

# Index to Volumes 1 and 2

- Absolute continuity:** II.9.  
**Absorbing state:** III.12.  
**Accessible stopping time:** VI.13-14.  
**Adapted process:** II.45.  
**Additive functional:** III.16; construction from  $\lambda$ -potential, III.16, I.17.  
**Affine group:** V.35.  
**Algebra:** II.1.  
**Almost surely:** II.14, III.9.  
**Announceable time:** VI.12.  
**Approximation to compensators:** VI.31.  
**Arcsine law:** II.34, III.24, V.53.  
**Arzelà–Ascoli theorem:** II.85.  
**Atlas:** V.34.  
**Atom:** II.88.  
**Awaiting the almost inevitable:** II.57.  
**Azéma’s martingale:** II.37.
- Backward equation:** I.4.  
**Barlow’s example:** V.41.  
**Basic integrands:** IV.5.  
**Bessel process:** IV.35, V.48; time reversal, III.49.  
**Bi-invariant metric:** V.35.  
**Birth process:** III.26, VI.14.  
**Blumenthal’s 0–1 Law:** for Brownian motion, I.12; for FD processes, III.9.  
**Bochner’s theorem:** I.24.  
**Bochner’s horizontal Laplacian:** V.34.  
**Borel–Cantelli Lemmas:** II.15.  
**Boundary points of one-dimensional diffusions:** V.47, V.51.  
**Boundary theory:** see Martin–Doob–Hunt theory.  
**Branch points:** definition, III.37; illustrative example, III.37; probabilistic significance III.41.  
**Brownian motion:** definition, I.1; on affine group, V.35; arcsine law, II.34, III.24, VI.53; Brownian bridge, I.25, II.91, IV.40; canonical, II.90; complex, see complex Brownian motion; and continuous martingales, I.2, IV.34; Dirichlet problem, I.22; elastic Brownian motion, III.24; of ellipses, V.36, V.37; energy of charge, I.22; excursion law, VI.50; exponential martingales, I.2, I.9; Feller Brownian motions, VI.57; on filtered probability space, I.2, II.72; first-passage distribution, I.9, I.14, III.10; Gaussian description, I.3; generator, I.4, III.6; Green function, I.22; iterated-logarithm laws, I.16; Kolmogorov’s

- backward and forward equations, I.4; Kolmogorov's test, I.13; Lévy's characterization, IV.33; on Lie groups, V.35; local time, I.5, I.14, III.16; on a manifold, V.30, V.31; martingales of, I.17; martingale characterisations, I.2; martingale representation, IV.36, IV.41; modulus of continuity, I.10; no-increase property, I.10; nowhere-differentiability, I.12; on the orthonormal frame bundle, V.30, V.33, V.34; path decomposition, VI.55; potential theory, I.22; quadratic variation, I.11, IV.2; Ray–Knight Theorems, VI.52; recurrence, I.3; reflecting Brownian motion, I.14, III.22, V.6; reflection principle, I.13; resolvent, III.3; rotational invariance, I.18; scaled Brownian excursion, IV.40; scaling, I.3; skew-product representation, V.31; on  $SO(3)$ , V.35; Skorokhod embedding, I.7, VI.51; slow points, I.10; strong Markov property, I.12; on a surface, V.4, V.31; time-reversed, II.38; transition density, I.4; unbounded variation, I.11, IV.2; wandering to infinity, I.18.
- Brownian sheet:** I.25.
- Burkholder–Davis–Gundy inequalities:** IV.42.
- Càdlàg maps:** see *R*-paths.
- Cameron–Martin–Girsanov change of measure:** IV.38–41, V.27.
- Canonical decomposition of a special semimartingale:** VI.40.
- Canonical process:** II.28, II.71, III.7.
- Capacity:** I.22.
- Carathéodory's Extension Theorem:** II.5.
- Carverhill's noisy North–South flow:** V.14.
- Cauchy law:** I.20.
- Cauchy process:** I.28, VI.2, VI.28.
- Change of time scale:** see time substitution.
- Chapman–Kolmogorov equations:** I.4, III.1.
- Characteristic exponent:** I.28.
- Characteristic operator:** III.12.
- Charge:** I.22, III.27; see also equilibrium charge.
- Chart:** V.34.
- Choquet capacitability theory:** III.76.
- Choquet representation of  $\lambda$ -excessive functions:** III.44.
- Choquet representation of 1-excessive probabilities:** III.38.
- Choquet's theorem on integral representations:** III.27.
- Christoffel symbols:** V.31, V.34.
- Cieselski–Taylor Theorem:** III.20, III.49.
- Clark's Theorem on Brownian martingale representation:** IV.41.
- Coffin state:** III.3.
- Comparison theorem:** V.43.
- Compensated Poisson process:** II.64.
- Compensator:** VI.29, VI.31; see also dual previsible projection.
- Completions:** II.75.
- Complex Brownian motion:** I.19; cone point, I.21; cut point, I.21; multiple points, I.21; Spitzer's theorem, I.20; windings of, I.20.
- Compound Poisson process:** I.28.
- Condition N:** III.54.
- Condition S:** III.55.
- Conditional expectations and probabilities:** II.41, II.44; regular, II.42, II.43.
- Conditional independence:** II.60.
- Cone point:** I.21.

