DEONTIC LOGIC AND LEGAL SYSTEMS

A considerable number of books and papers have analyzed normative concepts using new techniques developed by logicians; however, few have bridged the gap between the Continental (i.e., European) and Latin American traditions in legal philosophy. This book addresses this issue by offering an introductory study on the many possibilities that logical analysis offers the study of legal systems.

The volume is divided into two sections. The first section covers the basic aspects of classical logic and deontic logic and their connections, advancing an explanation of the most important topics of the discipline by comparing different systems of deontic logic and exploring some of the most important paradoxes in its domain. The second section deals with the role of logic in the analysis of legal systems by discussing in what sense deontic logic and the logic of norm-propositions are useful tools for a proper understanding of the systematic structure of law. Arguments are provided to stress the relevance of a systematic reconstruction of law as a necessary step in the identification of the truth conditions of legal statements and the reasons for accepting or rejecting the validity of logical consequences of enacted legal norms.

Pablo E. Navarro is a professor of philosophy of law at the National University of the South and Blaise Pascal University. He is also a researcher for the National Council for Research in Science and Technology (CONICET) in Argentina. Navarro has published several books and has written papers on legal theory and deontic logic for journals such as *Law and Philosophy*, *Ratio Juris*, *Rechtstheorie*, and *Theoria*. He has been a visiting professor in many European and Latin American universities. He obtained a Guggenheim Fellowship (2001–2002) and was recognized by the Konex Foundation (2006) in the discipline of legal philosophy and was awarded with the Bernardo Houssay Prize (2003).

Jorge L. Rodríguez is a professor of legal theory at the National University of Mar del Plata School of Law and a visiting researcher in the department of law at the University of Girona. He has published several books and articles in Argentina, Brazil, Canada, Colombia, Germany, Italy, Mexico, and the United Kingdom on legal theory and deontic logic. Rodríguez was awarded the Young Scholar Prize by the International Association for Philosophy of Law and Social Philosophy (1999) and was recognized by the Konex Foundation (2006) in the discipline of legal philosophy. He has served as a criminal judge in Mar del Plata since 2009.

CAMBRIDGE INTRODUCTIONS TO PHILOSOPHY AND LAW

Series Editors

Brian H. Bix
University of Minnesota

William A. Edmundson
Georgia State University

This introductory series of books provides concise studies of the philosophical foundations of law, of perennial topics in the philosophy of law, and of important and opposing schools of thought. The series is aimed principally at students in philosophy, law, and political science.

Matthew Kramer, *Objectivity and the Rule of Law* (2007)

Larry Alexander and Emily Sherwin, *Demystifying Legal Reasoning* (2008)

Larry Alexander, Kimberly Kessler Ferzan, and Stephen J. Morse, *Crime and Culpability* (2009)

Robin West, *Normative Jurisprudence* (2011)

William A. Edmundson, *An Introduction to Rights, 2nd edition* (2012)

Gregory S. Alexander and Eduardo S. Peñalver, *An Introduction to Property Theory* (2012)

Brian H. Bix, *Contract Law* (2013)

Liam Murphy, *What Makes Law* (2014)

Pablo E. Navarro and Jorge L. Rodríguez, *Deontic Logic and Legal Systems* (2014)

Deontic Logic and Legal Systems

PABLO E. NAVARRO

Blaise Pascal University, Argentina
National University of the South, Argentina

JORGE L. RODRÍGUEZ

National University of Mar del Plata, Argentina

With a Prologue by
Eugenio Bulygin

CAMBRIDGE
UNIVERSITY PRESS

. . . things are never odd *in logic, only* different.

P. T. Geach

. . . there is a good deal of unfinished business for analytical jurisprudence still to tackle, and this unfinished business includes a still much needed clarification of the meaning of the common assertion that laws belong to or constitute a system of laws . . .

H. L. A. Hart

CAMBRIDGE
UNIVERSITY PRESS

© Pablo E. Navarro and Jorge L. Rodríguez 2014

Contents

viii   *Contents*

PART II LOGIC AND LEGAL SYSTEMS

Prologue

by Eugenio Bulygin

Logic and law have a long history in common, but the influence has been mostly one-sided, except perhaps in the fifth and sixth centuries BC, when disputes at the marketplace or in tribunals in Greece seem to have stimulated a lot of reflection among sophistic philosophers on such topics as language and truth. Most of the time it was logic that influenced legal thinking, but in the past fifty years, logicians began to be interested in normative concepts, and hence in law.

