Probability

Theory and Examples

Fourth Edition

This book is an introduction to probability theory covering laws of large numbers, central limit theorems, random walks, martingales, Markov chains, ergodic theorems, and Brownian motion. It is a comprehensive treatment concentrating on the results that are the most useful for applications. Its philosophy is that the best way to learn probability is to see it in action, so there are 200 examples and 450 problems.

Rick Durrett received his Ph.D. in operations research from Stanford University in 1976. After nine years at UCLA and twenty-five at Cornell University, he moved to Duke University in 2010, where he is a professor of mathematics. He is the author of 8 books and more than 170 journal articles on a wide variety of topics, and he has supervised more than 40 Ph.D. students. He is a member of the National Academy of Science and the American Academy of Arts and Sciences and a Fellow of the Institute of Mathematical Statistics.

CAMBRIDGE SERIES IN STATISTICAL AND PROBABILISTIC MATHEMATICS

This series of high-quality upper-division textbooks and expository monographs covers all aspects of stochastic applicable mathematics. The topics range from pure and applied statistics to probability theory, operations research, optimization, and mathematical programming. The books contain clear presentations of new developments in the field and also of the state of the art in classical methods. While emphasizing rigorous treatment of theoretical methods, the books also contain applications and discussions of new techniques made possible by advances in computational practice.

Already Published

1. *Bootstrap Methods and Their Application*, by A. C. Davison and D. V. Hinkley
2. *Markov Chains*, by J. Norris
3. *Asymptotic Statistics*, by A. W. van der Vaart
4. *Wavelet Methods for Time Series Analysis*, by Donald B. Percival and Andrew T. Walden
5. *Bayesian Methods*, by Thomas Leonard and John S. J. Hsu
6. *Empirical Processes in M-Estimation*, by Sara van de Geer
7. *Numerical Methods of Statistics*, by John F. Monahan
8. *A User's Guide to Measure Theoretic Probability*, by David Pollard
9. *The Estimation and Tracking of Frequency*, by B. G. Quinn and E. J. Hannan
10. *Data Analysis and Graphics Using R*, by John Maindonald and John Braun
11. *Statistical Models*, by A. C. Davison
12. *Semiparametric Regression*, by D. Ruppert, M. P. Wand, and R. J. Carroll
13. *Exercise in Probability*, by Loic Chaumont and Marc Yor
14. *Statistical Analysis of Stochastic Processes in Time*, by J. K. Lindsey
15. *Measure Theory and Filtering*, by Lakhdar Aggoun and Robert Elliott
16. *Essentials of Statistical Inference*, by G. A. Young and R. L. Smith
17. *Elements of Distribution Theory*, by Thomas A. Severini
18. *Statistical Mechanics of Disordered Systems*, by Anton Bovier
19. *The Coordinate-Free Approach to Linear Models*, by Michael J. Wichura
20. *Random Graph Dynamics*, by Rick Durrett
21. *Networks*, by Peter Whittle
22. *Saddlepoint Approximations with Applications*, by Ronald W. Butler
23. *Applied Asymptotics*, by A. R. Brazzale, A. C. Davison, and N. Reid
24. *Random Networks for Communication*, by Massimo Franceschetti and Ronald Meester
25. *Design of Comparative Experiments*, by R. A. Bailey
26. *Symmetry Studies*, by Marlos A. G. Viana
27. *Model Selection and Model Averaging*, by Gerda Claeskens and Nils Lid Hjort
28. *Bayesian Nonparametrics*, by Nils Lid Hjort, Peter Müller, and Stephen G. Walker
29. *From Finite Sample to Asymptotic Methods in Statistics*, by Pranab K. Sen, Julio M. Singer, and Antonio C. Pedroso de Lima
30. *Brownian Motion*, by Peter Mörters and Yuval Peres

Probability

Theory and Examples

Fourth Edition

RICK DURRETT
Department of Mathematics, Duke University

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521765398

First edition published 1991 by Wadsworth Publishing
Second edition published 1995 by Duxbury Press
Third edition published 2004 by Duxbury Press
Fourth edition published 2010 by Cambridge University Press