- Conformal martingales: IV.34.  
 Connection: V.32, V.34.  
 Continuous Lévy processes, characterization of: I.28, III.14.  
 Continuous local martingale: pure local martingales, IV.34; quadratic-variation process IV.30; as time change of Brownian motion, IV.34.  
 Continuous mapping principle: II.84.  
 Continuous semimartingale: canonical decomposition, IV.30, VI.24; Itô's formula, IV.32; local time, IV.43.  
 Contraction resolvent: III.4; strongly continuous (SCCR), III.4.  
 Contraction semigroup: III.4; strongly continuous (SCCSG), III.4.  
 Control problems: see stochastic control.  
 Controlled variance problem: V.6, V.42.  
 Convergence of random variables: II.19.  
 Coupling inequality: V.54.  
 Coupling of one-dimensional diffusions: V.54.  
 Covariance of a diffusion: V.1.  
 Covariant differentiation: V.32, V.34.  
 Cumulative risk: II.64, VI.22.  
 Curvature: V.38.  
 Cut point: I.21.  
 Cylinder: II.25.
- d*-system: II.1.  
 Daniell–Kolmogorov Theorem: II.30, II.31; limitations of, II.34.  
 Début: of open set for R-process, II.74; of compact set of R-process, II.75; of progressive set, VI.3.  
 Début Theorem: II.76, III.9, VI.3.  
 De Finetti's Theorem: II.51.  
 Diffeomorphism: V.34.  
 Diffeomorphism Theorem: V.13.  
 Diffusion equation: I.4.  
 Diffusion: III.13, V.1, V.2; diffusion SDE, V.8; in one dimension, see one-dimensional diffusions; physical, I.23.  
 Directed set: II.80.  
 Dirichlet form: I.23; for Markov chains, III.59.  
 Dirichlet problem: I.22.  
 Distribution function: II.16.  
 Doléans' characterization of FV processes: VI.20, VI.25–27.  
 Doléans exponential: IV.19.  
 Doléans' proof of the Meyer Decomposition Theorem: VI.30.  
 Dominated-Convergence Theorem: II.8.  
 Donsker's Invariance Principle: I.8.  
 Doob decomposition of a submartingale: II.54.  
 Doob *h*-transform: III.29, III.45, IV.39.  
 Doss–Sussmann method: V.28.  
 Downcrossing Theorem (Lévy): I.14.  
 Drift of a diffusion: V.1.  
 Dual previsible projection: VI.1, VI.21, VI.23.  
 Dynkin's formula: III.10.  
 Dynkin's Local-Maximum Principle: III.13.

Dynkin's Isomorphism Theorem: I.27.

Dynkin's Maximum Principle: III.6.

Elastic boundary: III.24.

Elementary process: IV.6, IV.25.

Elworthy's example: V.13.

Empirical distribution: II.91.

Entrance laws: III.39.

Equilibrium charge and potential: I.22, III.48, VI.35.

Ergodic Theorem for one-dimensional diffusions: V.53.

Evanescence process: IV.13.

Excessive functions: III.27; representation, see Martin–Doob–Hunt theory; Riesz decomposition, III.27; uniformly  $\lambda$ -excessive, III.16.