From the fourth century BC until the nineteenth century AD, logic was basically Aristotelian logic. Aristotle was not only the founder of logic but also the first to formulate a theory of systems.[1] An important result of this influence was the theory of judicial syllogism. The justification of a judicial decision was regarded as a typical case of syllogistic reasoning, where from a normative and a factual premise the decision of the case was inferred by the judge. It was with the Enlightenment that the theory of judicial syllogism became dominant, based on two important ideas: the doctrine of the separation of powers (above all, the separation between the legislative and the judicial power) and a sharp distinction between the creation and the application of the law. The law is conceived of as a set of all general legal norms created by the legislative power (Parliament); the task of judges is limited to the application of the law to particular disputes. But to be able to fulfill this role assigned to judges, the law must provide solutions to all legal issues; it must contain one and only one solution for each legal problem, which entails that the law must be complete and consistent. If the law does not contain a norm solving the problem (i.e., if there is what traditionally is called a legal gap) or if the law contains two or more incompatible norms applying to the same

---

[1] E. W. Beth, *The Foundations of Mathematics*, Amsterdam, 1959, 31–39; C. E. Alchourrón and E. Bulygin, *Normative Systems*, Springer, 1971, Vienna–New York, 44–53.

case (conflict of laws), then the judge will not be able to solve the problem by mere application of the law. The codification of law by Napoleon was the first serious attempt to create a legal system that would allow judges to apply the law without modifying it. In the nineteenth and twentieth centuries, therefore, the logical ideas of completeness and consistency occupied a very important place in legal practice. But the treatment of these issues by legal thinkers was rather unsatisfactory; it was incomplete and sometimes even inconsistent. It was incomplete because they never defined satisfactorily the concept of incompleteness distinguishing between different kinds of "gaps," and it was inconsistent because some legal philosophers insisted that there were no gaps or inconsistencies because such situations always can be eliminated by interpretation.[2] Instead of asking such questions as what does it mean that a legal system is incomplete or inconsistent, it was proclaimed dogmatically that all legal systems are necessarily complete and consistent. Even such an outstanding and sharp legal philosopher as Hans Kelsen maintained during his whole (and rather long) life that all legal systems are necessarily (for conceptual reasons) complete, and he recognized the possibility of conflicts in the law only in 1962,[3] when he was already over eighty.

The second half of the nineteenth century began with, as is well known, an enormous development of logic, which was not followed by jurists. Symbolic logic remained for a long period practically unknown by legal writers and philosophers. This led to an almost complete isolation of law from logic. This regrettable situation lasted for about 100 years and began to change only in the second half of the twentieth century. The publication of Georg Henrik von Wright's famous paper "Deontic Logic" (1951) is generally regarded as the birth of a new branch of logic, deontic logic, and it constitutes the beginning of a new era in the relation of these two disciplines.

In the past sixty years, a considerable number of books and papers have analyzed normative concepts – such as norm, obligation, prohibition, and permission – using the new techniques developed by logicians. The very notion of a legal system became the center of concern for many legal philosophers. Books by G. H. von Wright (*Norm and Action*, 1963), Joseph Raz (*The Concept of a Legal System*, 1970), C. E. Alchourrón and E. Bulygin

[2] See Giorgio Del Vecchio, *Filosofía del Derecho*, Barcelona, 1947, 399; Luis Recaséns Siches, *Tratado General de Filosofía del Derecho*, México, 1959, 323–325; Carlos Cossio, *La Plenitud del Ordenamiento Jurídico*, Buenos Aires, 1947, 42.

[3] "[T]he science of law is just as incompetent to solve by interpretation existing conflicts between norms, or better, to repeal the validity of positive norms, as it is incompetent to issue legal norms": "Derogation," in R. Newman (ed.), *Essays in Jurisprudence in Honor of Roscoe Pound*, Indianapolis, New York, 1962, at 355.

(*Normative Systems*, 1971), and Lars Lindahl (*Position and Change*, 1977) exercised a considerable influence and were soon followed by a great number of publications dealing with logical aspects of the law. It is significant that this trend is almost exclusively limited to the Continental (i.e., European) and Latin American traditions in legal philosophy. The influence of symbolic logic on Anglo-American jurisprudence is still rather scant. In this sense, this book by Pablo E. Navarro and Jorge L. Rodríguez may be regarded as a new bridge between these two legal traditions. It is an updated introduction to deontic logic (Part I), and it shows the importance of the logical analysis of legal concepts, especially for the concept of a legal system (Part II).

Part I contains a survey of several systems of deontic logic, especially the Minimal, the Classical, and the Standard systems, and analyzes their main problems. The authors discuss some objections to the very possibility of deontic logic (i.e., the so-called Frege-Geach problem and Jørgensen's dilemma). They also analyze the main paradoxes of deontic logic (among others, Ross's paradox, the paradox of derived obligation, and several "contrary-to-duty" paradoxes).

Paradoxes are rather common in logic; they appear in different domains (propositional logic, modal logic), but it would be a mistake to think that they might pose a danger for logic. As von Wright puts it:

> The paradoxes traditionally belong to the most lively debated matters in logic. Attempts to "solve" them have contributed decisively to the development of logic after Frege. To me the fascination of the antinomies has been that they challenge reflection about the most basic ideas of logical thinking: property and proposition, truth and demonstration, the meaning of "contradiction." These ideas are intertwined in their roots. The antinomies make us aware of this. There is no unique way of untwisting the connections – and therefore no *one* way of "solving" the paradoxes either.[4]

This dictum fully applies to the paradoxes of deontic logic.

