Electromechanics and MEMS

Offering a consistent, systematic approach to capacitive, piezoelectric, and magnetic MEMS, this textbook equips students to design and develop practical, system-level MEMS models.

- Includes a concise yet thorough treatment of the underlying principles of electromechanical transduction.
- Makes extensive use of easy-to-interpret electrical and mechanical analogs, such as electrical circuits, electromechanical two-port models, and the cascade paradigm.
- Each chapter features extensive worked examples, and numerous homework problems.

Thomas B. Jones is Professor of Electrical Engineering at the University of Rochester. An experienced educator involved in teaching for over 40 years, his research has focused on electric field-mediated manipulation and transport of particles and liquids. He holds a Ph.D. from MIT, is the author of Electromechanics of Particles (Cambridge University Press, 1995) and is a Fellow of the IEEE.

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“This is an excellent textbook presenting the fundamentals of electromechanics required by every practicing MEMS engineer. The authors treat the arduous concepts of coupled electrical and mechanical systems simultaneously with lucidity and a thorough pedagogical rigor that comes from deep appreciation of the field and the love to impart that knowledge as a teacher. The book elucidates the concepts with very topical examples of microelectromechanical systems such as MEMS microphones, comb drive actuators, gyroscopes, energy harvesters, and piezoelectric and magnetic devices, including MATLAB models and a comprehensive set of problems at the end of each chapter.”

Srinivas Tadigadapa, The Pennsylvania State University

“A fantastic book for the student seeking a solid foundation in electromechanical device design and an essential reference for the expert MEMS engineer. Jones and Nenadic present the fundamental theory behind electromechanical transduction, with a focus on capacitive drive and sense microsystems. The authors systematically frame the device fundamentals into real world microscale device applications that provide relevance to the underlying physics. This book captures and dutifully explains the foundational physics at work in the MEMS devices we often unknowingly use daily in our automobiles, mobile phones and electronic devices.”

Chris Keimel, GE Global Research

“Electromechanics and MEMS is a thorough treatment of fundamental MEMS analysis for both the student and the practitioner. The readers are presented with the tools to methodically build system models that are comprehensive yet manageable.”

Eric Chojnacki, MEMSIC, Inc.
Electromechanics and MEMS

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Preface

The growing interest in microsystems, and particularly in MEMS technology, has reasserted electromechanics as a key discipline. This book fills the need for a textbook that presents the fundamentals of electromechanics, classifies structures according to their functional capabilities, develops systematic modeling methods for the design of MEMS devices integrated into electronic systems, and provides practical examples derived from selected microdevice technologies. It is written for engineering students and physical science majors who want to learn about such systems. A further ambition is that the book will find its way slowly onto the shelves of practicing engineers involved in MEMS design and development.

Organization

The organization proceeds from basics to systems-oriented applications. The first three chapters focus on fundamentals of circuits and lumped parameter electromechanics. Chapter 1 provides some historical context, introducing key terminology and then offering a general description of electromechanical transducers based on power and energy considerations. Chapter 2 introduces the crucial concept of circuit-based modeling. Because the vast majority of MEMS devices are capacitive, this chapter focuses on circuits with capacitors and resistors. Chapter 3, drawing heavily on Part 1 of H. H. Woodson and J. R. Melcher's text, *Electromechanical Dynamics*, presents the classic, energy-based formulation for electromechanical interactions. The treatment here differs from their text by concentrating on capacitive microelectromechanical devices and introducing the geometries and dimensions characteristic of MEMS technology.

Next, we present in detail a systematic method for modeling and then analyzing practical MEMS. Chapter 4 introduces the general small-signal transducer formalism found in H. K. P. Neubert's classic text, *Instrument Transducers*. This formalism serves as the robust backbone for the chapters that follow. Representing the coupled small-signal behavior – mechanical and electrical – of a MEMS device is the key to developing a systematic, integrated coverage of the transient dynamics, frequency response, and mechanical stability of transducers and sensors.¹ The critical design and performance

issues can be addressed in terms of a mechanical variable-dependent capacitance, $C(x)$, and its first two derivatives, i.e., $dC/dx$, and $d^2C/dx^2$. Small-signal analysis also facilitates the modeling of virtually any cascaded system-on-a-chip comprising both the MEMS device and the electronics that controls or monitors it.