Excessive measures: III.38.

Excursion intervals: VI.42.

Excursion law: VI.47, VI.50; for Brownian motion, VI.50, VI.55; for Markov chain, VI.43, VI.50.

Excursion theory: Ch.VI; censoring and reweighting of excursion laws, VI.58; characteristic measure, VI.47; excursion filtration, VI.59; for a finite Markov chain, VI.43; lifetime, VI.47; marked excursions, VI.49; for a Markov chain, III.57; Markovian character of excursion law, VI.48; path decomposition for Brownian excursions, VI.55; from a point which is not regular extremal, VI.50; Poisson point process, VI.43, VI.47; starred excursion, VI.49; by stochastic calculus, VI.59.

Excursion space: VI.43, VI.47.

Expectation: II.17.

Exponential map: V.34.

Exponential semimartingale: IV.19, IV.22, IV.37.

Extending the generator: III.4.

FD (Feller–Dynkin) diffusions: III.13, V.22; martingale representation, V.25.

FD processes: existence, III.7; strong Markov property, III.8, III.9.

FD semigroups: III.6.

FV: see finite-variation.

Fair stopping time: VI.12.

Fatou Lemma: II.8; for non-negative supermartingales, IV.14.

Feller Brownian motions: VI.57.

Feller property: III.6.

Feller–McKean chain: III.23, III.35.

Feynman–Kac formula: III.19; for Markov chains, IV.22.

Field: see algebra.

Fick's Law: I.23.

Filtering: VI.8; Bayesian approach, VI.10; change-detection filter, V.10, V.22; Kalman–Bucy filter, VI.9; robust filtering, VI.11.

Filtration: II.45, II.63; natural, II.45.

Finite-dimensional distributions: II.29, II.87.

Finite fuel control problem: V.7, V.15.

Finite-variation functions: II.13.

Finite-variation processes: IV.7; Doléans' characterization, VI.20.

First-approach times: II.74.

- First-entrance decomposition: III.52.  
 First-entrance times: see *début*.  
 First-hitting times: II.74.  
 Forward equation: I.4, I.23.  
 Freedman's interpretation of  $q_{bf}$ : III.57.  
 Fubini's Theorem: II.12.  
 Fundamental Theorem of Algebra: I.20.
- Gamma process: I.28.  
 Gaussian process: definition, I.3.  
 Gaussian random fields, isotropic: I.26.  
 Generator: see *infinitesimal generator*.  
 Geodesic: V.32, V.34.  
 Girsanov SDE: V.26.  
 Good  $\lambda$  inequality: IV.42.  
 Green function: I.22, III.27, III.30.  
 Gronwall's lemma: V.11.
- Harmonic function: III.31.  
 Hausdorff moment problem: III.28.  
 Hazard function: II.64.  
 Heat equation: I.4.  
 Helms–Johnson example: II.79, III.31, IV.14, VI.33.  
 Hermite polynomials: I.2.  
 Hewitt–Savage 0–1 law: II.51.  
 Hille–Yosida Theorem: III.5.  
 Hölder inequality: II.10.  
 Honest transition function: III.3.  
 Horizontal lift: V.34.  
 Horizontal vector field: V.34.  
 Hörmander's Theorem: V.38.  
 Hunt's Theorem: VI.35.  
 Hyperbolic plane: V.34, V.35, V.36.  
 Hyperboloid sheet: V.36.  
 Hypothèses droites: VI.46.
- Identical hitting-distributions: III.21.  
 Imbedding: V.34.  
 Independence: II.21, II.23.  
 Indistinguishable processes: II.36, IV.13.  
 Infinite divisibility: I.28.  
 Infinitesimal generator: III.2, III.4; **Brownian motion**, I.4, III.6; **one-dimensional diffusion**  
     V.47, V.50.  
 Inner measure: II.6.  
 Inner regularity of measures: II.80.  
 Innovations process: VI.8.  
 Instantaneous state of a Markov chain: III.51.  
 Integral: II.7.  
 Integrable-variation processes: IV.7.