In Chapter 2 the authors face what David Makinson has called the "fundamental problem of deontic logic,"[5] which is raised by the rather obvious feature of norms: their lack of truth values. This problem was clearly stated by the Danish philosopher and logician Jørg Jørgensen and is known as Jørgensen's dilemma, but it was regrettably ignored by many logicians. Navarro and Rodríguez analyze the different attempts to overcome this dilemma, from the "skeptical solution" (as norms lack truth values, there is no logic of norms; Kelsen), to the different substitutes for truth – satisfaction (Hofstadter and

---

[4] G. H. von Wright, *Philosophical Logic*, Basil Blackwell, Oxford, 1983, VII.
[5] D. Makinson, "On a Fundamental Problem of Deontic Logic," in P. McNamara and H. Prakken (eds.), *Norms, Logics and Information Systems*, IOS Press, Amsterdam et al., 1999.

McKinsey), validity as binding force, and validity as membership (Wein-berger); and they accept the proposal of Carlos Alchourrón and A. A. Martino – a logical system independent of the notions of truth and falsity, based on the idea of an abstract notion of consequence.[6] Navarro and Rodríguez reach the conclusion that "the only open alternatives to deal with Jørgensen's dilemma would be the two radical views . . . either accepting that, after all, norms are proposition-like entities, and thus susceptible of truth values, or abandoning the idea that logic is restricted to the realm of truth . . . and each of these views corresponds to two fundamentally different conceptions of norms." These two conceptions are the semantic and the pragmatic, which roughly correspond to the distinction between hyletic and expressive conceptions of norms.[7] But the authors hold that the semantic conception of norms as proposition-like enti-ties requires that norms have truth values, and in this case norm-propositions are not distinguishable from norms. In any case, they regard the pragmatic conception as the only one that offers a possibility of developing a genuine logic of norms.

Especially important seems to me Chapter 3, which deals with three much discussed problems of the logic of norms: (1) the distinction – crucial to my mind – between norms and norm-propositions (i.e., propositions about norms); (2) conditional norms; and (3) the problem of defeasibility.

Although norms and norm-propositions can be expressed by similar or even the same words, they are very different in nature, and so the logical structure of norms differs significantly from that of norm-propositions. This is shown by the role played by negation. When applied to norms, negation is analogous to ordinary negation as it is used in descriptive language; the negation of a norm is also a norm; for any norm there is only one negation-norm; they are reciprocal, mutually exclusive, and jointly exhaustive. But the negation of a norm-proposition is more complex. There are two ways to negate a norm-proposition. The negation can operate over the membership, or it may affect the norm itself. The negation of the norm-proposition "*p* is prohibited in *S*" can mean: (1) there is no norm in *S* prohibiting *p*, or (2) there is in *S* a norm that does not prohibit (i.e., permits) *p*. The distinction between the exter-nal and the internal negation of a norm-proposition allows one to detect the

6 C. E. Alchourrón and A. A. Martino, "Logic without Truth," *Ratio Juris* 3 (1990), 46–67, and
   C. E. Alchourrón, "Concepciones de la lógica," in Alchourrón et al. (eds.), *Lógica. Enciclo-
   pedia Iberoamericana de Filosofía*, vol. 7, Trotta, Madrid 1995, 11–48.
7 C. E. Alchourrón and E. Bulygin, "The Expressive Conception of Norms," in R. Hilpinen (ed.)
   New Studies in Deontic Logic, Reidel, Dordrecht-Boston-London, 1981, 95–124. Reprinted in
   S. L. Paulson and B. Litschewski Paulson (eds.) *Normativity and Norms: Critical Perspectives
   on Kelsenian Themes*, 383–410, Oxford University Press, Oxford, 1998.

ambiguity of "permission" in norm-propositions: *negative permission* as the mere absence of a norm prohibiting the action in question and *positive permission* as the presence of a norm permitting *p*. Therefore, we have three concepts of permission: one prescriptive, occurring in norms, and two descriptive, occurring in norm-propositions. This gives rise to two different logics: the logic of norms aiming to reconstruct the rationality of the activity of the legislator (i.e., the activity of enacting norms), whereas the logic of norm-propositions is concerned with the reconstruction of the logical consequences of a given set of norms, a normative system. The two logics are isomorphic only under the assumption of completeness and consistency.

These conceptual distinctions show that the principle "What is not legally prohibited is legally permitted" (which is often used to maintain that all legal systems are necessarily complete) is also ambiguous; if "permitted" means negative permission, then the principle is trivially true, for it only states that what is not prohibited is not prohibited. And if "permitted" means positive permission, then the principle is clearly contingent, for from the absence of a prohibition we cannot infer the existence of a permissive norm. In neither case can this principle be used as an argument that all legal systems are necessarily complete.

Once we clearly distinguish between norms and norm-propositions, we must face the problem of the nature of the logic of norms. If norms are conceived of as acts of command or permission, as is postulated by the pragmatic conception shared by Navarro and Rodríguez, then it seems that there can be no logic of norms, for there are no logical relations between acts of prescribing. Navarro and Rodríguez try to base the logic of norms on the idea that there are incompatibilities between certain acts of commanding or permitting. Von Wright was the first to elaborate on this idea.[8] Certain acts such as issuing such commands as *!p* and *!∼p* (to command *p* and its negation, that is, e.g., commanding one to open the window and not to open it) or *!p* and *¡p* (i.e., to command *p* and to reject *p*) are in normal circumstances regarded as irrational. Such relations are logical in a different sense, for they are based not on the idea of truth, but on the rationality of the activity of norm-giving. Therefore, the logic of norms may be regarded as a logic of rational legislation.

