntax 252, 253
 Szpilrajn's Theorem *244*

 tautology **252**
 terminating (program) **6**
 ternary discriminator *110*
 token **205**
 top (element), \top **15**, 16, 17, 34, 41, 202
 topological space 4, *30*, 45, **275**
 topology **247**, 275–279
 discrete 249, 256, **278**
 of prime ideal space 247, 257
 Scott *195*
 T_0 *30*
 topped \cap -structure **48–49**, 69, 138, 147–148, 160, 161, 204
 total map **7**
 total object **9**
 totally disconnected space **249**
 totally order-disconnected space **258**
 transitive closure *31*
 transitivity **2**
 tree *26*
 truth function **97**, 102–103
 truth table **97**
 Tychonoff's Theorem 237, *274*, 278

 ultrafilter(s) **233–234**
 existence of 237
 non-principal **234**, *245*
 on a set **234–235**, *245*
 universal algebra *110*, 131, 284

298

upper adjoint **156**, 161–162, 172upper bound(s) **33**, 156up-set **20**, 21–22, 31valuation **253**, 255, 268

vector space 51, 206, 231

weakest precondition **158**weakly atomic **242–243**, 246*Index*well-formed formula (wff) **96**

while-loop 8, 220

width 32, 127

witness 139

zero-dimensional space 267

zero element, 0 **41**Zorn's Lemma, (ZL) 16, 109, 188,
229, 230, 231–232, 235–237, 244,
245, 246, 284