

Contents

<i>Preface to the second edition</i>	<i>page</i> xxi
<i>Preface to the first edition</i>	xxv
Part I Introduction to gravity and supergravity	1
1 Differential geometry	3
1.1 World tensors	3
1.2 Affinely connected spacetimes	5
1.3 Metric spaces	9
1.3.1 Riemann–Cartan spacetime U_d	11
1.3.2 Einstein–Weyl spacetime EW_d	14
1.3.3 Riemann spacetime V_d	14
1.4 Tangent space	16
1.4.1 Weitzenböck spacetime A_d	20
1.5 Killing vectors	22
1.6 Duality operations	23
1.7 Differential forms and integration	25
1.8 Extrinsic geometry	27
2 Symmetries and Noether’s theorems	29
2.1 Equations of motion	29
2.2 Noether’s theorems	30
2.3 Conserved charges	34
2.4 The special-relativistic energy–momentum tensor	35
2.4.1 Conservation of angular momentum	36
2.4.2 Dilatations	40
2.4.3 Rosenfeld’s energy–momentum tensor	42

x	<i>Contents</i>	
2.5	The Noether method	44
2.6	Generic symmetries of field theories	47
2.6.1	Single vector field	48
2.6.2	The general case	53
2.6.3	Extension to higher dimensions and ranks	59
2.7	The embedding tensor formalism	62
3	A perturbative introduction to general relativity	70
3.1	Scalar SRFTs of gravity	71
3.1.1	Scalar gravity coupled to matter	72
3.1.2	The action for a relativistic massive point-particle	73
3.1.3	The massive point-particle coupled to scalar gravity	75
3.1.4	The action for a massless point-particle	76
3.1.5	The massless point-particle coupled to scalar gravity	78
3.1.6	Self-coupled scalar gravity	78
3.1.7	The geometrical Einstein–Fokker theory	80
3.2	Gravity as a self-consistent massless spin-2 SRFT	82
3.2.1	Gauge invariance, gauge identities, and charge conservation in the SRFT of a spin-1 particle	85
3.2.2	Gauge invariance, gauge identities, and charge conservation in the SRFT of a spin-2 particle	88
3.2.3	Coupling to matter	92
3.2.4	The consistency problem	101
3.2.5	The Noether method for gravity	103
3.2.6	Properties of the gravitational energy–momentum tensor $t_{\text{GR}}^{(0)\mu\sigma}$	110
3.2.7	Deser’s argument	114
3.3	General relativity	121
3.4	The Fierz–Pauli theory in a curved background	128
3.4.1	Linearized gravity	129
3.4.2	Massless spin-2 particles in curved backgrounds	134
3.4.3	Self-consistency	137
3.5	Final comments	137
4	Action principles for gravity	139
4.1	The Einstein–Hilbert action	140
4.1.1	Equations of motion	142
4.1.2	Gauge identity and Noether current	144
4.1.3	Coupling to matter	145
4.2	The Einstein–Hilbert action in different conformal frames	146
4.3	The first-order (Palatini) formalism	148
4.3.1	The purely affine theory	151
4.4	The Cartan–Sciama–Kibble theory	152
4.4.1	The coupling of gravity to fermions	153
4.4.2	The coupling to torsion: the CSK theory	156
4.4.3	Gauge identities and Noether currents	159

