

Cambridge University Press  
0521779197 - Spinning Tops: A Course on Integrable Systems  
Michele Audin  
Frontmatter  
[More information](#)

---

CAMBRIDGE STUDIES IN  
ADVANCED MATHEMATICS: 51

EDITORIAL BOARD

W. FULTON, T. TOM DIECK, P. WALTERS

## SPINNING TOPS

*Already published*

- 1 W.M.L. Holcombe *Algebraic automata theory*
- 2 K. Petersen *Ergodic theory*
- 3 P.T. Johnstone *Stone spaces*
- 4 W.H. Schikhof *Ultrametric calculus*
- 5 J.-P. Kahane *Some random series of functions, 2nd edition*
- 6 H. Cohn *Introduction to the construction of class fields*
- 7 J. Lambek & P.J. Scott *Introduction to higher-order categorical logic*
- 8 H. Matsumura *Commutative ring theory*
- 9 C.B. Thomas *Characteristic classes and the cohomology of finite groups*
- 10 M. Aschbacher *Finite group theory*
- 11 J.L. Alperin *Local representation theory*
- 12 P. Koosis *The logarithmic integral I*
- 13 A. Pietsch *Eigenvalues and s-numbers*
- 14 S.J. Patterson *An introduction to the theory of the Riemann zeta-function*
- 15 H.J. Baues *Algebraic homotopy*
- 16 V.S. Varadarajan *Introduction to harmonic analysis on semisimple Lie groups*
- 17 W. Dicks & M. Dunwoody *Groups acting on graphs*
- 18 L.J. Corwin & F.P. Greenleaf *Representations of nilpotent Lie groups and their applications*
- 19 R. Fritsch & R. Piccinini *Cellular structures in topology*
- 20 H. Klingens *Introductory lectures on Siegel modular forms*
- 22 M.J. Collins *Representations and characters of finite groups*
- 24 H. Kunita *Stochastic flows and stochastic differential equations*
- 25 P. Wojtaszczyk *Banach spaces for analysts*
- 26 J.E. Gilbert & M.A.M. Murray *Clifford algebras and Dirac operators in harmonic analysis*
- 27 A. Frohlich & M.J. Taylor *Algebraic number theory*
- 28 K. Goebel & W.A. Kirk *Topics in metric fixed point theory*
- 29 J.E. Humphreys *Reflection groups and Coxeter groups*
- 30 D.J. Benson *Representations and cohomology I*
- 31 D.J. Benson *Representations and cohomology II*
- 32 C. Allday & V. Puppe *Cohomological methods in transformation groups*
- 33 C. Soulé et al *Lectures on Arakelov geometry*
- 34 A. Ambrosetti & G. Prodi *A primer of nonlinear analysis*
- 35 J. Palis & F. Takens *Hyperbolicity, stability and chaos at homoclinic bifurcations*
- 37 Y. Meyer *Wavelets and operators I*
- 38 C. Weibel *An introduction to homological algebra*
- 39 W. Bruns & J. Herzog *Cohen-Macaulay rings*
- 40 V. Snaith *Explicit Brauer induction*
- 41 G. Laumon *Cohomology of Drinfeld modular varieties I*
- 42 E.B. Davies *Spectral theory and differential operators*
- 43 J. Diestel, H. Jarchow & A. Tonge *Absolutely summing operators*
- 44 P. Mattila *Geometry of sets and measures in Euclidean spaces*
- 45 R. Pinsky *Positive harmonic functions and diffusion*
- 46 G. Tenenbaum *Introduction to analytic and probabilistic number theory*
- 47 C. Peskine *An algebraic introduction to complex projective geometry I*
- 48 Y. Meyer & R. Coifman *Wavelets and operators II*
- 49 R. Stanley *Enumerative combinatorics*
- 50 I. Porteous *Clifford algebras and the classical groups*
- 51 M. Audin *Spinning tops*
- 52 V. Jurdjevic *Geometric control theory*
- 53 H. Voelklein *Groups as Galois groups*
- 54 J. Le Potier *Lectures on vector bundles*
- 55 D. Bump *Automorphic forms*
- 56 G. Laumon *Cohomology of Drinfeld modular varieties II*
- 57 D. Clarke & B.A. Davey *Natural dualities for the working algebraist*
- 59 P. Taylor *Practical foundations of mathematics*
- 60 M. Brodmann & R. Sharp *Local cohomology*
- 64 J. Jost & X. Li-Jost *Calculus of variations*

Cambridge University Press  
0521779197 - Spinning Tops: A Course on Integrable Systems  
Michele Audin  
Frontmatter  
[More information](#)

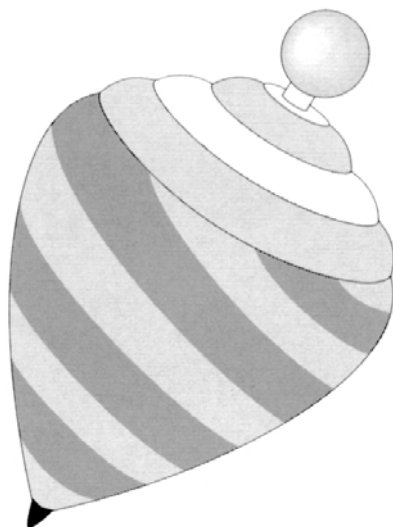
---

# Spinning tops

A Course on Integrable Systems

Michèle Audin

*Institute de Recherche Mathématique Avancée  
Université Louis Pasteur et CNRS*



**CAMBRIDGE**  
UNIVERSITY PRESS

Cambridge University Press  
0521779197 - Spinning Tops: A Course on Integrable Systems  
Michele Audin  
Frontmatter  
[More information](#)

---

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE  
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS

The Edinburgh Building, Cambridge CB2 2RU, UK <http://www.cup.cam.ac.uk>  
40 West 20th Street, New York, NY 10011-4211, USA <http://www.cup.org>  
10 Stamford Road, Oakleigh, Melbourne 3166, Australia  
Ruiz de Alarcón 13, 28014 Madrid, Spain

© Cambridge University Press 1996

This book is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without  
the written permission of Cambridge University Press.