A very important part of the book is dedicated to the analysis of conditional norms. The authors discuss the two main conceptions of conditional norms, the so-called *bridge conception*, in which the deontic operator affects only the

---

[8] G. H. von Wright, "Norms, Truth, and Logic" (1982) reprinted in *Practical Reason*, Basil Blackwell, Oxford, 1983, 130–209; cfr. also C. E. Alchourrón and E. Bulygin, "Pragmatic Foundations for a Logic of Norms," *Rechtstheorie* 15 (1984), 453–464.

consequent of the conditional ($p \rightarrow Oq$), and the *insular conception*, in which both the antecedent and the consequent of the conditional are within the scope of the deontic operator (e.g., $O(p \rightarrow q)$). C. Alchourrón has proposed both terms.[9] A thorough discussion of this issue leads the authors to the conclusion that each conception has its raison d'être, for there are two different concepts of conditional norms: the one admits the factual detachment (($p \rightarrow Oq \land p$) $\rightarrow Oq$) but not the deontic detachment (($O(p \rightarrow q) \land Op) \rightarrow Oq$), and the other admits the deontic but not the factual detachment. In natural languages, there are conditional norms that are better represented by one or another of these two different conceptions (the bridge and the insular conception). This, I think, is a very valuable insight.

Finally, Navarro and Rodríguez analyze the problem of defeasibility of legal norms. This is a much debated topic and a rather popular field of research in recent times, especially in legal philosophy.[10] The outcome of their discussion is that if rules are regarded as defeasible in the strong sense that they are subject to an open list of exceptions (which cannot be exhaustively listed), then this implies that general rules (and especially legal rules) are incapable of justifying any deontic qualification in a particular case and so lack inferential force and become useless for practical reasoning.

Part II is dedicated to the analysis of logical problems that are basically related to the systematic nature of law and so are of utmost importance for jurisprudence. Legal norms never appear in isolation, but form part of what jurists call a legal order or legal system. The term "system" is frequently used in legal contexts, but it is seldom clear what is meant by it. A legal system is often described as a set of all valid legal norms, where the term "valid" is even more ambiguous. By "validity" different authors understand different things: membership in a system, existence, or binding force of a norm. Even great legal philosophers do not always distinguish clearly between these concepts. Therefore, a conceptual distinction between these items is a necessary prolegomenon. This is what the authors do in the first chapters of the second part of the book. Their discussion of the lack of terminological and conceptual distinctions related to the notion of validity brings to light several difficulties, especially in the works of Kelsen, such as his theory of the alternative clause that proves to be incompatible with some of the main tenets of his Pure Theory of Law.

---

9 C. E. Alchourrón, "Detachment and Defeasibility in Deontic Logic," *Studia Logica*, 57 (1996), 5–18.
10 A good survey of publications on defeasibility can be found in J. Ferrer Beltrán and G. B. Ratti (eds.), *The Logic of Legal Requirements*, Oxford University Press, Oxford, 2012.

The distinction between generic and individual cases leads to the problem of the connection between general norms and the solution of individual cases. This relation is internal or conceptual; general norms regulate all individual cases belonging to the generic case. So the solution of an individual case can be determined by analyzing the logical consequences of general norms. But there is a grain of truth in Kelsen's contention that a judicial decision cannot be regarded as a "normative syllogism," because the connection between a general norm and an individual legal norm that regulates the individual case requires a normative act – that is, the decision of a judge. However, this does not mean that individual cases are not regulated by general norms. Navarro and Rodríguez distinguish between an individual case and a *judicial case* – that is, a particular controversy litigated in the courts, a practical problem that calls for an institutional solution. As both individual cases and judicial cases are particular cases, the question about the relation between general norms and particular cases becomes ambiguous. The answer to this question depends on the kind of case; the relation between general norms and individual cases is internal or conceptual, but the connection between a general norm and a judicial case is external or institutional. This leads to the distinction between the *internal* and the *external applicability* of a norm.

The introduction of the notion of applicability, which should not be confused with validity in the sense of membership, is of the utmost importance. Invalid norms can be applicable, and inapplicable norms can be valid. A derogated norm is no longer valid and does not belong to the system, but it can be applicable to certain cases.

The structure of a legal system is determined by internal relations between its norms. An important distinction must be made between *independent* and *dependent* norms.[11] Dependent norms are those that satisfy a relation of validity with other norms, but as the chain of validity cannot be infinite, it follows that for logical reasons there must be some independent norms in every system. Independent norms belong to the system not because they are created according to other norms, but by definition. They are the point of departure of a system of norms.

Two criteria for the validity of dependent norms have been analyzed by legal scholars: deducibility and legality. According to the first, a norm belongs to a legal system if it is a logical consequence of other norms of this system, and according to the second, a norm belongs to a legal system if it has been

---

[11] This terminology is from R. Caracciolo, *El sistema jurídico. Problemas actuales*, 31–33, Centro de Estudios Constitucionales, Madrid, 1988; von Wright uses the expression "sovereign norms" instead of independent norms.

created by a competent authority (i.e., if there is a valid norm that authorizes its creation). But Navarro and Rodríguez maintain that these two criteria cannot determine the membership of norms to the same entity; whereas deducibility determines the membership of norms to static sets of norms, legality determines the membership of sets of norms to a dynamic sequence of such sets. Therefore, we have two concepts of a legal system, a static and a dynamic one, although the two are deeply intertwined, and logic is essential not only for explaining the relation among norms but also for a reconstruction of legal dynamics.