- Integral curve: V.34.  
 Integration by parts: IV.2, VI.38; for continuous semimartingales, IV.32; for finite-variation processes, IV.18.  
 Integrators: IV.16.  
 Isometric imbedding: V.34.  
 Isotropic Gaussian random fields: I.26.  
 Itô's formula: IV.3, VI.39; for continuous semimartingales, IV.32; for convex functions IV.45, V.47; for FV processes, IV.18.  
 Itô integral: see stochastic integral.  
  
 Jensen's inequality: II.18, II.41, II.52.  
 Joint law: II.16.  
  
 Kalman–Bucy filter: VI.9.  
 Khasminskii's method for stability: V.37.  
 Khasminskii's test for explosion: V.52.  
 Killing: III.18  
 Kingman's Markov Characterization Theorem: III.58.  
 Knight's Theorem on continuous local martingales: IV.34.  
 Kolmogorov's backward equations: I.4.  
 Kolmogorov's forward equations: I.4.  
 Kolmogorov's lemma: I.25, II.85, IV.44.  
 Kolmogorov's test for Brownian motion: I.13.  
 Kolmogorov's 0–1 Law: II.50.  
 Krylov's example: V.29.  
 Kunita–Watanabe inequalities: IV.28.  
  
 L-process, L-path: IV, Introduction.  
 $\lambda$ -potential operator: see resolvent.  
 Laplace exponent: I.28, I.37.  
 Laplace–Beltrami operator: V.30, V.34.  
 Last-exit decomposition: III.56, VI.43, VI.48.  
 Last-exit distribution for Brownian motion: VI.35.  
 Law of the Iterated Logarithm: I.16.  
 Law of process: II.27.  
 Law of a random variable: II.16.  
 LCCB: locally compact Hausdorff space with countable base, II.6.  
 Lebesgue measure: II.5.  
 Lebesgue's thorn: III.9.  
 Left-invariant vector field: V.35.  
 Lévy Brownian motion: I.24.  
 Lévy's characterization of Brownian motion: I.2, IV.33.  
 Lévy–Doob 'Downward' Theorem: II.51.  
 Lévy kernels: III.57, IV.21.  
 Lévy–Hincin formula: I.28, VI.2.  
 Lévy measure: I.28, II.37, VI.2; Lévy system: VI.28.  
 Lévy process: I.28, I.29, I.30, II.37, VI.2.  
 Lévy's 'Upward' Theorem: II.50.  
 Lie algebra: V.35.  
 Lie bracket: V.34, 38.

- Lie group:** V.35.  
**Lifetime:** III.7.  
**Likelihood ratio:** II.79, IV.17; for Markov chains, IV.22.  
**Lipschitz square root:** V.12.  
**Local martingale:** IV.1, IV.14; on a manifold, V.30, V.33.  
**Local time:** for Brownian motion, I.5, I.14; for continuous semimartingales, IV.43-4; growth set, VI.45; for Lévy processes, I.30; Markovian local time, IV.43; as an occupation density, IV.45; for one-dimensional diffusions, V.49; at regular extreme point of a Ray process, VI.45; from upcrossings of Brownian motion, I.14, II.79.  
**Localization:** IV.9.  
**Locally bounded previsible process:** IV.10.  
**Lusin space:** II.31, II.82.  
**Lyapunov exponent:** V.37.
- Malliavin–Bismut integration-by-parts:** V.38.  
**Malliavin calculus:** V.38.  
**Manifold:** V.34.  
**Marked excursions:** VI.49.  
**Markov chains:** III.2; birth process, IV.26; Dirichlet form, III.59; Feller–McKean chain, III.23, III.35; Lévy's diagonal  $Q$ -matrix, III.35, IV.35; Martin boundary, III.48; martingale problem, IV.20-22; as Ray processes, III.50; stable and instantaneous states, III.51; see also  $Q$ -matrices, standard transition functions.  
**Markov  $p$ -function:** III.58.  
**Markov inequality:** II.18.  
**Markov processes:** III.1; see also FD processes, Ray processes.  
**Martin compactification:** III.28.  
**Martin kernel:** III.27; for Brownian motion in the unit ball, III.30.  
**Martin–Doob–Hunt theory:** for discrete-parameter chains, III.28, III.29, III.42; for Brownian motion, III.30, III.31.  
**Martingales:** definitions, II.46, II.63; for Brownian motion, I.17; convergence theorems, II.49, II.50, II.51, II.69; in  $L^p$ , II.53; regularity of paths, II.65, II.66, II.67; for FD processes, III.10; for Brownian motion, I.17.  
**Martingale inequalities:** Burkholder–Davis–Gundy inequality, IV.42; Doob's  $L^p$  inequality, II.52, II.70; Doob's submartingale inequality, II.52, II.54, II.70; Doob's Upcrossing Lemma, II.48.  
**Martingale problem:** V.19; existence of solutions, V.23; for Markov chains, IV.20; Markov property of solution, V.21; relationship to weak solutions of SDEs, V.19-20; Stroock–Varadhan Theorem, V.24; well-posed, V.19.  
**Martingale representation:** for Brownian motion, IV.36, 41; for FD diffusion, V.25; for Markov chains, IV.21.  
**Maximum-Modulus Theorem:** I.20.  
**Maximum Principle:** III.13.  
**McGill's Lemma:** VI.59.  
**Mean curvature:** V.4.  
**Measurable function:** II.2.  
**Measurable space:** II.1.  
**Measurable transition function:** III.3.  
**Measure space:** II.4.  
**Meyer decomposition:** III.17, VI.29, VI.32, VI.46.  
**Meyer's Previsibility Theorem:** VI.15.