First published 1996

First paperback edition 1999

*Typeface* Computer Modern 12/13pt *System* LaTeX [UPH]

*A catalogue record for this book is available from the British Library*

ISBN 0 521 56129 9 hardback

ISBN 0 521 77919 7 paperback

Transferred to digital printing 2004

## Contents

<b>Acknowledgements</b>	<b>vii</b>
<b>Introduction</b>	<b>1</b>
1 Completely integrable systems	2
2 The Arnold-Liouville theorem	5
3 A discourse on the method	6
4 About this book	11
5 Notation	13
<b>I The rigid body with a fixed point</b>	<b>15</b>
1 The equations	15
2 The question of integrability	18
3 The three-dimensional free rigid body and the Euler-Poinsot case	21
<b>II The symmetric spinning top</b>	<b>27</b>
1 Introduction to the symmetric spinning top	27
2 A Lax pair and what follows	32
<b>III The Kowalevski top</b>	<b>45</b>
1 Kowalevski's method	45
2 Lax pair and spectral curves	52
3 Lax pairs for generalised spinning tops and applications	60
<b>IV The free rigid body</b>	<b>65</b>
1 The Euler and Manakov equations	65
2 The dimension-3 free rigid body	66
3 Remarks on the dimension-4 rigid body	69
<b>V Non-compact levels: a Toda lattice</b>	<b>77</b>
1 The differential system and the spectral curve	77
2 The eigenvector mapping: the $n = 2$ case	83

Cambridge University Press  
0521779197 - Spinning Tops: A Course on Integrable Systems  
Michele Audin  
Frontmatter  
[More information](#)

---

vi

*Contents*

<b>Appendices</b>	<b>91</b>
Appendix 1 A Poisson structure on the dual of a Lie algebra	91
Appendix 2 $R$ -matrices and the “AKS theorem”	98
Appendix 3 The eigenvector mapping and linearising flows	104
Appendix 4 Complex curves, real curves and their Jacobians	113
Appendix 5 Prym varieties	124
<b>References</b>	<b>131</b>
<b>Index</b>	<b>137</b>

## Acknowledgements

Among the works in which I have learned the most, there are three papers that I wish to mention here. I think that I was very lucky to start my initiation to integrable systems by reading two very beautiful papers that are not among the most cited, those of Verdier [84] and Griffiths [36]. On the one hand, Verdier had the brilliant idea of illustrating his exposition of the work of Adler and van Moerbeke by the example of the symmetric top. On the other hand, Griffiths' main philosophical point was to look at a Lax equation, without specifying anything more. The advantage of this approach is that it allows one to start working without having first ingested loop algebras and the "AKS theorem". Moreover, in this sober presentation, the role played by the eigenvectors of the Lax matrices is amply brought to light. The algebraic geometry related to these eigenvectors is extremely well described in Reyman's paper [74] – which is the third paper to which I feel indebted.

The present text originates mainly from several talks<sup>1</sup> I have given on the examples here, in particular on the work I have done jointly with R. Silhol [15] and from a graduate course I taught in Strasbourg in 1992-93, jointly with J.-Y. Mérindol, on "Algebraic curves and integrable systems". The first version, *Toupies, un cours sur les systèmes intégrables* was written at the end of 1993. I am very pleased to acknowledge the influence of the very clear survey of Reyman & Semenov-Tian-Shanski [77] on the present version.

I have learned a lot in discussions with Jean-Yves Mérindol, Robert Silhol, and, especially, Alexei Reyman.

A lot of people have helped me to understand the material here, by criticism, questions or simply remarks during a talk, among which are, colleagues or students, Nicole Bopp, Nicole Desolneux-Moulis, Ljubomir Gavrilov, Sophie Gérardy, Bertrand Haas, Patrick Iglesias, Viatcheslav Kharlamov, Dimitri Markushevich, Nguyen Tien Dũng, Nitin Nitsure, Leonid Polterovich, Claude Sabbah, Jean-Marie Strelczyn, Jean Stutzmann and Pol Vanhaecke. I wish to thank them all.

The pictures in this book have been created by Raymond Seroul, whom I am also very pleased to thank.

Michèle Audin  
Strasbourg, January 30, 1996

---

<sup>1</sup>I thank all the people who have invited me to give or listened to me giving talks in Basel, Bochum, Bombay, Boston, Cambridge, Haifa, Lausanne, Luminy, Lyon, Montréal, Paris, Nantes, Strasbourg, Tel-Aviv.

Cambridge University Press  
0521779197 - Spinning Tops: A Course on Integrable Systems  
Michele Audin  
Frontmatter  
[More information](#)

---

viii

*Acknowledgements*

Last, but not least, I wish to thank the staff of Cambridge University Press, especially David Tranah, who kindly welcomed the book, and Susan Parkinson, who was very helpful in improving the English.

Michèle Audin  
Strasbourg, January 8, 1996

For the paperback edition, I have only corrected a few misprints.

Strasbourg, May 1, 1999