The next three chapters deal in some depth with small-signal modeling of practical MEMS geometries, such as cantilevered beams, membranes, and diaphragms, and practical systems, such as pressure sensors, actuators, and gyroscopes. Chapter 5 introduces some operational amplifier-based electronics topologies for capacitive sensing devices. While the treatment is elementary, some prior exposure to electronic circuits may be helpful. These circuits are employed with variable-gap and variable-area capacitive sensors configured in single-ended and half-bridge configurations, with both DC and AC excitation. The chapter includes a brief summary of amplitude- and double-sideband suppressed carrier modulated schemes for capacitive sensors. Then, after a very concise presentation of noise, the advantages of modulated schemes for circumventing the often-prevailing $1/f$ noise are revealed. The chapter concludes with a consideration of phase-locked loop drives for resonant actuators.

Chapter 6 is devoted to mechanical modeling of deformable continua, such as cantilevered beams and plates. It is shown by analytical approximation that beams and plates can be reasonably well represented by linear, single-degree-of-freedom models and formulated as electromechanical two-ports, as introduced in Chapter 4. The more general, multiple-degree-of-freedom modeling approach is then presented. With proper attention given to the existence of multiple resonant modes, reduced order modeling is again restored. Practical MEMS devices amenable to this approach include pressure sensors, microphones, mirror arrays, and energy harvesters. Chapter 7 focuses on a few important examples of MEMS devices that utilize identifiable mechanical continua, including pressure transducers, accelerometers, gyroscopes, and energy harvesters. Specific geometries for each of these devices are considered in turn, with small-signal lumped parameter models as outcomes of the modeling exercise. This approach provides an opportunity to introduce important design considerations and higher-order effects that influence practical MEMS.

Because piezoelectric technology is important in certain microdevice applications, and will probably remain so, Chapter 8 offers a brief presentation of this subject. First, the standard phenomenological models, expressed in terms of mechanical and electrical field variables, are presented for the three important piezoelectric effects, that is, longitudinal, transverse, and shear modes. Then, adhering to the strategy of the previous chapters, these models are reduced to small-signal, two-port circuits useful in analysis of actuator and sensor devices.

Chapter 9, the last, offers a concise, largely self-contained coverage of the electromechanics of microscale magnetic transducers and linear, small-signal models for them. This chapter was prepared to anticipate likely breakthroughs in materials processing and fabrication methods for high-aspect ratio magnetic MEMS devices.\(^2\) The

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MEMS device applications
- Examples of applications: pressure sensors, accelerometers, gyroscopes, energy harvester (Chap 7)

Mechanical side
- Mechanical continua: beams & plates (Chap. 6)
- Mechanical systems (App. B)
- Mechanics of solids (App. D)

Electromechanical conversion
- Capacitive transducers (Chap. 3)
- Linearized transducers (Chap. 4)
- Piezoelectric devices (Chap. 8)
- Magnetic transducers (Chap. 9)

Electrical side
- Circuit-based models (Chap. 2)
- Circuits (Chaps. 5, 8, 9)
- Electric/magnetic fields (App. A)

Review of fabrication basics
- App. C

A block diagram representation of the organization of this textbook.

organization largely parallels the far more extensive treatment of capacitive devices found in Chapters 3, 4, and 5. A brief presentation of the basics of lumped parameter magnetic transducers comes first. Then, Neubert’s small-signal formulation and the accompanying two-port circuit models are presented. The basic types of magnetic transducer are exemplified, as much as possible by introducing geometries that have already been or could be fabricated on the MEMS scale. The chapter concludes by presenting analyses of current bridge and linear variable differential transformer based sensors connected to operational amplifiers.

The diagram above reveals the organization of the text in a format mimicking standard block diagrams used throughout the book to represent general electromechanical systems. Thus, sections of the text that address the electromechanical conversion, the mechanical system, the electrical system, and MEMS device applications are separated into individual blocks.

The appendices found at the end of the text provide essential background and summaries of topics that intersect MEMS technology. Appendix A contains a very brief review of certain essentials from electromagnetic field theory. Appendix B covers mechanical systems, principally resonance of single- and multiple-degree-of-freedom systems. Appendix C offers a concise review of MEMS fabrication technology. The solid mechanics of typical MEMS structures, such as beams and plates, is covered in Appendix D.

