

Topics in Finite and Discrete Mathematics

Written for a broad audience of students in mathematics, computer science, operations research, statistics, and engineering, this textbook presents a short, lively survey of several fascinating noncalculus topics in modern applied mathematics. Coverage includes probability, mathematical finance, graphs, linear programming, statistics, computer science algorithms, and groups. A key feature is the abundance of interesting examples not normally found in standard finite mathematics courses, such as options pricing and arbitrage, tournaments, and counting formulas.

The author assumes a level of mathematical sophistication at the beginning calculus level; that is, students should have had at least a course in precalculus, and the added sophistication attained from studying calculus would be useful.

Sheldon M. Ross is a professor in the Department of Industrial Engineering and Operations Research at the University of California at Berkeley. He received his Ph.D. in statistics at Stanford University in 1968 and has been at Berkeley ever since. He has published nearly 100 articles as well as a variety of textbooks in the areas of statistics and applied probability. He is the founding and continuing editor of the journal *Probability in the Engineering and Informational Sciences*, a fellow of the Institute of Mathematical Statistics, and a recipient of the Humboldt U.S. Senior Scientist Award.

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SHELDON M. ROSS
University of California at Berkeley



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Preface

This text surveys many of the topics taught in discrete and finite mathematics courses. The topics chosen are widely applied in present-day industry and, at the same time, are mathematically elegant. Chapter 1 begins with such preliminaries as sets, mathematical induction, functions, and the division algorithm of algebra. Chapters 2 and 3 present combinatorics and probability. Chapter 4 introduces the modern approach to finance; it presents the concept of arbitrage and the arbitrage theorem and then uses them to analyze the no-arbitrage costs of options. Chapters 5 and 6 deal with graphs and their many applications. Chapter 7 introduces linear programming. Among other applications, we use the duality theorem to derive the arbitrage theorem as well as the min-max theorem of game theory. Chapter 8 presents sorting and searching techniques that are useful in computer science. Chapter 9 introduces the subject matter of statistics, presenting both its descriptive and inferential side. Chapter 10 deals with groups and permutations.

This book can be used for a course in discrete mathematics, or for one in finite mathematics, or for any course dealing with non-calculus-based applied mathematics. Calculus itself is not required, and a pre-calculus course should suffice as a prerequisite; the added mathematical sophistication attained from studying calculus would be useful. The text evolved from a seminar designed to introduce first-year undergraduates with a strong quantitative bent to the possibilities inherent in mathematics. Consequently, a key feature of the course, as well as of the text, is the emphasis on interesting examples.