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978-0-521-77598-4 - Geometric Approaches to Differential Equations

Edited by Peter J. Vassiliou and Ian G. Lisle

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Edited by

Peter J. Vassiliou & Ian G. Lisle

*School of Mathematics and Statistics,*

*University of Canberra*



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## Preface

This book began as a series of lectures given by various speakers during the “*Miniworkshop in Geometry and Differential Equations*”, held at the University of Canberra in May, 1995. The purpose of the workshop was to bring together people interested in the interplay between geometry and differential equations; a subject that has experienced a revival, particularly over the last 30 years.

Rather than having many short and highly specialised contributions, it was decided that a smaller number of more broadly based expository talks would better serve the purposes of the workshop. To that end, a small number of speakers were invited to give two hour long “microcourses” each of which would delineate an interesting area of current research in a clear, concise and elementary way. The lectures were to be accessible to graduate students as well as to the more experienced who may be from other fields. At the conclusion of the workshop, it was realised that the lectures might be of interest to a wider audience and workshop speakers were invited to prepare their lectures for publication. In order to give the ultimate publication more currency and breadth, a number of other experts who did not speak at the workshop were invited to contribute to the book as well.

In preparing this volume, every effort has been made to avoid the production of a “conference proceedings”. Such collections, of course, have their place but it isn’t what we were trying to create. We would like to thank the authors of the individual chapters most sincerely for rising to this challenge. Apart from the aforementioned attempt to keep the exposition as crisp and accessible as possible, authors have tried, where appropriate, to cross reference thereby lending the collection greater coherence than is usual for a typical conference proceedings. This end is hopefully further assisted by the inclusion of an introductory chapter which, among other things, aims to set each chapter in context and to provide even more motivation for the ideas presented, especially historical motivation. In addition, the introductory chapter aims to give a very brief and elementary introduction to some of the basic ideas in the subject as a whole.

In this respect, two basic historical threads within the subject can be discerned. Firstly, an approach to differential equations arising from the classical geometry of surfaces in three-dimensional Euclidean space. This work is associated with nineteenth century geometers such as G. Darboux, L. Bianchi and A.V. Backlund. Secondly, the introduction of the notion of “transformation group” by S. Lie and F. Klein in the late nineteenth century and profoundly developed by É. Cartan, E. Vessiot and others in this century leading to the emergence of a “geometry of differential equations”. These two threads are far from independent. In the main, the contributions of the various authors represented in this volume reflect this historical development. A particular focus concerns the application of geometric ideas to the study of completely integrable systems both finite and infinite dimensional. Indeed, the study of infinite dimensional completely integrable systems is one of the major new ideas introduced into the subject in this century.

It should be emphasised that no attempt has been made to offer a comprehensive account of the field – this would be impossible within the compass of a single volume. Neither do we claim that it covers the most interesting aspects.

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We have instead tried to offer the reader an introduction to some of the “basic topics” such as *exterior differential systems* as well as a taste of some of the more “exotic” aspects such as *twistor theory*. In this way, the book provides some foundational material as well as perspectives within the subject. In addition, numerous references have been included allowing the reader to explore further.

This volume is offered in the belief that geometric approaches to differential equations represent a rich and rewarding field of mathematical endeavour with many beautiful ideas and intriguing open problems. We hope you find the presentation informative and stimulating.

Peter Vassiliou, Ian Lisle,  
Editors.

## Acknowledgements

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