	<i>Contents</i>	xi
4.4.4	The first-order Vielbein formalism	161
4.5	Gravity as a gauge theory	166
4.6	Teleparallelism	170
4.6.1	The linearized limit	172
5	Pure $N = 1, 2, d = 4$ supergravities	175
5.1	Gauging $N = 1, d = 4$ superalgebras	176
5.2	$N = 1, d = 4$ (Poincaré) supergravity	180
5.2.1	Local supersymmetry algebra	184
5.3	$N = 1, d = 4$ AdS supergravity	184
5.3.1	Local supersymmetry algebra	186
5.4	Extended supersymmetry algebras	186
5.4.1	Central extensions	190
5.5	$N = 2, d = 4$ (Poincaré) supergravity	191
5.5.1	The local supersymmetry algebra	195
5.6	$N = 2, d = 4$ “gauged” (AdS) supergravity	195
5.6.1	The local supersymmetry algebra	197
5.7	Proofs of some identities	197
6	Matter-coupled $N = 1, d = 4$ supergravity	199
6.1	The matter supermultiplets	200
6.2	The ungauged theory	202
6.2.1	Examples	206
6.3	The gauged theory	208
6.3.1	The global symmetries	209
6.3.2	Example: symmetries of the axion–dilaton model	212
6.3.3	The gauging of the global symmetries	214
6.3.4	Examples of gauged $N = 1, d = 4$ supergravities	218
7	Matter-coupled $N = 2, d = 4$ supergravity	220
7.1	The matter supermultiplets	221
7.2	The ungauged theory	222
7.2.1	Examples	226
7.3	The gauged theory	238
7.3.1	The global symmetries	238
7.3.2	Examples	241
7.3.3	The gauging of the global symmetries	247
7.3.4	Examples of gauged $N = 2, d = 4$ supergravities	252
8	A generic description of all the $N \geq 2, d = 4$ SUEGRAs	256
8.1	Generic supermultiplets	256
8.2	The theories	259
9	Matter-coupled $N = 1, d = 5$ supergravity	263
9.1	The matter supermultiplets	264

xii	<i>Contents</i>	
9.2	The ungauged theory	265
9.2.1	Examples	267
9.3	The gauged theory	270
9.3.1	The global symmetries	270
9.3.2	Examples	271
9.3.3	The gauging of the global symmetries	272
9.3.4	Examples of gauged $N = 1, d = 5$ supergravities	274
10	Conserved charges in general relativity	275
10.1	The traditional approach	276
10.1.1	The Landau–Lifshitz pseudotensor	278
10.1.2	The Abbott–Deser approach	280
10.2	The Noether approach	283
10.3	The positive-energy theorem	284
	Part II Gravitating point-particles	289
11	The Schwarzschild black hole	291
11.1	The Schwarzschild solution	292
11.1.1	General properties	293
11.2	Sources for the Schwarzschild solution	305
11.3	Thermodynamics	307
11.4	The Euclidean path-integral approach	312
11.4.1	The Euclidean Schwarzschild solution	313
11.4.2	The boundary terms	315
11.5	Higher-dimensional Schwarzschild metrics	316
11.5.1	Thermodynamics	317
12	The Reissner–Nordström black hole	318
12.1	Coupling a scalar field to gravity and no-hair theorems	319
12.2	The Einstein–Maxwell system	323
12.2.1	Electric charge	326
12.2.2	Massive electrodynamics	331
12.3	The electric Reissner–Nordström solution	332
12.4	Sources of the electric RN black hole	343
12.5	Thermodynamics of RN black holes	345
12.6	The Euclidean electric RN solution and its action	348
12.7	Electric–magnetic duality	351
12.7.1	Poincaré duality	354
12.7.2	Magnetic charge: the Dirac monopole and the Dirac quantization condition	355
12.7.3	The Wu–Yang monopole	361
12.7.4	Dyons and the DSZ charge-quantization condition	362
12.7.5	Duality in massive electrodynamics	365