- Minkowski inequality:** II.10.  
**Moderate function:** IV.42.  
**Modification:** II.36.  
**Monotone-Class Theorems:** II.3.  
**Monotone Convergence Theorem:** II.8.  
**Multiple points of Brownian motion:** I.21.  
**Multiplicative functional:** see PCHMF.
- Nagasawa's formula:** III.42, III.46.  
**Natural scale:** V.46.  
**Net:** II.80.  
**Normal coordinates:** V.34.  
**Normal transition function:** III.3.
- Observation process:** VI.8  
**Occupation density formula:** I.5, IV.45.  
**One-dimensional diffusions:** I.5, V.44–54, absorbing, inaccessible, reflecting end-points, V.47, V.51; exit, entrance boundary points, V.51; infinitesimal generator, V.47, V.50; natural scale, V.46; regular diffusion, V.45; resolvent, V.50, VI.54; scale function, V.46; speed measure, V.47; time substitution, V.47.  
**One-parameter subgroup:** V.35.  
**Optional processes,  $\sigma$ -algebra:** VI.4.  
**Optional projection:** VI.7.  
**Optional-Sampling Theorem:** II.59, II.77.  
**Optional Section Theorem:** VI.5.  
**Optional-Stopping Theorem:** II.57.  
**Optional time:** VI.4; see also stopping time.  
**Orthonormal frame bundle:** V.30, V.33, V.34.  
**Ornstein–Uhlenbeck process:** I.23, V. 5; spectral measure, I.24.  
**Outer measure:** I.6, II.35.
- $\pi$ -system:** II.1.  
**Parallel-displacement:** V.32, 34.  
**Parallel transport:** V.32.  
**Path decomposition:** III.49, VI.55.  
**Path regularization:** II.67, III.7.  
**Path-space:** II.28, V.8.  
**Pathwise-exact SDE:** see SDE.  
**Pathwise uniqueness:** V.9, V.17; Nakao theorem, V.41; Yamada–Watanabe Theorem, V.40.  
**PCHAF (perfect, continuous, homogeneous, additive functional):** III.16.  
**PCHMF (perfect, continuous, homogeneous, multiplicative functional):** III.18.  
**PFA theorem:** VI.12, VI.16.  
**Picard's Theorem:** I.20.  
**Polish space:** II.82.  
**Poisson measures:** II.37.  
**Potential (supermartingale):** II.59.  
**Potential theory:** I.22; see also Dirichlet problem, Martin–Doob–Hunt theory.  
**Poisson kernel for the half plane:** I.19.  
**Poisson kernel for the unit ball:** III.30.



