

An Introduction to Radiative Transfer

Methods and applications in astrophysics

Astrophysicists have developed several very different methodologies for solving the radiative transfer equation. *An Introduction to Radiative Transfer* presents these techniques as applied to stellar atmospheres, planetary nebulae, supernovae and other objects with similar geometrical and physical conditions. Accurate methods, fast methods, probabilistic methods and approximate methods are all explained, including the latest and most advanced techniques. The book includes the different methods used for computing line profiles, polarization due to resonance line scattering, polarization in magnetic media and similar phenomena. Exercises at the end of each chapter enable these methods to be put into practice, and enhance understanding of the subject. This textbook will be of great value to graduates, postgraduates and researchers in astrophysics.

ANNAMANENI PERAIAH obtained his doctorate in radiative transfer from Oxford University. He was formerly a Senior Professor at the Indian Institute of Astrophysics, Bangalore, India. He has held positions in India, Canada, Germany and the Netherlands. His research interests include developing solutions to the radiative transfer equation in stellar atmospheres and line formation in expanding atmospheres with different physical and geometrical conditions.

An Introduction to Radiative Transfer

Methods and applications in astrophysics

Annamaneni Peraiah



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Preface

Astrophysicists analyse the light coming from stellar atmosphere-like objects with widely differing physical conditions using the solution of the equation of radiative transfer as a tool. A method of obtaining the solution of the transfer equation developed to suit a given physical condition need not necessarily be useful in a situation with different physical conditions. Furthermore, each individual has his/her preferences to a particular type of methodology. These factors necessitated the development of several widely differing methods of solving the transfer equation.

In the second half of the twentieth century several books were written on the subject of radiative transfer: one each by Chandrasekhar, Kourganoff and Sobolev, two books by Mihalas, two by Kalkofen and more recently two books by Sen and Wilson. These books, which describe the developments of the transfer theory, will remain milestones. They will be of great value to the researcher in this field. A beginner needs to understand the basic concepts and the initial development of the subject to proceed to use the latest advances. It is felt that it is necessary to have a book on radiative transfer which presents a comprehensive view of the subject as applied in astrophysics or more particularly in stellar atmospheres and objects with similar geometrical and physical conditions. This book serves such a purpose. Several methods are presented in the book so that the students of radiative transfer can familiarise themselves with the techniques old and new.

It became a daunting task to include all the existing techniques in the book as there is a restriction on its size. This resulted in leaving out a few methods that are of equal interest as those that appear in the book. I apologize to the authors of these methods in advance. The subject matter of the book assumes of the student a knowledge of basic mathematics and physics at the undergraduate level. This book

is intended to be included in the advanced course work of undergraduate students, and the course work of graduate students. Several exercises have been included at the end of each chapter for practising the concepts described in the chapter. These problems are straightforward and can be solved by direct application of the theory. Some of them involve just supplying the intermediate steps in the derivations of the chapter.

The material in the book is largely drawn from the books mentioned earlier and from various other references cited at the end of each chapter. If there are any errors these are mine and I shall be grateful if these are brought to my attention. Any suggestions for improvements and corrections are welcome.

It is a pleasure to thank Dr W. Kalkofen for a brief discussion on the subject matter of the book. I am grateful to Professor K. K. Sen for not only giving a few tips on writing books but also for going through the first draft and pointing out several typographical errors and adding a few conceptual points. This book would not have been possible without the active help from Mr Baba Anthony Varghese who very patiently typed the text. His phenomenal computer expertise enabled the book to rapidly and easily take its present form. It is pleasure to thank him for all this. I thank Drs A. Vagiswari and Christina Louis for their magnanimous and kind help in securing me any reference that I needed. Further, I thank Mr M. Srinivasa Rao, Mr S. Muthukrishnan and Mrs Pramila Kaveriappa for helping me in various ways during the writing of the book.

There is one person whose memory always lingers on in my mind – that of Professor M. K. Vainu Bappu. From him I have learnt several aspects not only of science but also of life. I fondly cherish the memory of my association with him.

I am grateful to my wife Jayalakshmi and my children Rajani (Vaidhyanathan), Chandra (Edith) and Usha (Madhusudan) – spouses in brackets – for the love and affection shown to me.

Finally I thank the staff of Cambridge University Press who have been connected with the publication of the book, especially Dr Simon Mitton and Miss Jacqueline Garget for clearing my doubts from time to time and Ms Maureen Storey, who very patiently went through the manuscript and suggested several corrections.

Bangalore
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Annamaneni Peraiah