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0521772001 - Physiology by Numbers: An Encouragement to Quantitative Thinking,
Second Edition

Richard F. Burton

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Thinking quantitatively about physiology is something many students find difficult. However, it is fundamentally important to a proper understanding of many of the concepts involved. In this enlarged second edition of his popular textbook, Richard Burton gives the reader the opportunity to develop a feel for values such as ion concentrations, lung and fluid volumes, blood pressures, etc. through the use of calculations that require little more than simple arithmetic for their solution. Much guidance is given on how to avoid errors and the usefulness of approximation and 'back-of-envelope sums'. Energy metabolism, nerve and muscle, blood and the cardiovascular system, respiration, renal function, body fluids and acid–base balance are all covered, making this book essential reading for students (and teachers) of physiology everywhere, both those who shy away from numbers and those who revel in them.

RICHARD F BURTON is Senior Lecturer in the Institute of Biomedical and Life Sciences at the University of Glasgow, Scotland, UK. *Biology by Numbers* by the same author is also published by Cambridge University Press.

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An Encouragement to Quantitative Thinking

SECOND EDITION

RICHARD F. BURTON

University of Glasgow, Glasgow



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PREFACE TO THE SECOND EDITION

When I started to write the first edition of this book, I particularly had in mind readers somewhat like myself, not necessarily skilled in mathematics, but interested in a quantitative approach and appreciative of simple calculations that throw light on physiology. In the end I also wrote, as I explain more fully in my original Preface, for those many students who are ill at ease with applied arithmetic. I confess now that, until I had the subsequent experience of teaching a course in 'quantitative physiology', I was not fully aware of the huge problems so many present-day students have with this, for so many are reluctant to reveal them. Part of my response to this revelation was *Biology by Numbers* (Burton 1998), a book which develops various simple ideas in quantitative thinking while illustrating them with biological examples. In revising *Physiology by Numbers*, I have retained the systematic approach of the first edition, but have tried to make it more accessible to the number-shy student. This has entailed, amongst other things, considerable expansion of the first chapter and the writing of a new chapter to follow it. In particular, I have emphasized the value of including units at all stages of a calculation, both to aid reasoning and to avoid mistakes. I should like to think that the only prior mathematics required by the reader is simple arithmetic, plus enough algebra to understand and manipulate simple equations. Logarithms and exponents appear occasionally, but guidance on these is given in Appendix B. Again I thank Dr J. D. Morrison for commenting on parts of the manuscript.

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PREFACE TO THE FIRST EDITION

Let us therefore take it that in a man the amount of blood pushed forward in the individual heartbeats is half an ounce, or three drams, or one dram, this being hindered by valves from re-entering the heart. In half an hour the heart makes more than a thousand beats, indeed in some people and on occasion, two, three or four thousand. Now multiply the drams and you will see that in one half hour a thousand times three drams or two drams, or five hundred ounces, or else some such similar quantity of blood, is transfused through the heart into the arteries – always a greater quantity than is to be found in the whole of the body.

But indeed, if even the smallest amounts of blood pass through the lungs and heart, far more is distributed to the arteries and whole body than can possibly be supplied by the ingestion of food, or generally, unless it returns around a circuit.

William Harvey, De Motu Cordis, 1628 (from the Latin)

In more familiar terms, if the heart beats, say, 70 times a minute, ejecting 70 ml of blood into the aorta each time, then more fluid is put out in half an hour (147l) than is either ingested in that time or contained in the whole of the body. Therefore the blood must circulate. Thus may the simplest calculation bring understanding. I invite the reader to join me in putting two and two together likewise, hoping that my collection of simple calculations will also bring enlightenment.

Although my main aim is to share some insights into physiology obtained through calculation, I have written also for those many students who seem to rest just on the wrong side of an educational threshold – knowing calculators and calculus, but shy of arithmetic; drilled in accuracy and unable to approximate; unsure what to make of all those physiological concentrations, volumes and pressures that are as meaningless as telephone numbers until toyed with,

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combined, or re-expressed. As ‘an encouragement to quantitative thinking’ I also offer, for those ill at ease with arithmetic, guidance on how to cheat at it, cut corners, and not be too concerned for spurious accuracy. Harvey’s calculations illustrate very well that a correct conclusion may be reached in spite of considerable inaccuracy. In his case it was the estimate of cardiac output that was wrong; it is now known to be about two and a half ounces per beat. (There are eight drams to the ounce.)

