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Tim Bedford and Roger Cooke
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Preface

We have written this book for numerate readers who have taken a first university course in probability and statistics, and who are interested in mastering the conceptual and mathematical foundations of probabilistic risk analysis. It has been developed from course notes used at Delft University of Technology. An MSc course on risk analysis is given there to mathematicians and students from various engineering faculties. A selection of topics, depending on the specific interests of the students, is made from the chapters in the book. The mathematical background required varies from topic to topic, but all relevant probability and statistics are contained in Chapters 3 and 4.

Probabilistic risk analysis differs from other areas of applied science because it attempts to model events that (almost) never occur. When such an event does occur then the underlying systems and organizations are often changed so that the event cannot occur in the same way again. Because of this, the probabilistic risk analyst must have a strong conceptual and mathematical background.

The first chapter surveys the history of risk analysis applications. Chapter 2 explains why probability is used to model uncertainty and why we adopt a subjective definition of probability in spite of its limitations. Chapters 3 and 4 provide the technical background in probability and statistics that is used in the rest of the book. The remaining chapters are more-or-less technically independent of each other, except that Chapter 7 must follow Chapter 6, and 14 should follow 13. The final chapter gives a broad overview of risk measurement problems and looks into the future of risk analysis.

Almost all the chapters are concluded with exercises. The answers to these exercises are not given in the book, but bona fide teachers who are using the book in conjunction with their courses may contact David Tranah (dtranah@cambridge.org) and ask for a PDF file with the solutions to the problems.

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