<i>Contents</i>		xiii
12.8	Magnetic and dyonic RN black holes	366
12.9	Higher-dimensional RN solutions	369
13	The Taub–NUT solution	374
13.1	The Taub–NUT solution	375
13.2	The Euclidean Taub–NUT solution	378
13.2.1	Self-dual gravitational instantons	379
13.2.2	The BPST instanton	381
13.2.3	Instantons and monopoles	384
13.2.4	The BPST instanton and the KK monopole	388
13.2.5	Bianchi IX gravitational instantons	389
13.3	Charged Taub–NUT solutions and IWP solutions	390
14	Gravitational pp-waves	394
14.1	pp-waves	394
14.1.1	Hpp-waves	395
14.2	Four-dimensional pp-wave solutions	397
14.2.1	Higher-dimensional pp-waves	399
14.3	Sources: the AS shock wave	399
15	The Kaluza–Klein black hole	402
15.1	Classical and quantum mechanics on $\mathbb{R}^{1,3} \times S^1$	403
15.2	KK dimensional reduction on a circle S^1	408
15.2.1	The Scherk–Schwarz formalism	411
15.2.2	Newton’s constant and masses	415
15.2.3	KK reduction of sources: the massless particle	418
15.2.4	Electric–magnetic duality and the KK action	422
15.2.5	Reduction of the Einstein–Maxwell action and $N = 1, d = 5$ SUGRAs	425
15.3	KK reduction and oxidation of solutions	431
15.3.1	ERN black holes	432
15.3.2	Dimensional reduction of the AS shock wave: the extreme electric KK black hole	435
15.3.3	Non-extreme Schwarzschild and RN black holes	438
15.3.4	Simple KK solution-generating techniques	441
15.4	Toroidal (Abelian) dimensional reduction	446
15.4.1	The 2-torus and the modular group	451
15.4.2	Masses, charges, and Newton’s constant	454
15.5	Generalized dimensional reduction	454
15.5.1	Example 1: a real scalar	456
15.5.2	Example 2: a complex scalar	459
15.5.3	Example 3: an $SL(2, \mathbb{R})/SO(2)$ σ -model	461
15.5.4	Example 4: Wilson lines and GDR	462
15.6	Orbifold compactification	463

16	Dilaton and dilaton/axion black holes	464
16.1	Dilaton black holes: the a -model	465
16.1.1	The a -model solutions in four dimensions	469
16.2	Dilaton/axion black holes	474
16.2.1	The general SWIP solution	479
16.2.2	Supersymmetric SWIP solutions	481
16.2.3	Duality properties of the SWIP solutions	482
17	Unbroken supersymmetry I: supersymmetric vacua	484
17.1	Vacuum and residual symmetries	485
17.2	Supersymmetric vacua and residual (unbroken) supersymmetries	487
17.2.1	Covariant Lie derivatives	490
17.2.2	Calculation of supersymmetry algebras	493
17.3	$N = 1, 2, d = 4$ vacuum supersymmetry algebras	494
17.3.1	The Killing spinor integrability condition	497
17.3.2	The vacua of $N = 1, d = 4$ Poincaré supergravity	498
17.3.3	The vacua of $N = 1, d = 4$ AdS ₄ supergravity	499
17.3.4	The vacua of $N = 2, d = 4$ Poincaré supergravity	503
17.3.5	The vacua of $N = 2, d = 4$ AdS supergravity	506
17.4	The vacua of $d = 5, 6$ supergravities with eight supercharges	507
17.4.1	$N = (1, 0), d = 6$ supergravity	507
17.4.2	$N = 1, d = 5$ supergravity	508
17.4.3	Relation to the $N = 2, d = 4$ vacua	510
18	Unbroken supersymmetry II: partially supersymmetric solutions	512
18.1	Partially supersymmetric solutions	513
18.1.1	Partially unbroken supersymmetry, supersymmetry bounds, and the superalgebra	514
18.1.2	Examples	519
18.2	Tod's program	522
18.2.1	The Killing spinor identities	525
18.3	All the supersymmetric solutions of ungauged $N = 1, d = 4$ supergravity	526
18.3.1	Supersymmetric configurations	527
18.3.2	Supersymmetric solutions	530
18.4	All the supersymmetric solutions of ungauged $N = 2, d = 4$ supergravity	533
18.4.1	The timelike case: supersymmetric configurations	533
18.4.2	The timelike case: supersymmetric solutions	538
18.4.3	The null case	540
18.5	The timelike supersymmetric solutions of $N = 2, d = 4$ SEYM theories	541
18.5.1	Supersymmetric configurations	541
18.5.2	Supersymmetric solutions	543
18.6	All the supersymmetric solutions of ungauged $N \geq 2, d = 4$ supergravity	544
18.7	All the supersymmetric solutions of ungauged $N = 1, d = 5$ supergravity	549
18.7.1	The timelike case: supersymmetric configurations	552
18.7.2	The timelike case: supersymmetric solutions	555