Much of physiology requires precise computation, so I must not appear too much the champion of error and slapdash. There are, however, situations where even the roughest of calculations may suffice. Consider the generalization (see Section 3.10) that small mammals have higher metabolic rates per unit body mass than do large ones: taking the case of a hypothetical mouse with the relative metabolic rate of a steer, Max Kleiber (1961) calculated that to keep in heat balance in an environment at 3°C its surface covering, if like that of the steer, would need to be at least 20 cm thick! Arguments of this kind appear below. Be warned, however, that improbable answers are not always wrong, as exemplified by Rudolph Heidenhain’s calculation of glomerular filtration rate in 1883 (Section 6.5).

The book is based on an assortment of questions to be answered by calculation, together with some introductory and background information and comment on the answers. (The answers are given at the back of the book, together with notes and references.) Such a quantitative approach is more suited to some areas of physiology than to others and the coverage of the book naturally reflects this. The book is neither a general guide to basic physiology, nor a collection of brain-teasers or practice calculations. It rarely strays from shopkeeper’s arithmetic and it is not a primer of mathematical physiology or of mathematics for physiologists. Rather, it is supplementary thinking for those who have done, or are still doing, at least an elementary course in Physiology. I have learned much myself from the calculations and hope that other mature students may learn from them too.

Except where otherwise stated, the calculations refer to the human body. This is often taken as that of the physiologist’s standard 70-kg adult man and many ‘standard’, textbook quantities are used here. This is partly to reinforce them in the reader’s memory and build bridges from one to another, but such standard values are also a natural starting point for back-of-envelope calculations. Indeed, if there is any virtue to learning these quantities, it is surely helpful to exercise them and put them to use. Thus may one hope to bring life to numbers – and not just numbers to Life.

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The link between the learning and usefulness of quantities may be viewed the other way round. A student may memorize many of them for examinations and for future clinical application, but which are most profitably learnt for the better understanding of the body? Those with most uses? In how many elementary contexts is it helpful to know the concentration of sodium in extracellular fluid? Is that of magnesium as useful? Or manganese? Such questions of priority are as important for those inclined to overtax their memories unreasonably as for the lazy. This book may help both with these decisions and with the learning process itself.

Partly for reasons just indicated, many of my 'numbers' come from textbooks. Working on this text, however, I came increasingly to realize how hard it may be to find what one supposes to be well-known quantities. Textbooks have less and less room for these as other knowledge accumulates, of course, and there is a laudable tendency for concepts to displace quantitative detail. So do not disdain the older books! Diem (1962) has been a very useful source. Sometimes when a quantitative argument seems frustrated through lack of reliable figures, the solution is to turn it on its head, depart from the natural sequence of calculations, and defer the uncertainties to the end. The reader may spot where I was able to rescue items that way. Only once have I resorted to original data; I am very grateful to Dr Andrew Chappell for dissecting and weighing human muscles for me (Section 9.4).

I thank also all my colleagues who read portions of draft manuscript or otherwise gave of their time and wisdom, and in particular Dr F. L. Burton, Professor J. V. G. A. Durnin, Dr M. Holmes, Dr O. Holmes, Professor S. Jennett, Dr D. J. Miller, Dr J. D. Morrison, Dr G. L. Smith and Dr N. C. Spurway.

R. F. Burton

HOW TO USE THIS BOOK

Understand the objectives as stated in the Preface to the first edition; be clear what the book is – and what it is not. Since it is written for readers of widely varying physiological knowledge and numerical skills, read selectively. Chapters 3–9, and their individual subsections, need not be read in sequence.

Although the book is primarily about physiology, another objective is to encourage and facilitate quantitative thinking in that area. If such thinking does not come easily to you, pay particular attention to Chapter 1. Note too that the calculations are not intended to be challenging. Indeed, many are designed for easy mental, or back-of-envelope, arithmetic – and help is always to hand at the back of the book, in ‘Notes and Answers’. The notes often deal with points considered either too elementary or too specialized for the main text.

Consider carefully the validity of all assumptions and simplifications. If you try guessing answers before calculating them, you are more likely to be rewarded, in some cases, with a surprise.

If you are unfamiliar with exponents or logarithms, note the guidance given in Appendix B. The mathematics of exponential time courses are not dealt with in a single place, but most of the essentials are covered incidentally (see pages 13–16, 80–81, 98–100, 210–211, 219).