Calculus

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Cambridge University Press
978-0-521-77541-0 - Calculus
Ken Binmore and Joan Davies
Frontmatter
More information

Dedications

To Jim Clunie — Ken Binmore

To my family
Roy, Elizabeth, Sarah and Marion — Joan Davies

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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of
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www.cambridge.org
Information on this title: www.cambridge.org/9780521775410

First published 2001
9th printing 2015

Printed in the United Kingdom by Bell and Bain Ltd, Glasgow

A catalogue record of this publication is available from the British Library

ISBN 978-0-521-77541-0 Paperback

Contents

◇ Revision material
♣ More challenging material

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Preface

Ancient accountants laid pebbles in columns on a sand tray to help them do their sums. It is thought that the impression left in the sand when a pebble is moved to another location is the origin of our symbol for zero.

The word calculus has the same source, since it means a pebble in Latin. Nowadays it means any systematic way of working out something mathematical. We still speak of a calculator when referring to the modern electronic equivalent of an ancient sandtray and pebbles. However, since Isaac Newton invented the differential and integral calculus, the word is seldom applied to anything else. Although there are pebbles on its cover, this book is therefore about differentiating and integrating.

Students who don't already know what derivatives and integrals are would be wise to start with another book. Our aim is to go beyond the first steps to discuss how calculus works when it is necessary to cope with several variables all at once.

We appreciate that some readers will be rusty on the basics, and others will be doubtful that they ever really understood what they can remember. We therefore go over the material on the calculus of one variable in a manner that we hope will offer some new insights even to those rare souls who feel confident of their mathematical prowess. However, we strongly recommend against using this material as a substitute for a first course in calculus. It goes too fast and offers too much detail to be useful for this purpose.

It should be emphasised that this is not a cook book containing a menu of formulas that students are expected to learn by rote in order to establish their erudition at examination time. We see no point in turning out students who can write down the formal derivation of the Slutsky equations, but have no idea what the mathematical manipulations they have learned to reproduce actually mean. When one teaches *how* things are done without explaining *why*, one does worse than fill the heads of the weaker students with mumbo-jumbo, one teaches the stronger students something very wrong – that mathematics is a list of theorems and proofs that have no practical relevance to anything real.

The attitude that mathematics is a menu of formulas that ordinary mortals can only admire from afar is very common among those who know no mathematics at all. Research mathematicians are often greeted with incredulity when they say what they do for a living. People think that inventing a new piece of mathematics would be like inventing a new commandment to be added to the ten that Moses brought down from the mountain. Such awe of mathematics creates a form of hysterical paralysis that must be overcome before a student can join the community of those of us who see mathematics as an ever changing box of tools that educated people can use to make sense of the world around them.

Within this community, a model is not expressed in mathematical form to invite the applause of those who are easily impressed, or to obfuscate the issues in order to immunise the model from criticisms by uninitiated outsiders. Instead the community we represent is always anxious to find the *simplest* possible model that captures how a particular aspect of some physical or social process works.

For us, mathematical sophistication is pointless unless it serves to demystify things that we would not otherwise be able to understand. We do not see mathematical modelling as some grandiose activity that can only be carried out by professors at the blackboard. Mathematical modelling is what *everybody* should do when seeking to make sense of a problem. Of course, beginners will only be able to construct very simple models – but a good teacher will only ask them to solve very simple problems.

Such an attitude to solving problems is not possible with students whose intellectual processes freeze over at the mention of an equation. The remedy for this species of mathematical paralysis lies in teaching that mathematics is something one *does* – not something that one just appreciates. Rather than offering them a cook book, one needs to teach students to put together simple recipes of their own. We need to build their confidence in their own ability to think coherent mathematical thoughts *all by themselves.*

Such confidence comes from involving students in the mathematics as it is developed, using the traditional method of demanding weekly answers to carefully chosen sets of problems. The problems must not be too hard – but nor must they be too easy. Nobody gets their confidence boosted by being asked to jump through hoops held too low. On the contrary, if we only ask students to solve problems that they can see are trivial, we merely confirm to them that their own low opinion of their mathematical ability is shared by their teacher. Some hoops have to be held high enough that students get to feel they have achieved something by jumping through them.

We feel particularly strongly on this latter point, having watched the confidence of our student intake gradually diminish over the years as cook book teaching has taken over our schools under the pretence that old fashioned rote learning is being replaced by progressive methods that emphasise the underlying concepts.

As this successor to the first version of *Calculus* shows, mathematicians don't mind adjusting the content of their courses as school syllabuses develop over time. We are even willing to welcome less mathematics being taught in school if this means that more children become numerate. But the kind of cook book teaching that leaves students helpless if a problem does not fit one of a small number of narrow categories seems to us inexcusable.

Our hostility to cook book teaching should not be taken to imply that this book is a rigorous work of mathematical analysis. It is a how to do it book, which contains no formal theorems at all. But we always explain why the methods work, because there is no way anybody can know *how* to use a method to tackle a new kind of problem unless they know *why* the method works.

Our approach to explaining why a method works is largely geometrical. To this end, the book contains an unusually large number of diagrams – even more than in the first version. The availability of colour and computer programs like *Mathematica* means that the diagrams are also better.

In addition, the already large number of examples and exercises has been augmented, with a view to increasing understanding by illustrating some of the things that can go wrong in cases that are usually passed over without comment.

Finally, we include examples of how the mathematics we are teaching gets used in practice. The mathematics is the same wherever it is applied, but the applications to economics on which we concentrate are often particularly instructive because of the need to be especially careful about what is being kept constant during a differentiation. Students whose prime interest is in the hard sciences may find that these applications are a lot more fun than examples from physics that they will have seen in some form before.

With our attitudes to the teaching of mathematics, it will come as no surprise that we view the exercises as an integral part of the text. There is no point in trying to read this or any other mathematics book without making a serious commitment to tackle as many of the exercises as time allows. Indeed, being able to solve a fair number of the exercises without assistance is the basic test of whether you understand the material. You may think you understand the concepts, but if you can't do any of the exercises, then you don't. You may think you *don't* understand the concepts, but if you can do most of the exercises, then you do. Either way, you need to attempt the exercises to find out where you stand.

A lot of work went into tailoring this successor to *Calculus* to the needs of today's students. Our readers will have to work equally hard to enjoy its benefits. We hope that they will also share our feeling of having done something genuinely worthwhile.

Ken Binmore
Joan Davies

Acknowledgements

Joan Davies is grateful to the following:

firstly to Michele Harvey for her support and formative suggestions during the early stages of the project,

to Gillian Colkin and especially to David Cartwright for their invaluable comments on drafts of the manuscript and for their help in testing exercises,

to Mark Baltovic for a student's view of the book and to Dean Ives and James Ward for useful comments,

to Roy Davies for help in coping with the idiosyncrasies of various software packages.

Ken Binmore gratefully acknowledges the support of the Leverhulme Foundation and the Economic and Social Research Council.