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication data

Durrett, Richard, 1951–
Probability : theory and examples / Rick Durrett. – 4th ed.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-76539-8 (hardback)
1. Probabilities. I. Title.
QA273.D865 2010
519.2–dc22 2010013387

ISBN 978-0-521-76539-8 Hardback

Contents

Preface

In 1989 when the first edition of this book was completed, my sons David and Greg were 3 and 1, and the cover picture showed the Dow Jones at 2650. The past 20 years have brought many changes, but the song remains the same. The title of the book indicates that as we develop the theory, we will focus our attention on examples. Hoping that the book would be a useful reference for people who apply probability in their work, we have tried to emphasize the results that are important for applications, and have illustrated their use with roughly 200 examples. Probability is not a spectator sport, so the book contains almost 450 exercises to challenge readers and to deepen their understanding.

This fourth edition has two major changes (in addition to a new publisher):

(i) The book has been converted from TeX to LaTeX. The systematic use of labels should eventually eliminate problems with references to other points in the text. In addition, the picture environment and graphicx package has allowed the figures lost from the third edition to be reintroduced and a number of new ones to be added.

(ii) Four sections of the old appendix have been combined with the first three sections of Chapter 1 to make a new first chapter on measure theory, which should allow the book to be used by people who do not have this background without making the text tedious for those who have.

Acknowledgments. I am always grateful to the many people who sent me comments and typos. Helping to correct the first edition were David Aldous, Ken Alexander, Daren Cline, Ted Cox, Robert Dalang, Joe Glover, David Griffeath, Phil Griffin, Joe Horowitz, Olav Kallenberg, Jim Kuelbs, Robin Pemantle, Yuval Peres, Ken Ross, Steve Samuels, Byron Schmuland, Jon Wellner, and Ruth Williams.

The third edition benefited from input from Manel Baucells, Eric Blair, Zhen-Qing Chen, Finn Christensen, Ted Cox, Bradford Crain, Winston Crandall, Amir Dembo, Neil Falkner, Changyong Feng, Brighten Godfrey, Boris Granovsky, Jan Hannig, Andrew Hayen, Martin Hildebrand, Kyoungmun Jang, Anatole Joffe, Daniel Kifer, Steve Krone, Greg Lawler, T. Y. Lee, Shlomo Levental, Torgny Lindvall, Arif Mardin, Carl Mueller, Robin Pemantle, Yuval Peres, Mark Pinsky, Ross Pinsky, Boris Pittel, David Pokorny, Vinayak Prabhu, Brett Presnell, Jim Propp,

Yossi Schwarzfuchs, Rami Shakarchi, Lian Shen, Marc Shivers, Rich Sowers, Bob Strain, Tsachy Weissman, and Hao Zhang.

New helpers for the fourth edition include John Angus, Phillipe Charmony, Adam Cruz, Ricky Der, Justin Dyer, Piet Groeneboom, Vlad Island, Elena Kosygina, Richard Laugesen, Sungchul Lee, Shlomo Levental, Ping Li, Fredddy López, Lutz Mattner, Piotr Milos, Davey Owen, Brett Presnell, Igal Sason, Alex Smith, Laurent Tournier, Harsha Wabgaonkar, John Walsh, Tsachy Weissman, Neil Wu, Ofer Zeitouni, Martin Zerner, and Andrei Zherebtsov. I apologize to those whose names have been omitted or are new typos.

Family update. David graduated from Ithaca College in May 2009 with a degree in print journalism, and like many of his peers is struggling to find work. Greg has one semester to go at MIT and is applying to graduate schools in computer science. He says he wants to do research in "machine learning," so perhaps he can write a program to find and correct the typos in my books.

After 25 years in Ithaca, we moved to Durham in June 2010 and I have taken a position in the math department at Duke. Everyone seems to focus on the fact that we are trading very cold winters for hotter summers and a much longer growing season, but the real attraction is the excellent opportunities for interdisciplinary research in the Research Triangle.

The more things change, the more they stay the same: inevitably there will be typos in the new version. You can email me at rtd@math.duke.edu

Rick Durrett, July 2010