<i>Contents</i>		xv
18.7.3	The null case: supersymmetric configurations	555
18.7.4	The null case: supersymmetric solutions	557
18.7.5	Solutions with an additional isometry	558
19	Supersymmetric black holes from supergravity	562
19.1	Introduction	563
19.2	The supersymmetric black holes of ungauged $N = 2, d = 4$ supergravity	565
19.2.1	The general recipe	565
19.2.2	Single-black-hole solutions	568
19.2.3	Multi-black-hole solutions	571
19.2.4	Examples of single-SBHSs: stabilization equations	575
19.2.5	Two-center SBHS of the axion–dilaton model	581
19.3	The supersymmetric black holes of $N = 2, d = 4$ SEYM	582
19.3.1	The general recipe	582
19.3.2	Examples	584
19.4	The supersymmetric black holes of $N = 8, d = 4$ supergravity	588
19.4.1	The duality group of $N = 8, d = 4$ SUEGRA and its invariants	589
19.4.2	The metric function	592
19.4.3	Single supersymmetric black-hole solutions	593
19.5	The supersymmetric black holes of $N = 1, d = 5$ supergravity	594
19.5.1	The general recipe	594
19.5.2	Single, static, black-hole solutions	596
19.5.3	Examples	599
19.5.4	Some stationary solutions of pure $N = 1, d = 5$ supergravity	601
Part III	Gravitating extended objects of string theory	605
20	String theory	607
20.1	Strings	611
20.1.1	Superstrings	614
20.1.2	Green–Schwarz actions	617
20.2	Quantum theories of strings	619
20.2.1	Quantization of free-bosonic-string theories	620
20.2.2	Quantization of free-fermionic-string theories	624
20.2.3	D-branes and O-planes in superstring theories	626
20.2.4	String interactions	627
20.3	Compactification on S^1 : T duality and D-branes	628
20.3.1	Closed bosonic strings on S^1	628
20.3.2	Open bosonic strings on S^1 and D-branes	630
20.3.3	Superstrings on S^1	631
21	The string effective action and T duality	632
21.1	Effective actions and background fields	632
21.1.1	The D-brane effective action	637

xvi	<i>Contents</i>	
21.2	T duality and background fields: Buscher's rules	637
21.2.1	T duality in the bosonic-string effective action	638
21.2.2	T duality in the bosonic-string worldsheet action	641
21.2.3	T duality in the bosonic Dp -brane effective action	645
21.3	Example: the fundamental string (F1)	647
22	From eleven to four dimensions	650
22.1	Dimensional reduction from $d = 11$ to $d = 10$	652
22.1.1	Eleven-dimensional supergravity	652
22.1.2	Reduction of the bosonic sector	655
22.1.3	Magnetic potentials	661
22.1.4	Reduction of fermions and the supersymmetry rules	664
22.2	Romans' massive $N = 2A, d = 10$ supergravity	666
22.3	Further reduction of $N = 2A, d = 10$ SUEGRA to nine dimensions	669
22.3.1	Dimensional reduction of the bosonic RR sector	669
22.3.2	Dimensional reduction of fermions and supersymmetry rules	671
22.4	The effective field theory of the heterotic string	672
22.5	Toroidal compactification of the heterotic string	674
22.5.1	Reduction of the action of pure $N = 1, d = 10$ supergravity	674
22.5.2	Reduction of the fermions and supersymmetry rules of $N = 1, d = 10$ SUGRA	678
22.5.3	The truncation to pure supergravity	680
22.5.4	Reduction with additional $U(1)$ vector fields	681
22.5.5	Trading the KR 2-form for its dual	683
22.6	T duality, compactification, and supersymmetry	685
23	The type-IIB superstring and type-II T duality	688
23.1	$N = 2B, d = 10$ supergravity in the string frame	689
23.1.1	Magnetic potentials	690
23.1.2	The type-IIB supersymmetry rules	691
23.2	Type-IIB S duality	691
23.3	Dimensional reduction of $N = 2B, d = 10$ SUEGRA and type-II T duality	694
23.3.1	The type-II T-duality Buscher rules	697
23.4	Dimensional reduction of fermions and supersymmetry rules	698
23.5	Consistent truncations and heterotic/type-I duality	700
24	Extended objects	703
24.1	Introduction	703
24.2	Generalities	704
24.2.1	Worldvolume actions	704
24.2.2	Charged branes and Dirac charge quantization for extended objects	708
24.2.3	The coupling of p -branes to scalar fields	712
24.3	General p -brane solutions	715
24.3.1	Schwarzschild black p -branes	715

