HYDRODYNAMIC INSTABILITIES

The instability of fluid flows is a key topic in classical fluid mechanics because it has huge repercussions for applied disciplines such as chemical engineering, hydraulics, aeronautics, and geophysics. This modern introduction is written for any student, researcher, or practitioner working in the area, for whom an understanding of hydrodynamic instabilities is essential.

Based on a decade's experience of teaching postgraduate students in fluid dynamics, this book brings the subject to life by emphasizing the physical mechanisms involved. The theory of dynamical systems provides the basic structure of the exposition, together with asymptotic methods. Wherever possible, Charru discusses the phenomena in terms of characteristic scales and dimensional analysis. The book includes numerous experimental studies, with references to videos and multimedia material, as well as over 150 exercises which introduce the reader to new problems.

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Cover photo: manifestation of the Kelvin–Helmholtz instability between two atmospheric layers with different velocity, above Laramie, Wyoming, USA. © Brooks Martner, NOAA, Environmental Technology Laboratory.

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To my father

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Foreword

Hydrodynamic instabilities occupy a special position in fluid mechanics. Since the time of Osborne Reynolds and G. I. Taylor, it has been known that the transition from laminar flow to turbulence is due to the instability of the laminar state to certain classes of perturbations, both infinitesimal and of finite amplitude. This paradigm was first displayed in a masterful way in the studies of G. I. Taylor on the instability of Couette flow generated by the differential rotation of two coaxial cylinders. From then on, the theory of hydrodynamical instability has formed a part of the arsenal of techniques available to the researcher in fluid mechanics for studying transitions in a wide variety of flows in mechanical engineering, chemical engineering, aerodynamics, and in natural phenomena (climatology, meteorology, and geophysics).

The literature on this subject is so vast that very few researchers have attempted to write a pedagogical text which describes the major developments in the field. Owing to the enormity of the task, there is a temptation to cover a large number of physical situations at the risk of repetition and of wearying the reader with just a series of methodological approaches. François Charru has managed to avoid this hazard and has risen to the challenge. With this book he fills the gap between the classical texts of Chandrasekhar and Drazin and Reid, and the more recent book of Schmid and Henningson.

Classical instability theory essentially deals with quasi-parallel or parallel shear flows such as mixing layers, jets, wakes, Poiseuille flow in a channel, boundary-layer flow, and so on. Such configurations are the focus of the books by Drazin and Reid and by Schmid and Henningson, and they are of particular interest to researchers of a "mechanical" bent. François Charru has chosen to follow an approach which synthesizes these classical situations, while carefully avoiding the treatment of the critical layer in all these states (*cf.* Drazin and Reid), known to be a source of difficulties. He opens perspectives on the most recent developments in the study of transition in shear flows, for example, the

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phenomena of nonmodal growth, "by-pass" transition, and convective or absolute instabilities.

During the last 25 years, our understanding of instabilities has evolved significantly under the combined influence of physicists and mathematicians specializing in nonlinear phenomena and the theory of dynamical systems. In particular, the influx of physicists studying macroscopic phenomena into the ideal playing field of fluid mechanics has led to a profound renewal of our discipline. It is therefore important to introduce the student to the essential concepts without getting mired in technical details. Here also François Charru has succeeded in attractively presenting the most important ideas which have become standard tools of any specialist in instabilities. The fundamentals of the spatio-temporal dynamics of dissipative structures are also introduced in such a way that they can be further pursued using the works by Manneville and by Godrèche and Manneville. By now, many authors have shown that the study of model amplitude equations of the Ginzburg–Landau or the nonlinear Schrödinger type can reveal the nature of the weakly nonlinear dynamics near the instability threshold. It is also known that these toy models remain relevant far from threshold, in a regime which is essentially supercritical, for extracting the generic characteristics of instabilities such as the Benjamin–Feir or the Eckhaus instability, and for testing methodological tools such as the phase dynamics of dissipative textures.

Finally, I would like to draw the attention of the reader to the two chapters at the heart of this book devoted to the surface instabilities of films and the instabilities governing the formation of ripples and dunes. The author has in his own research contributed very significantly in these two areas, and he discusses these topics from his own point of view. Here it should be emphasized that the law governing the behavior of granular media has not yet been set in stone. Experimental observation of the instabilities occurring in such complex media will make it possible to confirm or reject any particular behavior postulated in the theoretical models. In his discussion of the trends in current research, the author helps the student appreciate the vitality and current relevance of this field.

