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978-0-521-76840-5 — Ergodic Control of Diffusion Processes
Ari Arapostathis , Vivek S. Borkar , Mrinal K. Ghosh
Index
More Information

Index of symbols

$\mathcal{A}$: infinitesimal generator, 10
B_R: open ball of radius R, 60
$\mathcal{B}(E)$: set of bounded measurable maps on E, xv
$\mathcal{B}_0$, 10
$\mathscr{B}(E)$: Borel σ-field of E, xv
$C_0^2(\mathbb{R}^d)$, 82
$C_{\rm pol}$: continuous functions of polynomial growth, 260
$\mathscr{C}$: a countable subset of $C_c^2(\mathbb{R}^d)$ which is dense in $C_0^2(\mathbb{R}^d)$, 82
$\mathfrak{C}$: class of running cost functions, 107
$\mathcal{D}([0,\infty);S)$: space of right-continuous functions with left limits from $[0,\infty)$ to S, 3
$\mathscr{D}(\mathcal{A})$: domain of the operator $\mathcal{A}$, 10
$\mathfrak{d}$: Euclidean distance, 119, 155, 207
$\mathfrak{d}_s$: Skorohod metric, 3
$\mathbb{E}$: expectation, xv
$\mathcal{G}$: set of ergodic occupation measures, 87
$\bar{\mathcal{G}}$, 93
$\mathcal{G}_{\mathcal{U}}$, 89
$\mathcal{H}_T^p$, 32
$\mathscr{H}$: set of invariant probability measures, 87
$\mathscr{H}_{\mathcal{U}}$, 89
$\mathbb{I}_A$: indicator function of the set A, xv
$\mathcal{I}$, 22
$\mathfrak{I}$, 17, 19, 25
J_α^U: α-discounted cost, 83
$\mathcal{K}(\varrho)$, 114
$\mathcal{L}^u$: controlled extended generator, 41
$\mathscr{L}(X)$: law of the random variable X, 15
$\mathfrak{L}(\gamma)$, $\mathfrak{L}_0(\gamma)$, 303
$\langle\!\langle \mathscr{L}(X,U) \rangle\!\rangle$: marginal class, 236
$\mathcal{M}_T(E)$: set of invariant probability measures of T, 16
$\mathcal{M}_v$, 72
$\mathfrak{M}_s(E)$: space of finite signed measures on E, 10
$\mathfrak{M}(\mathbb{R}^d)$: space of finite nonnegative measures on $\mathbb{R}^d$, 282
$\mathbb{N}$: set of natural numbers, xv
$\mathscr{O}$, $\mathfrak{o}$, 124
$\mathscr{O}(t)$, 103
$\mathfrak{p}$: Poisson random measure, 195
$\mathbb{P}$: probability, xv
$\mathscr{P}(S)$: set of probability measures on S, 1
$\mathfrak{Q}(\gamma)$: class of quasilinear operators, 107
Q_λ: resolvent, 310
$\mathbb{R}$: set of real numbers, xv
$\mathbb{R}_+$: set of nonnegative real numbers, xv
$\bar{\mathbb{R}}^d$: one-point compactification of $\mathbb{R}^d$, 93
$\mathcal{R}_\lambda$: resolvent, 11
$\mathscr{R}(\mathcal{A})$: range of the operator $\mathcal{A}$, 220
$\mathbb{S}$: discrete set, 194
T_t^v, 80
$\mathbb{U}$: action space, 30
$\mathfrak{U}$: set of admissible controls, 31
$\mathfrak{U}_{\rm sd}$: set of precise stationary Markov controls, 47
$\mathfrak{U}_{\rm sm}$: set of stationary Markov controls, 47
$\mathfrak{U}_{\rm ssd}$: set of precise stable stationary Markov controls, 60
$\mathfrak{U}_{\rm ssm}$: set of stable stationary Markov controls, 60
$\mathfrak{U}_{\rm ws}$: set of wide-sense admissible controls, 282
$\tilde{\mathfrak{U}}_{\rm ssd}$, 121
$\tilde{\mathfrak{U}}_{\rm ssm}$, 134
$\mathscr{U}$: space of control trajectories, 50
$\mathscr{U}_s$, 55
$\mathcal{V}$: Lyapunov function, 61
$\mathscr{V}$: class of Lyapunov functions, 137
V_α: α-discounted value function, 82
$\mathscr{W}^{k,p}$: Sobolev space, xvi
$\mathbb{Z}$: set of integers, xv
$\Subset$, xv
Γ_ν, 233
Γ'_ν, 236
$\Gamma(\kappa, R_0)$, 115
δ_q: Dirac measure at q, 47
$\bar{\zeta}_{\nu,t}^U$: mean empirical measure, 62
ζ_t^U: process of empirical measures, 102