<i>Contents</i>		xvii
24.3.2	The p -brane a -model	717
24.3.3	Sources for solutions of the p -brane a -model	720
25	The extended objects of string theory	724
25.1	String-theory extended objects from duality	725
25.1.1	The masses of string- and M-theory extended objects from duality	728
25.2	String-theory extended objects from effective-theory solutions	734
25.2.1	Extreme p -brane solutions of string and M theories and sources	736
25.2.2	The M2 solution	737
25.2.3	The M5 solution	739
25.2.4	The fundamental string F1	741
25.2.5	The S5 solution	742
25.2.6	The Dp -branes	743
25.2.7	The D-instanton	745
25.2.8	The D7-brane and holomorphic $(d - 3)$ -branes	746
25.2.9	Some simple generalizations	752
25.3	The masses and charges of the p -brane solutions	753
25.3.1	Masses	753
25.3.2	Charges	755
25.4	Duality of string-theory solutions	756
25.4.1	$N = 2A, d = 10$ SUEGRA solutions from $d = 11$ SUGRA solutions	757
25.4.2	$N = 2A/B, d = 10$ SUEGRA T-dual solutions	760
25.4.3	S duality of $N = 2B, d = 10$ SUEGRA solutions: pq -branes	761
25.5	String-theory extended objects from superalgebras	762
25.5.1	Unbroken supersymmetries of string-theory solutions	765
25.6	Intersections	769
25.6.1	Brane-charge conservation and brane surgery	773
25.6.2	Marginally bound supersymmetric states and intersections	774
25.6.3	Intersecting-brane solutions	775
25.6.4	The $(a_1 - a_2)$ -model for p_1 - and p_2 -branes and black intersecting branes	776
26	String black holes in four and five dimensions	780
26.1	Composite dilaton black holes	781
26.2	Black holes from branes	783
26.2.1	Black holes from single wrapped branes	783
26.2.2	Black holes from wrapped intersecting branes	785
26.2.3	Duality and black-hole solutions	794
26.3	Entropy from microstate counting	796
27	The FGK formalism for (single, static) black holes and branes	798
27.1	The $d = 4$ FGK formalism	799
27.1.1	FGK theorems and the attractor mechanism	804