The next step is the analysis of formal properties of static legal systems, completeness and consistency, or, rather, of their formal defects: gaps and conflicts. There is, in the book, an exhaustive discussion of the concepts of normative and axiological gaps. Even if the authors follow the steps of previous analyses, they manage to introduce many new developments, especially in discussing the idea that there are no gaps in the case of the silence of law (Raz) and such notions as (descriptive and prescriptive, positive and negative) normative relevance and irrelevance, leading to considerable refinement of the concept of axiological gaps.

One of the main problems of deontic logic is the notion of inconsistency. Are the norms Op and $\sim Op$ (commanding p and not commanding p) inconsistent (contradictory)? If $\sim Op$ means $P\sim p$, then it seems reasonable to assume that these two norms are incompatible, for the obligation of p and the permission of its omission are indeed incompatible in the sense that the fulfillment of the obligation makes it impossible to use the permission, and vice versa. Similarly, the norms Op and $O\sim p$ (obligation and prohibition of the same action) cannot both be obeyed. But this only shows that the norm contents p and $\sim p$ are inconsistent, not that the norms Op and $O\sim p$ (or $\sim Op$) are inconsistent, for the norms Pp and $P\sim p$ are perfectly consistent. This shows that the problem lies in the normative operator and not in the norm-contents. So the inconsistency of norm-contents proves to be a necessary but not a sufficient condition for the inconsistency of norms. The authors adopt the characterization of inconsistency proposed by Carlos Alchourrón,[12] who gives separate criteria for sets of O-norms, for sets of P-norms, and for mixed sets of O- and P-norms. These criteria are based on two ideas: inconsistency of norm-contents and the logical impossibility of complying with all such norms. The authors also discuss several problems not identical to logical contradiction but related to it, such as inconsistency via certain facts and conflicts of instantiation,

[12] C. E. Alchourrón, "Conflicts of Norms and Revision of Normative Systems," *Law and Philosophy*, 10, 413–425.

in which the impossibility of complying stems not from logical incompatibility, but from factual circumstances.

The sixth and last chapter of the book is particularly fascinating. It deals with legal dynamics, a very difficult and complicated topic of legal theory. Legal dynamics means change: the changing of norms (as a consequence of incorporating new norms and eliminating existing norms) and the changing of systems of norms as a consequence of the change of norms. These are the two main problems of legal dynamics.

These problems have not escaped the attention of legal philosophers and theoreticians, but they have not been successfully analyzed until very recent times, and even today there is no complete agreement on several topics.

Legislation as the deliberate incorporation and elimination of legal norms is certainly the main source of change in the law; consequently, the authors concentrate on acts of legislation (promulgation, amendment, and derogation) and the consequences that such acts produce in a legal system. They express their hope that other kinds of change stemming from custom or precedent can be analyzed in a similar way. Moreover, as amendment is nothing more than the combination of derogation and promulgation, we can dispense with it.

Navarro and Rodríguez analyze the acts of promulgation and derogation of norms and the consequent indeterminacy of the resulting system that is produced under certain conditions, following the lines of the analysis of C. Alchourrón and E. Bulygin. But they simplify considerably the whole issue by rejecting the idea that the logical consequences of promulgated norms are also valid norms of the system. In short, in their view, derived legal norms are not necessarily valid, but they must necessarily be taken into account in the application of legal norms and for the explanation of legal dynamics; they belong to the set of applicable norms and play an important part in the dynamics of law. I do not quite agree with this tenet, but the arguments they produce in its support certainly deserve close attention.

Perhaps the main problem of legal dynamics is the characterization of the concept of a legal order that, in spite of change in its contents, preserves its identity over the course of time. Whereas the notion of a legal system understood as a set of legal norms correlated to a given temporal point (momentary system in the terminology of Joseph Raz) is a static concept, the notion of a legal order is dynamic. It is a temporal sequence of legal systems (a family – that is, a set of sets of norms). Its identity is given by the identity of the criteria for the identification of norms belonging to the systems of this sequence. Therefore, to give an account of the structure of law, the interplay of three different concepts is necessary: the momentary legal system (a set of legal norms valid at a certain temporal moment), the applicable system (a set of

legal norms relative to the solution of a certain individual case), and the legal order (a sequence of momentary systems).

A few words about the authors of this book are in order. They belong to a relatively young generation of Argentinean legal philosophers, but they are already well known internationally. Pablo Navarro has been teaching in Córdoba (Argentina), in Barcelona (Spain), and in México. He is now a full professor in Bahía Blanca, Argentina and a member of the Research Council of Argentina (CONICET), and he also teaches at Blaise Pascal University (Cordoba). Jorge Rodríguez is a professor at the University of Mar del Plata, Argentina, and was awarded the Young Scholar Prize of the IVR (International Association for Legal and Social Philosophy) in 1999. Both of them have published several books[13] and a considerable number of papers in well-known philosophical journals, and they have participated in many international conferences in Europe and in America.