The resolutely "physical" approach adopted by the author is an essential characteristic of this work. For each instability class François Charru presents, by means of dimensional analysis and elegant physical arguments, the mechanism responsible for amplifying the perturbations. This type of reasoning and evaluation of orders of magnitude is carried out before any systematic mathematical treatment is undertaken. The author also makes a special effort to present examples of laboratory experiments which allow confirmation of the theoretical results. This type of exposition familiarizes the student with both the theoretical and the experimental aspects of the research process.

The reader is therefore encouraged to adopt the concepts and methods presented in this book, and to become immersed in the author's approach in which an important role is played by intuition and physical understanding of the phenomena. He or she will then have an ideal jumping-off point for the discovery of more beautiful hydrodynamic instabilities.

Patrick Huerre

Preface

La raison a tant de formes, que nous ne sçavons à laquelle nous prendre ;
l'experience n'en a pas moins.
Montaigne, *Essais*, Livre 3, 13.

Reason has so many forms that we know not to which to take;
experience has no fewer.
Montaigne, *Essays, XXI. Of Experience*, tr. Charles Cotton.

For over a century now, the field of hydrodynamic instabilities has been constantly and abundantly renewed, and enriched by a fruitful dialogue with other fields of physics: phase transitions, nonlinear optics and chemistry, plasma physics, astrophysics and geophysics. Observation and analysis have been stimulated by new experimental techniques and numerical simulations, as well as by the development and adaptation of new concepts, in particular, those related to asymptotic analysis and the theory of nonlinear dynamical systems. Ever since the observations of Osborne Reynolds in 1883, there has been unflagging interest in the fundamental problem of the transition to turbulence. This topic has been given new life by concepts such as convective instabilities, transient growth, and by the recognized importance of unstable nonlinear solutions. New problems have emerged, such as flows involving fluid–structure interactions, granular flows, and flows of complex fluids – non-Newtonian and biological fluids, suspensions of particles, bubbly flows – where constitutive laws play an essential role.

This book has been written over the course of 10 years of teaching postgraduate students in fluid dynamics at the University of Toulouse. It is intended for any student, researcher, or engineer already conversant with basic hydrodynamics, and interested in the questions listed above. As far as possible, the phenomena are discussed in terms of characteristic scales and dimensional analysis in order to elucidate the underlying physical mechanisms or, in Feynman's words, the "qualitative

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content of the equations."[1] This approach blends well with the theory of dynamical systems, bifurcations, and symmetry breaking, which provides the basic structure for our exposition. Asymptotic methods also play an important role. Their power and success, sometimes well beyond the region where they should apply, are always amazing. Numerous experimental studies are discussed in detail in order to confirm the theoretical developments or, conversely, to display their shortcomings.

The first part of the book (Chapters 1 to 7) is essentially devoted to linear stability, while the second part (Chapters 8 to 11) deals with nonlinear aspects. The first chapter presents an introduction to the theory of dynamical systems, including numerous examples of "simple" hydrodynamical problems; there we also introduce the idea of transient growth. In the second chapter we present the general methodology of a stability analysis: perturbation of a base state, linearization, normal modes, and the dispersion relation; we illustrate these techniques by the classical problems of thermal, capillary, and gravitational instabilities.

Chapters 3 to 5 give the classical analyses of instabilities in open flows (the instability criterion, convective and absolute instabilities, temporal and spatial growth) and then instabilities of parallel flows: inviscid instabilities are discussed in Chapter 4 (the Rayleigh inflection-point theorem, the Kelvin–Helmholtz instability) and viscous ones in Chapter 5 (the Orr–Sommerfeld equations, Tollmien–Schlichting waves in boundary layers and Poiseuille flow).

In Chapters 6 and 7 we discuss problems that are barely touched on in the classical textbooks: (i) instabilities at small Reynolds number, which arise, in particular, in the presence of deformable interfaces (liquid films falling down an inclined plane or sheared by another fluid, flows of superposed layers), and (ii) the instabilities of granular beds flowing down a slope (avalanches) or eroded by a flow, which give rise to the growth of surface waves, ripples, and dunes. Chapter 7 also presents a (very sketchy) introduction to the physics of granular media, and illustrates how stability is strongly affected by the modeling, in particular, by the introduction of relaxation phenomena.