xviii	<i>Contents</i>	
	27.1.2 The FGK formalism for $N = 2, d = 4$ supergravity	808
	27.1.3 Flow equations	811
27.2	The general FGK formalism	813
	27.2.1 FGK theorems for static flat branes	818
	27.2.2 Inner horizons	819
	27.2.3 FGK formalism for the black holes of $N = 1, d = 5$ theories	820
	27.2.4 FGK formalism for the black strings of $N = 1, d = 5$ theories	821
27.3	The H-FGK formalism	822
	27.3.1 For the black-hole solutions of $N = 1, d = 5$	824
	27.3.2 For $N = 2, d = 4$ black holes	826
	27.3.3 Freudenthal duality	828
	Appendix A Lie groups, symmetric spaces, and Yang–Mills fields	830
A.1	Generalities	830
A.2	Yang–Mills fields	834
	A.2.1 Fields and covariant derivatives	834
	A.2.2 Kinetic terms	836
	A.2.3 $SO(n_+, n_-)$ gauge theory	838
A.3	Riemannian geometry of group manifolds	841
	A.3.1 Example: the $SU(2)$ group manifold	842
A.4	Riemannian geometry of homogeneous and symmetric spaces	843
	A.4.1 H-covariant derivatives	846
	A.4.2 Example: round spheres	847
	Appendix B The irreducible, non-symmetric Riemannian spaces of special holonomy	849
	Appendix C Miscellanea on the symplectic group	851
C.1	The symplectic group	851
	Appendix D Gamma matrices and spinors	858
D.1	Generalities	858
	D.1.1 Useful identities	866
	D.1.2 Fierz identities	867
	D.1.3 Eleven dimensions	868
	D.1.4 Ten dimensions	870
	D.1.5 Nine dimensions	871
	D.1.6 Eight dimensions	871
	D.1.7 Two dimensions	872
	D.1.8 Three dimensions	872
	D.1.9 Four dimensions	872
	D.1.10 Five dimensions	874
	D.1.11 Six dimensions	875
D.2	Spaces with arbitrary signatures	876
	D.2.1 AdS_4 gamma matrices and spinors	879

	<i>Contents</i>	xix
D.3	The algebra of commuting spinor bilinears	883
D.3.1	Four-dimensional case	883
D.3.2	Five-dimensional case	889
	Appendix E Kähler geometry	893
E.1	Complex manifolds	893
E.1.1	Hermitian connections	896
E.1.2	Holomorphic isometries of complex manifolds	897
E.2	Almost complex structures and manifolds	898
E.3	Kähler manifolds	899
E.3.1	Holomorphic isometries of Kähler manifolds	901
	Appendix F Special Kähler geometry	905
F.1	Special Kähler manifolds	905
F.2	The prepotential	909
F.3	Holomorphic isometries of special Kähler manifolds	911
	Appendix G Quaternionic-Kähler geometry	914
G.1	Triholomorphic isometries of quaternionic-Kähler spaces	918
G.1.1	Alternative notation for the $d = 5$ case	921
	Appendix H Real special geometry	923
H.1	The isometries of real special manifolds	925
	Appendix I The generic scalar manifolds of $N \geq 2, d = 4$ SUEGRAs	928
	Appendix J Gauging isometries of non-linear σ-models	933
J.1	Introduction: gauging isometries of Riemannian manifolds	934
J.2	Gauging holomorphic isometries of complex manifolds	939
J.3	Kähler-Hodge manifolds	939
J.4	Gauging holomorphic isometries of special Kähler manifolds	943
J.5	Gauging isometries of quaternionic-Kähler manifolds	945
J.5.1	Alternative notation for the $d = 5$ case	947
J.6	Gauging isometries of real special manifolds	947
	Appendix K n-spheres	949
K.1	S^3 and S^7 as Hopf fibrations	951
K.2	Squashed S^3 and S^7	952
	Appendix L Palatini's identity	953
	Appendix M Conformal rescalings	954
	Appendix N Connections and curvature components	955
N.1	For a $d = 3$ metric	955
N.2	For some $d = 4$ metrics	955

xx	<i>Contents</i>	
	N.2.1 General static, spherically symmetric metrics (I)	955
	N.2.2 General static, spherically symmetric metrics (II)	956
	N.2.3 $d = 4$ IWP-type metrics	957
	N.2.4 The $d = 4$ conformastationary metric	958
N.3	For some $d > 4$ metrics	959
	N.3.1 $d > 4$ general static, spherically symmetric metrics	959
	N.3.2 The $d = 5$ conformastationary metric	960
	N.3.3 A general metric for (single, black) p -branes	961
	N.3.4 A general metric for (composite, black) p -branes	962
	N.3.5 A general metric for extreme p -branes	963
	N.3.6 Brinkmann metrics	964
N.4	A five-dimensional metric with a null Killing vector	965
	Appendix O The harmonic operator on $\mathbb{R}^3 \times S^1$	967
	<i>References</i>	969
	<i>Index</i>	1002