- Poisson measure, process: II.37, VI.2.  
 Polish space: II.82; characterization of, II.82.  
 Probability triple: II.14.  
 Pre-Brownian motion: II.32, II.68.  
 Pre-Poisson set function: II.33.  
 Pre- $T$   $\sigma$ -algebra: II.58, II.73, VI.17.  
 Previsible: II.47; path functionals, V.8; processes,  $\sigma$ -algebra, IV.6; Section Theorem, VI.19; stopping time, VI.12.  
 Previsible projection: VI.19.  
 Product  $\sigma$ -algebras: II.11; product measures, II.12, II.22.  
 Progressive process,  $\sigma$ -algebra: II.73, VI.3.  
 Prohorov's Theorem: II.83.  
 Pseudo-Riemannian metric: V.36.  
 Pure local martingale: IV.34, IV.35, V.28.  
 Purely discontinuous martingales: IV.24.
- $Q$ -matrices: III.1; DK conditions, III.53; local-character condition, III.54; probabilistic significance, III.57, IV.21; of symmetric chains, III.59; of totally instantaneous chains, III.55.  
 Quadratic-covariation process: IV.26.  
 Quadratic variation: I.11.  
 Quadratic-variation process: IV.26, VI.36; for continuous local martingales, IV.30; previsible angle-bracket process, VI.34.  
 Quantum fluctuations: V.5.  
 Quasi-left-continuity: for FD processes, III.11; of filtrations, VI.18; for Ray processes III.41, III.50.  
 Quasimartingales: VI.41.  
 Quaternions: V.35.
- R-filtered space: II.67.  
 R-path, R-process: II.62, II.63, Introduction to Chapter IV.  
 R-regularisation: II.67.  
 R-supermartingale convergence theorem: II.69.  
 Radon–Nikodým Theorem: II.9.  
 Random field: I.24.  
 Random walk, Martin boundary of: III.28.  
 Ray–Knight compactification: III.35.  
 Ray–Knight Theorem on local times: VI.52.  
 Ray processes: III.36; application to chains, III.50.  
 Ray resolvent: III.34.  
 Ray's Theorem: III.36, III.38.  
 Reducing sequence: IV.11.  
 Reduction: IV.11, IV.29.  
 Reflecting Brownian motion: see Brownian motion.  
 Reflection principle: I.13.  
 Regular conditional probabilities: existence theorem, II.89; counterexample, II.43.  
 Regular class (D) submartingale: VI.31.  
 Regular diffusion: V.45.  
 Regular increasing process: VI.21.  
 Regular point: I.22.

- Regular function:** III.27.  
**Regularizable path:** II.62.  
**Resolvent:** III.2, III.3; of Brownian motion, III.3.  
**Resolvent equation:** III.2, III.3.  
**Reuter's Theorem on drifting Brownian motion:** IV.39.  
**Reversed Martingale Convergence Theorem:** II.51.  
**Riemannian connection:** V.32, V.34.  
**Riemannian manifold:** V.31.  
**Riemann mapping theorem:** I.19.  
**Riemannian metric:** V.34.  
**Riemannian structure induced by non-singular diffusion:** V.34.  
**Riesz decomposition of excessive functions:** III.27.  
**Riesz decomposition of a UI supermartingale:** II.59.  
**Riesz representation Theorem:** II.80, III.6.  
**Rolling without slipping:** V.33.
- $\sigma$ -additivity: II.4.  
 $\sigma$ -algebra: II.1; countably-generated, II.88.  
 $\sigma$ -field: see  $\sigma$ -algebra.  
**SDE:** of diffusion type, V.8; exact, V.9, V.17; Itô's Theorem on existence and uniqueness of solutions, V.11; links with martingale problem, V.19-20; with (locally) Lipschitz coefficients, V.11-13; Markov property of solutions, V.13; pathwise uniqueness, V.9, V.17; strong solution, V.10; Tanaka's SDE, V.16; time-reversal, V.13; Tsirel'son's SDE, V.18; uniqueness in law, V.16; weak solution, V.16.  
**Scale function:** V.28, V.46.  
**Scheffé's Lemma:** II.8.  
**Section Theorem:** II.76; Optional Section Theorem, VI.5; Previsible Section Theorem, VI.19.  
**Semimartingale:** IV.15; continuous, see continuous semimartingale; as integrator, IV.16; local time, IV.43; in a manifold, IV.15.  
**Signal process:** VI.8.  
**Skew product of Brownian motion:** IV.35.  
**Skorokhod embedding:** I.7, VI.51.  
**Skorokhod's equation:** V.6.  
**Special semimartingale:** VI.40.  
**Spectral measure of stationary Gaussian process:** I.24.  
**Spitzer–Rogozin identity for Lévy processes:** I.29.  
**Splitting time:** III.49.  
**Stable process:** I.28.  
**Stable subspace:** IV.24.  
**Standard process:** III.49.  
**'Standard' transition matrix function:** III.2.  
**Stochastic control, optimality principle:** V.15.  
**Stochastic development:** V.33.  
**Stochastic differential equation:** see SDE.  
**Stochastic differentials:** IV.32, V.1.  
**Stochastic flows:** V.13.  
**Stochastic integral:** IV.27, VI.36-38; Riemann-sum approximation, IV.47.  
**Stochastic partial differential equations:** VI.11.  
**Stochastic process:** II.27.