Chapters 8 to 10 present an introduction to weakly nonlinear dynamics, where the method of multiple scales plays an essential role. In Chapter 8 we discuss nonlinear oscillators and the "canonical" nonlinear effects such as amplitude saturation and frequency correction (the Landau equation), and frequency locking for forced oscillators. Next, the analysis of systems that are governed by partial differential equations but which are spatially confined reveals how the dynamics near the instability threshold is controlled by the weakly unstable "master mode." Chapter 9 is devoted to dispersive nonlinear waves, the canonical model of which is the Stokes gravity wave, and to the Benjamin–Feir instability. The latter is analyzed from two

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[1] *The Feynman Lectures on Physics*, Volume 2. Electromagnetism, §41.6, Addison Wesley Longman, 1970.

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viewpoints: in terms of resonances with side-band wave numbers (described by amplitude equations), and in terms of modulations of the envelope of the wave packet (described by the nonlinear Schrödinger equation). Chapter 10 presents the dynamics of dissipative systems in the supercritical and subcritical cases, typically Rayleigh–Bénard convection or Couette–Taylor flow for the former, and Poiseuille and boundary-layer flow for the latter. Then for the supercritical case we analyze the secondary instabilities of the Eckhaus type, or the Benjamin–Feir–Eckhaus type in the case of waves. Finally, we study the situation where, owing to a particular invariance (Galilean, or an invariance associated with a conservation law), the mode of zero wave number is marginal, leading to a nontrivial coupling of two nearly neutral phase modes.

The final chapter is devoted to a more mathematical exposition of bifurcation theory (the central manifold theorem, normal forms, bifurcations of codimension greater than unity), providing a systematic treatment of the ideas introduced in the earlier chapters. Finally, in the Appendix we derive the depth-averaged Saint-Venant equations, which offer a simple framework for analyzing problems where the gradients in the flow direction are small.

A list of videos and multimedia material that illustrate the phenomena covered by this book is given below. At the end of each chapter we suggest some exercises which often serve as introductions to new problems. Finally, dispersed throughout the text are 11 short biographies of some of the most important figures in the study of instabilities: Bagnold, Chandrasekhar, Helmholtz, Kapitza, Kelvin, Landau, Poincaré, Rayleigh, Reynolds, Stokes, and Taylor.

This work does not pretend to be exhaustive: choices had to be made from among the enormous diversity of advances in the subject. Important topics like wakes and vortices, and ideas like transient growth and global modes, are only briefly addressed or completely omitted, but some general bibliographical information is provided.*

The author would like to thank his colleagues and friends who, in numerous conversations, have contributed to enriching this work, in particular, B. Andreotti, A. Bottaro, G. Casalis, G. Iooss, E. J. Hinch, P. Luchini, P. Brancher, and J. Magnaudet.

Finally, I would like to warmly thank Bud Homsy for his kind and fruitful help. His thorough reading of the initial translation of the French book, the fine questions he raised, the detailed modifications he proposed, and his contribution to the final writing significantly improved this book.

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Video resources

There are increasing numbers of videos and multimedia material available that illustrate various instability phenomena covered in this book. Below is a list of resources that were available at the time this book went to press. It is easily anticipated that the number of resources will only increase, so the reader is encouraged to search the popular websites: efluids.com, YouTube.com, etc.

The NSF/NCFMF Series. Between the years 1961–1969, the US National Science Foundation supported the production of a series of movies by the National Committee for Fluid Mechanics Films under the leadership of the late Asher Schapiro. These can be accessed at the MIT website: http://web.mit.edu/hml/ncfmf.html as part of MIT's iFluids program, requiring only the use of RealPlayer software. Of particular interest are the films on *Flow Instabilities*, *Turbulence*, and *Boundary Layers*, although others will also contain material related to hydrodynamic instabilities.

Multimedia Fluid Mechanics. Between the years 1998–2008 the US National Science Foundation supported the production of the DVD, *Multimedia Fluid Mechanics* (G. M. Homsy *et al.*, Cambridge University Press, 2007). It is available from the publisher at reasonable cost and contains over 800 movies and animations illustrating fluid phenomena. These media pieces are displayed on explanatory pages and also collected in a "Video Library." Below we list those relevant to hydrodynamic stability by their number in the Video Library. Links from there will take the interested viewer to the page where further explanation may be found.

