

During 1996–97, the Mathematical Sciences Research Institute held a full academic year program on combinatorics, with special emphasis on the connections with other branches of mathematics, such as algebraic geometry, topology, commutative algebra, representation theory, and convex geometry.

The rich combinatorial problems arising from the study of various algebraic structures are the subject of this book, which represents work done or presented at seminars during the program. It contains contributions on matroid bundles, combinatorial representation theory, lattice points in polyhedra, bilinear forms, combinatorial differential topology and geometry, Macdonald polynomials and geometry, enumeration of matchings, the generalized Baues problem, and Littlewood–Richardson semigroups.

These expository articles, written by some of the most respected researchers in the field, present the state of the art to researchers and graduate students in combinatorics as well as algebra, geometry, and topology.





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New Perspectives in Algebraic Combinatorics

Edited by

Louis J. Billera

 $Cornell\ University$

Anders Björner

Royal Institute of Technology, Stockholm

Curtis Greene

Haverford College

Rodica E. Simion

George Washington University

Richard P. Stanley

Massachusetts Institute of Technology





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Preface

Algebraic combinatorics involves the use of techniques from algebra, topology and geometry in the solution of combinatorial problems, or the use of combinatorial methods to attack problems in these areas. Problems amenable to the methods of algebraic combinatorics arise in these or other areas of mathematics, or from diverse parts of applied mathematics. Because of this interplay with many fields of mathematics, algebraic combinatorics is an area in which a wide variety of ideas and methods come together.

During 1996–97 MSRI held a full academic year program on Combinatorics, with special emphasis on algebraic combinatorics and its connections with other branches of mathematics, such as algebraic geometry, topology, commutative algebra, representation theory, and convex geometry. Different periods of the year were devoted to research in enumeration, extremal questions, geometric combinatorics and representation theory.

The rich combinatorial problems arising from the study of these various areas are the subject of this book, which represents work done or presented at seminars during the program. It contains contributions on matroid bundles, combinatorial representation theory, lattice points in polyhedra, bilinear forms, combinatorial differential topology and geometry, Macdonald polynomials and geometry, enumeration of matchings, the generalized Baues problem, and Littlewood-Richardson semigroups. These expository articles, written by some of the most respected researchers in the field, present the state-of-the-art to graduate students and researchers in combinatorics as well as algebra, geometry, and topology.

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