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Index
More Information

η_v: invariant probability measure under control v, 87
θ_t: shift operator, 25
$\xi^U_{\nu,\alpha}$: α-discounted occupation measure, 62
ρ_w: Wasserstein metric, 72
$\tau(D)$: first exit time from a domain D, 59
τ_R: first exit time from B_R, 60
$\check{\tau}_R$: first hitting time of B_R, 60

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978-0-521-76840-5 — Ergodic Control of Diffusion Processes
Ari Arapostathis , Vivek S. Borkar , Mrinal K. Ghosh
Index
More Information

Subject index

T-concatenation, 239
$\mathcal{L}$-harmonic function, 304
$\mathcal{L}$-superharmonic function, 304
α-discounted
  control problem, 83
  cost, 82
  value function, 82
λ-resolvent, 307
φ-irreducible, 290

a priori estimate, 304
accessible atom, 290
adjoint equation, 81
Alexandroff–Bakelman–Pucci theorem, 304
  elliptic systems, 201
Arrow–Barankin–Blackwell theorem, 159
associated system, 163
asymptotic flatness, 255, 258
averaged system, 164

barycenter, 18
Beneš measurable selection theorem, 46
Birkhoff ergodic theorem, 17
bounded in probability, 63
bounded pointwise convergence, 220
bp-closed, 220

Cameron–Martin–Girsanov theorem, 43
Caratheodory's theorem, 160
chattering lemma, 47
Chentsov's criterion, 4
Choquet
  –Meyer theorem, 19
  simplex, 19
  theorem, 18
Clark–Davis pathwise filter, 284
continuity
  Kolmogorov criterion, 3
  modulus of, 3
  stochastic, 3
control
  admissible
    wide-sense, 282
  admissible, 31
    strict-sense, 280
  average-cost optimal, 134
  feedback, 40
  Markov, 45
  precise, 47
  relaxed, 47
  stable, 60
  stationary Markov, 45
  uniformly stable, 97
controlled infinitesimal generator, 56
controlled nonlinear filter, 281
cost potential, 120

decomposition
  Doeblin, 27
  ergodic, 19
diffusion
  asymptotically flat, 254
  controlled, 30
    periodic coefficients, 172
    reflected, 173
    switching, 194
  degenerate, 33
  matrix, 27
  nondegenerate, 33
  process, 27
diffusion matrix, 31
Dirichlet problem, 305
discount factor, 83, 107
discounted cost, 107
  optimal, 110
dissipative operator, 221
domain in $\mathbb{R}^d$, 59
drift, 27
drift vector, 31
duality gap, 248
dynamical system
  deterministic, 4
  stochastic, 4

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CAMBRIDGE
Cambridge University Press & Assessment
978-0-521-76840-5 — Ergodic Control of Diffusion Processes
Ari Arapostathis , Vivek S. Borkar , Mrinal K. Ghosh
Index
More Information

elliptic
  equations, 304
  operators, 303
    quasilinear, 107
  regularity, 306
  systems, 199
  uniformly, 33
embedding, 307
  compact, 307
  constant, 307
  continuous, 307
empirical measure, 102
  mean, 62, 288
energy-efficient scheduling, 174
equation
  Chapman–Kolmogorov, 5
  Duncan-Mortensen-Zakai, 283
  Fokker–Planck, 28
  Fujisaki–Kallianpur–Kunita, 284
  Hamilton–Jacobi–Bellman, 86
  Kolmogorov, 28
  Poisson, 305, 307
ergodic
  decomposition, 19
  occupation measure, 87, 223
    ε-optimal, 101
    optimal, 101
  theorem, 17
  theory
    of Markov processes, 15
    of random processes, 21
ergodic control problem
  almost sure formulation, 84
  average formulation, 84
  with constraints, 154
exterior cone condition, 67
extremal law, 236

