

Contents

<i>Preface</i>	<i>page</i> xiii
<i>Useful Abbreviations</i>	xvii
1 Introduction	1
1.1 Modern Finance: A Brief History	1
1.2 Financial Engineering and Computation	1
1.3 Financial Markets	2
1.4 Computer Technology	4
2 Analysis of Algorithms	7
2.1 Complexity	7
2.2 Analysis of Algorithms	8
2.3 Description of Algorithms	9
2.4 Software Implementation	10
3 Basic Financial Mathematics	11
3.1 Time Value of Money	11
3.2 Annuities	14
3.3 Amortization	15
3.4 Yields	17
3.5 Bonds	24
4 Bond Price Volatility	32
4.1 Price Volatility	32
4.2 Duration	34
4.3 Convexity	41
5 Term Structure of Interest Rates	45
5.1 Introduction	45
5.2 Spot Rates	46
5.3 Extracting Spot Rates from Yield Curves	47
5.4 Static Spread	49
5.5 Spot Rate Curve and Yield Curve	50
5.6 Forward Rates	50
5.7 Term Structure Theories	56
5.8 Duration and Immunization Revisited	60

viii **Contents**

6 Fundamental Statistical Concepts	64
6.1 Basics	64
6.2 Regression	69
6.3 Correlation	71
6.4 Parameter Estimation	72
7 Option Basics	75
7.1 Introduction	75
7.2 Basics	76
7.3 Exchange-Traded Options	77
7.4 Basic Option Strategies	78
8 Arbitrage in Option Pricing	84
8.1 The Arbitrage Argument	84
8.2 Relative Option Prices	85
8.3 Put–Call Parity and Its Consequences	86
8.4 Early Exercise of American Options	88
8.5 Convexity of Option Prices	89
8.6 The Option Portfolio Property	90
9 Option Pricing Models	92
9.1 Introduction	92
9.2 The Binomial Option Pricing Model	93
9.3 The Black–Scholes Formula	104
9.4 Using the Black–Scholes Formula	111
9.5 American Puts on a Non-Dividend-Paying Stock	113
9.6 Options on a Stock that Pays Dividends	114
9.7 Traversing the Tree Diagonally	118
10 Sensitivity Analysis of Options	123
10.1 Sensitivity Measures (“The Greeks”)	123
10.2 Numerical Techniques	127
11 Extensions of Options Theory	131
11.1 Corporate Securities	131
11.2 Barrier Options	137
11.3 Interest Rate Caps and Floors	140
11.4 Stock Index Options	141
11.5 Foreign Exchange Options	143
11.6 Compound Options	147
11.7 Path-Dependent Derivatives	148
12 Forwards, Futures, Futures Options, Swaps	155
12.1 Introduction	155
12.2 Forward Contracts	156
12.3 Futures Contracts	161
12.4 Futures Options and Forward Options	168
12.5 Swaps	173

Contents ix

13 Stochastic Processes and Brownian Motion	177
13.1 Stochastic Processes	177
13.2 Martingales (“Fair Games”)	179
13.3 Brownian Motion	183
13.4 Brownian Bridge	188
14 Continuous-Time Financial Mathematics	190
14.1 Stochastic Integrals	190
14.2 Ito Processes	193
14.3 Applications	197
14.4 Financial Applications	201
15 Continuous-Time Derivatives Pricing	206
15.1 Partial Differential Equations	206
15.2 The Black–Scholes Differential Equation	207
15.3 Applications	211
15.4 General Derivatives Pricing	220
15.5 Stochastic Volatility	221
16 Hedging	224
16.1 Introduction	224
16.2 Hedging and Futures	224
16.3 Hedging and Options	228
17 Trees	234
17.1 Pricing Barrier Options with Combinatorial Methods	234
17.2 Trinomial Tree Algorithms	242
17.3 Pricing Multivariate Contingent Claims	245
18 Numerical Methods	249
18.1 Finite-Difference Methods	249
18.2 Monte Carlo Simulation	255
18.3 Quasi–Monte Carlo Methods	262
19 Matrix Computation	268
19.1 Fundamental Definitions and Results	268
19.2 Least-Squares Problems	273
19.3 Curve Fitting with Splines	278
20 Time Series Analysis	284
20.1 Introduction	284
20.2 Conditional Variance Models for Price Volatility	291
21 Interest Rate Derivative Securities	295
21.1 Interest Rate Futures and Forwards	295
21.2 Fixed-Income Options and Interest Rate Options	306
21.3 Options on Interest Rate Futures	310
21.4 Interest Rate Swaps	312

22	Term Structure Fitting	321
22.1	Introduction	321
22.2	Linear Interpolation	322
22.3	Ordinary Least Squares	323
22.4	Splines	325
22.5	The Nelson–Siegel Scheme	326
23	Introduction to Term Structure Modeling	328
23.1	Introduction	328
23.2	The Binomial Interest Rate Tree	329
23.3	Applications in Pricing and Hedging	337
23.4	Volatility Term Structures	343
24	Foundations of Term Structure Modeling	345
24.1	Terminology	345
24.2	Basic Relations	346
24.3	Risk-Neutral Pricing	348
24.4	The Term Structure Equation	350
24.5	Forward-Rate Process	353
24.6	The Binomial Model with Applications	353
24.7	Black–Scholes Models	359
25	Equilibrium Term Structure Models	361
25.1	The Vasicek Model	361
25.2	The Cox–Ingersoll–Ross Model	364
25.3	Miscellaneous Models	370
25.4	Model Calibration	371
25.5	One-Factor Short Rate Models	372
26	No-Arbitrage Term Structure Models	375
26.1	Introduction	375
26.2	The Ho–Lee Model	375
26.3	The Black–Derman–Toy Model	380
26.4	The Models According to Hull and White	384
26.5	The Heath–Jarrow–Morton Model	388
26.6	The Ritchken–Sankarasubramanian Model	395
27	Fixed-Income Securities	399
27.1	Introduction	399
27.2	Treasury, Agency, and Municipal Bonds	399
27.3	Corporate Bonds	401
27.4	Valuation Methodologies	406
27.5	Key Rate Durations	412
28	Introduction to Mortgage-Backed Securities	415
28.1	Introduction	415
28.2	Mortgage Banking	416
28.3	Agencies and Securitization	417
28.4	Mortgage-Backed Securities	419

Contents xi

28.5 Federal Agency Mortgage-Backed Securities Programs	422
28.6 Prepayments	423
29 Analysis of Mortgage-Backed Securities	427
29.1 Cash Flow Analysis	427
29.2 Collateral Prepayment Modeling	440
29.3 Duration and Convexity	444
29.4 Valuation Methodologies	446
30 Collateralized Mortgage Obligations	451
30.1 Introduction	451
30.2 Floating-Rate Tranches	452
30.3 PAC Bonds	453
30.4 TAC Bonds	457
30.5 CMO Strips	457
30.6 Residuals	457
31 Modern Portfolio Theory	458
31.1 Mean–Variance Analysis of Risk and Return	458
31.2 The Capital Asset Pricing Model	464
31.3 Factor Models	470
31.4 Value at Risk	474
32 Software	480
32.1 Web Programming	480
32.2 Use of <i>The Capitals</i> Software	480
32.3 Further Topics	482
33 Answers to Selected Exercises	484
<i>Bibliography</i>	553
<i>Glossary of Useful Notations</i>	585
<i>Index</i>	587