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978-0-521-76491-9 - Theory of Finite Simple Groups II: Commentary on the Classification Problems

Gerhard Michler

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Theory of Finite Simple Groups II

Commentary on the Classification Problems

This second volume provides a coherent explanation for the existence of the known 26 sporadic simple groups originally arising out of many unrelated contexts. Chapter 1 presents a new algorithm constructing centralizers of 2-central involutions of finite simple groups from indecomposable subgroups of the general linear groups $GL_n(2)$ using the representation theoretic and algorithmic methods developed in the first volume. It is shown that 25 sporadic simple groups can be constructed by this algorithm. The smallest Mathieu group M_{11} can be omitted for theoretical reasons. The algorithm is not restricted to sporadic simple groups as is shown explicitly in Chapters 2 and 10.

Here the author describes the constructions of Conway's groups Co_3 , Co_2 , Co_1 , Fischer's groups Fi_{22} , Fi_{23} , Fi'_{24} , Janko's group J_4 , McLaughlin's group McL , Rudvalis' group Ru , Lyons' group Ly , Suzuki's group Suz , and O'Nan's group ON . Their uniqueness is proved whenever possible. The computational existence proofs are documented in the accompanying DVD. Chapter 16 outlines such theoretically possible constructions for the baby monster and the monster from well determined indecomposable subgroups of $GL_9(2)$ and $GL_{10}(2)$, respectively. The other sporadic groups were constructed in volume I.

The present mathematical literature does not contain an accessible proof of the announced classification theorem asserting that there are exactly 26 sporadic simple groups. On the other hand the literature has not paid much attention to R. Brauer's warning (published in 1979) that there may be infinitely many sporadic groups. Therefore the author describes Brauer's ideas on a general classification scheme in Chapter 1 and states several related open problems in Chapter 16. Some require new experiments with the author's algorithm.

GERHARD MICHLER is an Emeritus Professor of the Institute of Experimental Mathematics at the University of Duisburg-Essen and Adjunct Professor at Cornell University.

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In Memory

of

Ruth I. Michler

1967 (Ithaca, NY)–2000 (Boston, MA)

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