Feynman–Kac formula, 285
filtration, 2
  natural, 2
  right-continuous, 2
first exit time, 59
fully coupled
  controlled switching diffusion, 200
  elliptic operator, 200

generator
  controlled extended, 42
  extended, 15
Green's function, 309

Harnack's inequality, 304
  elliptic systems, 202
Hille–Yosida theorem, 12
hitting time, 60

ideal point, 160
inf-compact, 61
invariant
  T-, 16
  measure, 16
    infinitesimally, 81
invariant probability measure, 22
Ionescu-Tulcea theorem, 7
Itô
  –Krylov formula, 45
  formula, 33
  stochastic integral, 29

Kallianpur–Striebel formula, 283
Kolmogorov
  backward equation, 28
  continuity criterion, 3
  extension theorem, 2
  forward equation, 28
Krylov–Bogolioubov theorem, 16

Laplace operator, 14
Lebesgue point, 236
linear growth condition, 31
linear program, 247
  dual, 247
  infinite dimensional, 247
  primal, 247
local time, 173

map
  lower semicontinuous, 102
marginal class, 236
Markov
  family, 9
  process, 5
  property, 5
    strong, 5
Markov selection, 249
martingale dynamic programming principle, 272
  separated control, 294
martingale problem, 15
  controlled, 220
    relaxed, 220
  well-posed, 15
martingales
  strong law of large numbers, 105
maximum principle
  strong, 304
  weak, 304
measurable
  progressively, 2
metric
  bounded Lipschitz, 73
  Skorohod, 3
  Wasserstein, 72
minorization condition, 289
multi-objective problems, 154, 159

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CAMBRIDGE
Cambridge University Press & Assessment
978-0-521-76840-5 — Ergodic Control of Diffusion Processes
Ari Arapostathis , Vivek S. Borkar , Mrinal K. Ghosh
Index
More Information

near-monotone cost, 101
  strong, 255
non-anticipative, 30
nondegeneracy
  partial, 266
occupation measure
  α-discounted, 62
  ergodic, 87
  infinitesimal, 63
operator
  dissipative, 12
  domain of, 10
Pareto point, 159
partial nondegeneracy, 254
partial observations, 280
Polish space, 1
probability space, 1
  complete, 1
process
  adapted, 2
  diffusion, 27
  Feller, 5
    strong, 5
  innovations, 283
  Markov, 5
    strong, 5
  modification of, 2
  null recurrent, 60
  observations, 280
  positive recurrent, 60
  r.c.l.l., 3
  recurrent, 60
  stationary, 21
  transient, 60
  version of, 2
Prohorov's theorem, 1
pseudo-atom, 290
recurrence time, 60
resolvent
  equation, 11
  of a semigroup, 11
running cost, 82
saddle point, 159
Schauder fixed point theorem, 16
Scheffé's theorem, 58
semigroup, 9
  infinitesimal generator of, 10
  resolvent, 11
  strongly continuous, 10
separated control problem, 281
shadow minimum, 160
singular perturbations, 160
Skorohod topology, 4
Sobolev spaces, xvi
sojourn times, 13
solution
  strong, 40
  weak, 40
split chain, 289
stationary
  distribution, 22
  process, 21
stochastic differential equation, 29
stopping time, 2
storage function, 143
strong formulation, 40
strong solution
  unique, 40
support of a measure, 262
tail σ-field, 25
transformation
  ergodic, 16
  measure preserving, 15, 16
transition probability
  density, 28
  kernel, 5
Tsirelsen's counterexample, 42
uniform
  positive recurrence, 96
  stability, 97
unnormalized conditional law, 283
Urysohn's theorem, 288
utility function, 159
value function, 144
vanishing discount, 82, 107, 113
viscosity solution, 259
weak solution
  unique, 41

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