- Stone–Weierstrass Theorem: II.80.  
 Stopping time: II.56, II.73.  
 Strassen's Law: I.16; invariance principle, I.16.  
 Stratonovich calculus: IV.46; switch to Itô, V.30.  
 Strong Law of Large Numbers: II.51.  
 Strong Markov property: for Brownian motion, I.12; for FD processes, III.8, III.9; for Ray processes, III.40; under time reversal, III.47.  
 Strong reduction: VI.37.  
 Structural constants for Lie groups: V.35.  
 Structural equations: V.34.  
 Subadditive Ergodic Theorem: I.22.  
 Submanifold: V.34; regular submanifold, V.31.  
 Sub-Markov semigroup: III.3.  
 Submartingale: II.46, II.63.  
 Subordinator: I.28, II.37, VI.43.  
 Summation convention: V.1.  
 Superharmonic function: III.31.  
 Supermartingale: convergence theorem, II.49; definition, II.46, II.63; sup of a sequence of, II.78.  
 Supermedian function: III.34.  
 Symmetrisable  $Q$ -matrix, transition matrix function: III.59.
- Taboo probabilities: III.52.  
 Tanaka's formula: IV.43.  
 Tanaka's SDE: V.16.  
 Tangent bundle: V.34.  
 Tangent vector: V.30, V.34.  
 Tchebychev inequality: II.18.  
 Terminal time: III.18.  
 Tightness: II.83, II.85.  
 Time change: see time substitution.  
 Time reversal: III.42, III.47, III.49.  
 Time-reversed Brownian motion: II.38.  
 Time substitution: III.21, IV.30, V.26.  
 Torsion: V.34.  
 Totally inaccessible stopping time: V.21, VI.13–14.  
 Tower property of conditional expectation: II.41.  
 Transition function: III.1; measurable, III.3.  
 Trotter's Theorem: I.5.  
 Tsirel'son's SDE: V.18.
- Uniform asymptotic negligibility: I.28.  
 UI: see uniform integrability.  
 Uniform integrability: II.20, II.29, II.21 II.44.  
 Uniqueness in law: V.16.  
 Universal completion: III.9.  
 Upcrossings: II.62.  
 Upcrossing Lemma: II.48.  
 Usual augmentation: II.67, II.75.  
 Usual conditions: II.67, IV, Introduction.

Cambridge University Press

978-0-521-77594-6 - Diffusions, Markov Processes, and Martingales Volume 1: Foundations, 2nd Edition

L. C. G. Rogers and David Williams

Index

[More information](#)

386

## INDEX TO VOLUMES 1 AND 2

Volkonskii's formula: III.21.

Volkonskii–Šur–Meyer Theorem: III.16, III.17.

Volume element: V.34.

Von Mises distribution: IV.39.

Weak convergence: II.83; Prohorov's Theorem, II.83; in  $W$ , II.85; Skorokhod's interpretation, II.84, II.86.

Weak\* topology: II.80.

Weyl's Lemma: V.38.

Whittle's flypaper example: V.7, V.15.

Wiener–Hopf factorisation of Lévy processes: I.29 .

Wiener measure: I.6.

Wiener process: see Brownian motion.

Wiener's Theorem: I.6; proofs, I.6, II.71.

Yor's addition formula: IV.19.

Yor's Theorem on semimartingale local time: IV.44.

Zvonkin's observation: V.18, V.28.