They have not been, technically speaking, my students, but in an extended sense they can be regarded as such. At least I regard them as my former students, who have the disagreeable property of having surpassed their teacher.

An interesting feature of legal philosophy in countries with Latin tradition, especially in Argentina, Spain, and Italy, is the relatively large number of joint publications, not found as frequently in other disciplines. This highlights friendship, frequent dialogue, and intense discussions and is (at least partly) responsible for the high level of philosophical production of the younger generation of legal philosophers.

[13] The following books deserve special mention: José Juan Moreso and Pablo E. Navarro, *Orden jurídico y sistema jurídico*, Centro de Estudios Constitucionales, Madrid, 1993; Jorge L. Rodríguez, *Lógica de los sistemas jurídicos*, Centro de Estudios Constitucionales, Madrid, 2002; Jordi Ferrer Beltrán and Jorge Rodríguez, *Jerarquías normativas y dinámica de los sistemas jurídicos*, Marcial Pons, Madrid-Barcelona-Buenos Aires, 2011.

Preface

In *The Critique of Pure Reason*, Kant claims that from the time of Aristotle until his time, logic had been "unable to take a single step forward, and therefore seems to all appearance to be finished and complete."[1] Moreover, he adds that some alleged improvements were only minor changes or, even worse, confusing and full of misunderstandings. But contrary to this vision, the last two centuries have witnessed an extraordinary rebirth of logic. New approaches to classical problems, as well as new horizons opened to logical exploration (e.g., modal logic, the logic of relevance, the logic of action), have gained a legitimate reputation in contemporary philosophy.

One of these new logical domains is *deontic logic*, the branch of logic that offers a formal analysis of normative discourse. Law is one of the most important normative fields, and deontic logic constitutes an invaluable aid for legal scholars and philosophers in the analysis of fundamental legal concepts. More specifically – as we try to show – deontic logic can be regarded as an essential tool to understand both the systematic structure of law and its dynamic nature. Undoubtedly, deontic logic is also useful for the evaluation of moral discourse, but in this book we limit our attention to the legal domain, with very few and merely incidental remarks on morality.

Are legal norms prescriptions or propositions? Is it possible to develop a logical system referred to objects that are not proposition-like entities? What does it mean to claim that norm N1 is a logical consequence of another norm N2? Can legal arguments be grounded on the fact that a certain solution is implicit in the content of explicitly enacted legal norms? Is logic relevant for understanding the dynamic nature of law? These are the kind of questions that are central in our analysis, and their answers reveal part of the relevance of deontic logic in law and legal theory. Two related aspects are particularly

[1] Kant 1781: 106.

important here. On the one hand, deontic logic is a necessary conceptual device used to make clear the *implicit* content of law. In a certain sense, law is not exhausted by the explicit material provided by legal sources, but also includes the consequences that follow from explicitly enacted norms. On the other hand, the *structure* of legal systems is determined by relations that connect their elements, and to the extent that logical consequences are regarded as legally binding, deontic logic seems unavoidable in the explanation of the systematic nature of law.

In *The Concept of Law*, H. L. A. Hart mentions some recurrent issues that are responsible for the persistence of the debate about the concept of law. One issue concerns the relations between law and morality.[2] He points out three things that relate law and morality: a shared vocabulary, coincident contents, and practical normative force. These connections explain to a certain extent our bewilderments when we try to determine the (conclusive) solutions that a particular legal system offers to certain recalcitrant cases. Law and logic exhibit at least two of these three coincidences that relate law and morality: a shared vocabulary and practical normative force. First, law and logic have a rich vocabulary in common. Expressions such as "rules," "reasoning," "justification," "interpretation," "validity," "systems," "coherence," "syllogism," "proof," and "decision" are basic concepts of both disciplines; of course, one can wonder whether this fact is actually something more than a "linguistic accident."

Although the origin of logic was connected to the control of legal arguments,[3] it is somewhat ironic that in modern times both law and legal reasoning have been often regarded as not being governed by logical structures and forms.[4] By contrast with this skeptical view, in this book we claim that a better understanding of deontic logic and logical analysis is of the utmost importance in the study of law and legal theory. The relations between logic and legal theory have followed two different perspectives that can be sketched as follows:

(1) *The logical study of norms and normative systems*. The central issues from this perspective are the existence of norms, the distinctive features of normative actions, the systematic structure of normative sets, the formal properties of normative systems, and so on. This approach has been developed by

---

[2] See Hart 1961: 7. Other recurrent issues are the relations between law and force and the relations between law and rules (see Hart 1961: 6–13).

[3] See von Wright 1993: 10–11.

[4] This view is reflected in the famous words of O. W. Holmes in the opening paragraph of *The Common Law*: "the life of the law has not been logic; it has been experience" (Holmes 1881: 5).

philosophers von Wright, Weinberger, Alchourrón and Bulygin, and Soeteman, among others. In this view, the fundamental challenges derive from questions like: Is a logic of norms possible? What is the logical structure of conditional norms? What are the logical properties of normative systems?