- 4, 702, Steady and spiral Couette–Taylor instabilities
- 84, Boundary layer flow showing both laminar and turbulent boundary layers
- 172, 173, 174, 180, 455, 643, 645, Pipe Flow – a series of experiments on transition in pipe flow conducted on Osborne Reynolds' original apparatus
- 392, 393, In- and out-of-phase vortex shedding from pairs of aligned cylinders
- 636, Wake instabilities and vortex shedding from a cylinder

- 484, 638, Tollmien–Schlichting waves, spanwise instabilities and turbulent spots in a boundary layer
- 3489, 3490, Plateau–Rayleigh instability of a water jet
- 3487, Instability of a soap film
- 3489, Viscous Rayleigh–Taylor instability of a thin liquid film
- 3584, Gravity–capillary waves
- 3587, 4305, Plateau–Rayleigh instability of an annular film on a wire
- 3518, 4053, Formation and instability of a soap film catenoid
- 3599, Famous film by Breidenthal showing the instability and “mixing transition” in a free shear layer
- 3600, Turbulent streaks in a boundary layer
- 3696, 3697, 3698, 3699, 3703, 3704, Taylor vortices in Couette–Taylor flow exhibiting turbulent bursts, drifting, and intermittency
- 3805, Famous film by Brown and Roshko showing instability of the free shear layer
- 3832, 3838, The rivulet instability in climbing Marangoni films
- 3915, Hexagonal Marangoni–Bénard convection cells
- 3936, Onset of Rayleigh–Bénard convection
- 3976, 3978, Simulation of turbulent Rayleigh–Taylor instability
- 4013, 4015, Dewetting instability of thin films of nonwetting liquids
- 4412, Simulation of thermal convection in the Sun
- 4548, 4818, Kelvin–Helmholtz instability of a jet
- 5396, Turbulent mixing in Rayleigh–Taylor instability
- 5104, Axial instability of a vortex pair

efluids Media Gallery. There are many static images and movies of fluid phenomena posted in the Media Gallery at www.efluids.com (then link to “galleries”). Of particular interest are:

1. Breakup of a liquid jet:
   http://media.efluids.com/galleries/all?medium=717
2. Viscous fingering of an elastic liquid:
   http://web.mit.edu/nnf/people/jbico/exp89.mov
3. Fractal viscous fingering:
   http://media.efluids.com/galleries/all?medium=581
4. von Kármán vortex street:
   http://media.efluids.com/galleries/all?medium=578
5. Piano waves in a vibrated granular media:
   http://media.efluids.com/galleries/all?medium=507
6. Shear layer instabilities:
   http://media.efluids.com/galleries/youtube?medium=579

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7. Flow around a cylinder:
   http://media.efluids.com/galleries/instability?medium=417
8. Wake of a low aspect ratio pitching plate:
   http://media.efluids.com/galleries/instability?medium=332
9. Helical instability in a compressible jet:
   http://media.efluids.com/galleries/instability?medium=424
10. Ferrofluid instability:
    http://media.efluids.com/galleries/instability?medium=3
11. Collapse of a soap bubble:
    http://media.efluids.com/galleries/all?medium=723
12. Richtmeyer–Meshkov instability:
    http://media.efluids.com/galleries/all?medium=707

AIP Gallery of Fluid Motion. The Division of Fluid Dynamics of the American Physical Society (APS/DFD) chooses winners of the annual Gallery of Fluid Motion. The winning images and videos are available at: http://scitation.aip.org/pof/gallery/archives.jsp. Of particular interest are:

1. Viscous fingering in microgravity:
   http://scitation.aip.org/pof/gallery/video/2001/915109phfVFfilm.mov
2. Faraday jets and sand:
   http://scitation.aip.org/pof/gallery/2003-sandtke.jsp#video
3. Turbulent Rayleigh–Taylor instability:
   http://scitation.aip.org/pof/gallery/video/2005/908509phfenhanced.mov
4. Rayleigh–Taylor instability of an evaporating liquid:
   http://scitation.aip.org/pof/gallery/2009-Dehaeck.jsp#video
5. Helical instability of a rotating jet:
   http://scitation.aip.org/pof/gallery/2008-Weidman.jsp#video
6. Water bell and sheet instabilities:
   http://scitation.aip.org/pof/gallery/2006-Bush.jsp#video
7. Breakdown modes of swirling jets:
   http://scitation.aip.org/pof/gallery/2002-ruith.jsp#video