(2) The logical study of normative reasoning. The central issues from this perspective are the logical analysis of normative reasoning and the formal reconstruction of normative positions (rights, responsibility, competence, etc.). The main contributions to this field have been made by Hohfeld, Kanger, Lindahl, Hintikka, Prakken, Makinson, A. Jones, Sartor, and others. In this view, the fundamental challenges derive from questions such as: What are the relations between the different normative positions (e.g., rights and duties)? What is the most adequate formal structure to account for the set of normative relations involved in the notion of legal power or right? What are the differences between conclusive and *prima facie* obligations? What logical rules must follow a sound defeasible legal argument?

Both approaches are related through many common problems, but despite those connections, it seems wise to deal with each of these perspectives in a relatively independent way. In this book we favor the first approach, and the main reason for choosing it is that it seems to offer a more natural perspective for some basic philosophical problems of deontic logic, such as the nature of norms and the kind of fact that makes the statement that a certain norm exists true.[5] The answers to those questions seem more philosophical than logical and, consequently, it seems more convenient to introduce deontic logic by means of a conceptual analysis of the basic concepts of law and legal theory. As we try to show, no genuine progress on the ontology of norms can be achieved without a careful reconstruction of other more general concepts. Accordingly, the problem of the fruitful application of deontic logic to a theory of norms cannot be entirely detached from other classical problems of legal philosophy.

Over the past decades, the nature of law has been explained in terms of normative systems, and two ideas have been widely accepted. First, municipal laws, like Argentinean law or U.S. law, are mainly legal *systems*. Second, although legal norms are the most important elements of a municipal law, they also contain other elements. In other words, legal systems are *normative* systems, but their structure is more complex than the one presented by classical legal theories (e.g., Bentham, Austin, Kelsen).[6]

---

5 Von Wright 1969: 89. See also von Wright 1980: 404.
6 See Gardner 2004.

In contemporary legal theory the analysis of legal systems as a methodological device for understanding law was decisively influenced by two important works: Joseph Raz's *The Concept of a Legal System* and Alchourrón and Bulygin's *Normative Systems*.[7] From different perspectives, both books explicitly defend the priority of a systematic reconstruction of law over other traditional approaches, and both also insist on the role played by laws that are not norms (e.g., definitions, derogatory clauses). However, there are also important differences between them, which arise from the fact that the systematic nature of law is explained differently by two traditions in legal theory. Those traditions may be called the *logical model* and the *institutional model*. Whereas Alchourrón and Bulygin emphatically subscribe to the first, Joseph Raz is the most important defender of the second.

These different traditions sharply disagree on the role that logic plays in law and legal systems. Following the logical model, authors like Weinberger, Alchourrón and Bulygin, Kanger, and Lindahl understand the law as a *deductive system* (i.e., legal systems include all the logical consequences of enacted norms). According to this conception – widely accepted by deontic logicians[8] – law is much more than a set of explicitly enacted norms, as it also encompasses *implicit* norms that can be logically derived from a specific normative base. The conceptual content of such a base cannot be fully grasped without deriving its logical consequences.

According to the institutional model, legal norms are internally related by different criteria, the most important of them being the *genetic* criterion (i.e., that a certain norm belongs to a legal system if it has been created by a competent authority). This criterion helps explain the dynamic character of legal systems, the hierarchical structure of law, and its institutional nature. However, Raz stresses that genetic relations are not the only relevant internal connection between legal norms. In his opinion, genetic relations only identify raw legal materials but are unable to show the legal positions that law attributes to individuals at a particular time, which are determined by specific principles of individuation of laws reconstructed by legal philosophers. These principles of individuation show the internal connections between laws that define the *operative structure* of a legal system at a certain time.[9]

7   Raz 1970; Alchourrón and Bulygin 1971.
8   For example, see Gärdenfors 1992b: 195.
9   One of the most important insights of Raz is that legal systems are *momentary* systems, and that their structure (i.e., *operative* structure) mainly depends on punitive and regulative relations that we can identify at a particular moment. On the contrary, genetic relations show the development of the legal history of a certain community; they determine the structure of

Both models are deeply entrenched in our legal culture. Their main func-
tion seems to be connected to our understanding of the truth-conditions of
legal statements. The institutional model helps us explain the *legal* nature
of our rights and duties, as something different from other normative quali-
fications (e.g., *moral* ones). On the other hand, the logical model shows the
reason why some norm-propositions can be true even if no explicit decision
of legal authorities justifies such a claim. Thus, these approaches seem to be
in mutual tension, for the institutional model emphasizes that some norms
are not legally valid because they have not been enacted by legal authorities,
whereas the logical model emphasizes why some norms are legally valid even
though legal authorities have not (explicitly) prescribed them.

Several arguments explaining the conceptual connection between law and
genetic relations among norms (e.g., Kelsen, Hart) have been offered. How-
ever, a similar argument showing the relevance of the logical consequences
of legal norms is actually missing. Defenders of the deductive model seem to
have confused the problem of the possibility of logical relations among norms
with the different question of the legal validity of logical consequences of
enacted legal norms (i.e., they seem to believe that a positive answer to the
former question is also sufficient to ground a positive answer to the latter).
However, this is a fallacy; a distinct argument is needed to move from the
premise that there are logical relations among norms to the conclusion that
those consequences are valid in a legal system. Therefore, the role of logic in
an adequate explanation of the systematic nature of law is an open question
in legal theory.

Our purpose in this book is to offer an introduction to deontic logic and legal
systems. We will be more interested in the philosophical aspects of deontic
logic (i.e., philosophical logic) than in its technical refinements. Important
achievements during the past decades have been made in the domain of
deontic logic, through the application of sophisticated analytical techniques
to control normative arguments, especially those advanced by judges and
legal scholars in the interpretation of legal sources and the justification of
legal decisions. However, these developments usually focus their attention on
the formal complexities of legal reasoning that go far beyond the ordinary
comprehension of lawyers, and thus have generated few applicable results
(apart from the very remarkable fact that they contribute to the development
of logic itself). For this reason, and in spite of the progress that deontic logic
registers in the formal analysis of legal reasoning, we believe that a book

non-momentary legal systems (see Raz 1970). Unfortunately, as we try to show, Raz's analysis
of the logical relations between momentary and non-momentary systems is defective.

devoted to the philosophical problems of deontic logic is the most natural way for introducing legal scholars into the formal aspects of norms and legal systems. Hence, this book aspires to be an analytical exercise on the logic of norms and legal systems.[10]

Although there is a rich literature on logic and law, it is not easy to find introductory texts. In general terms, an introductory philosophical text can be seen from two different perspectives: it can work as a *preliminary analysis* of a complex philosophical issue, or it can be presented as a *basic* text concerning a certain discipline. In the first case, the introductory nature means a *partial* approach to a given problem; it offers a necessary step for analyzing other, more intricate problems. Such limitation says nothing about the philosophical complexity of the work at hand, and frequently the reading of these kinds of texts requires a great deal of information and philosophical skill. In the second case, an introductory philosophical text makes almost no assumptions regarding a certain discipline; it offers to the nonspecialized reader a conceptual map for exploring some problems. In general, the virtues of this type of text relate to the clarity and simplicity of the analysis, the organization of a complex agenda of problems, and the emphasis on central questions rather than the examination of details. This is precisely one of the aims of our project: to provide a general and basic introduction to the logic of norms to demonstrate its relevance and utility for a rational reconstruction of some fundamental legal concepts. Its introductory character will be seen in the abundance of quotations and bibliographical notes, the preference for arguments that do not require formalization, and the avoidance of technicalities.

The book is divided in two parts, each part is composed of three chapters, and each chapter is divided in five sections. In the first part we offer a brief reminder of the basic ideas on classical logic (i.e., propositional and predicate calculus). Only after this basis has been presented do we move on to deontic logic. We provide a brief reference to the origins of deontic logic, the discussion of the problem of its foundations (i.e., its contested possibility), and advance an explanation of the most important topics of the discipline, comparing different systems of deontic logic (e.g., *minimal*, *classic*, and *standard* systems), exploring some of the most important paradoxes of deontic logic, and so on. Special attention is given to the distinction between a genuine logic of norms

---

[10] Following von Wright, we understand by philosophical logic "the analysis of concepts which are peculiar to logic proper – such as, for example, consistency and entailment – and the application of the formal apparatus of logic for clarifying clusters of concepts" (von Wright 1993: 42). For a more general approach to philosophical logic, see Grayling 1997.

and a logic of propositions *about* norms, and to the problematic character of conditional norms.

In the second part, we analyze legal systems. Our concern is focused on three objectives: (1) to underline the relevance of a systematic reconstruction of law as a necessary step in the identification of the truth-conditions of legal statements; (2) to revise the role that logic plays in the analysis of legal systems; and (3) to discuss the reasons for accepting or rejecting the validity of logical consequences of enacted legal norms.

We are aware that writing an introductory text is a risky effort. It is always a simplification of problems that deserve a refined treatment and, even worse, in some cases it conceals the philosophical relevance of details that are essential for understanding a certain discipline. Even being conscious of those perils, we are firmly convinced that a philosophical analysis of both the logic of norms and the systematic structure of law can be presented without unnecessary logical technicalities. Thus, although many problems that need a careful analysis in an advanced book have been put aside, we expect that this introductory text promotes the curiosity necessary for dealing with deeper levels of both deontic logic and legal theory.

Many colleagues and friends have read different parts of this book and their criticisms and suggestions have been essential for improving both the content and style of our work. We would especially like to thank Eugenio Bulygin, Luís Duarte d'Almeida, Riccardo Guastini, Bonnie Litschewski Paulson, Juliano Maranhão, Leticia Morales, José Juan Moreso, Claudina Orunesu, Stanley L. Paulson, Pablo Perot, Giovanni Ratti, Cristina Redondo, and Hugo Zuleta. We are also highly indebted to Shailan Patel and Stanley L. Paulson, who have revised the English-language manuscript, for their invaluable services. Last but not least, we are greatly indebted to Professors Brian Bix and William A. Edmundson, editors of *Cambridge Introductions to Philosophy and Law*, for having encouraged the publication of this book and their careful reading, comments, and criticism on an earlier version of the manuscript.