Numerous examples and end-of-chapter problems reinforce and extend the important principles. A few more challenging design-oriented exercises have been included in some of the later chapters, and these are specially marked. Some of these exercises are good choices for assignment to teams composed of students from different disciplines.
The highly interdisciplinary nature of MEMS is daunting to newcomers. Our challenge in writing this text has been to offer a cohesive treatment of the subject using material chosen to strike the right balance between theory and practice. The book is intended for fourth-year undergraduates and beginning graduate students in mechanical, electrical, optical, and biomedical engineering, plus physics. To maintain vital linkages to basic models and principles as new topics are introduced and as extra detail becomes warranted, the text returns again and again to certain capacitor geometries, viz., variable-gap and variable-area structures, and to the circuit constraints of constant voltage and constant charge. Many examples and end-of-chapter problems take advantage of coverage of the two capacitor types introduced in the earlier chapters. For this reason, it is probably best to go through the book in sequence, at least as far as Chapter 7.

Preparation in basic circuit theory, complex numbers, first- and second-order differential equations, some linear algebra and matrices, plus standard physics courses on mechanics, electricity, and magnetism are essential. In a typical setting, one might expect electrical and mechanical engineering students to dominate a class population, the electrical engineers having more understanding of circuits and linear systems and the mechanical engineers bringing their knowledge of mechanics and materials. Such an interdisciplinary mix is fertile. We are confident that industrious students will be able to overcome their respective “deficiencies” and gain deep understanding of the important basics of microelectromechanical interactions and how real MEMS devices are integrated into real systems. Our experience in teaching this material is that motivated students have little difficulty in overcoming any initial unfamiliarity with topic areas falling outside their undergraduate preparation. In fact, engineering and mechanical engineers and other students quickly form effective study teams and assist each other quite effectively in learning the material.

The sections with headings flagged by an asterisk provide details on related but peripheral topics. These sections are optional and may be passed over safely in a one-semester course.

Some limitations

While the acronym MEMS now seems to cover virtually all micromechanical devices, whether or not their actuation or transduction mechanism is really electromechanical in nature, this book is limited to devices with a true electromechanical mechanism. Thus, there is no coverage of magnetoresistive, bimetallic, or other thermally actuated devices. There is clearly a need for such a text, but we are not the ones to provide it.

Our coverage of mechanical continua is highly focused. A special effort has been made to demonstrate that the performance of the most common mechanical continua, e.g., the cantilevered beam and the circular plate diaphragm, can be reasonably approximated by reduced-order, lumped parameter models. Such models will prove useful to the systems
engineer whose ambition is to integrate a MEMS device with the drive electronics in a real system.

The fabrication of MEMS is a diverse and ever-evolving enterprise. Because of the pace of change and development in the field, this text limits its coverage to the very concise summary of microfabrication found in Appendix C. Critical terminology and basic processing methods are introduced but MEMS packaging is not covered. We believe that reliance on the fine reference books already available is the best strategy for those ready to learn how to build MEMS devices.

Guide to the use of this text

The study of MEMS is a very broad and highly interdisciplinary subject. That being the case, the length of this text is more a testament to the breadth of the subject than to authorial diligence or ambitions for thoroughness. Indeed, we had great difficulty – and some vigorous arguments – about what to include and what to leave out. The instructor trying to decide whether or not to adopt this book faces a related dilemma; namely, does this text adequately cover the material for an established MEMS course with given objectives. The table on page xviii, which identifies the sections, examples, and end-of-chapter problems relevant to many of the important MEMS technologies, provides guidance in making the right decision. The information is organized by the set of technologies appearing in the leftmost column. This set is no doubt incomplete but, we think, reasonably illustrative.

Selected references on MEMS

Advanced students, design engineers, and researchers might not find adequate coverage of certain topics relevant to their specific interests. These individuals can refer to the many fine texts, monographs, and reference volumes available for assistance. These books, listed chronologically by date of publication and accompanied by very brief synopses of their contents, should be of help. Students are urged to familiarize themselves with some of these MEMS resources.

An early MEMS compendium is Micromechanics and MEMS, Classic and Seminal Papers to 1990 (IEEE Press, 1997), edited by W. D. Trimmer and containing seminal MEMS papers. This is required reading for students entering the field.

A general MEMS reference volume, Gregory T. Kovacs’ Micromachined Transducers Sourcebook (McGraw-Hill, 1998), offers a very broad survey of sensors and actuators, including some MEMS devices.

On the subject of MEMS microwave systems, Microelectromechanical (MEM) Microwave Systems (Artech House, 1999), by Héctor de los Santos, offers a concise treatment of MEMS devices and mechanisms applied in microwave systems. Microsystem Design (Kluwer Academic, 2001) by Stephen D. Senturia provides broad coverage of a large amount of material. Since its publication, practicing engineers in
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Guide to the use of this textbook organized by the more well-recognized MEMS technologies. Relevant sections found in the text itself, plus examples and end-of-chapter problems, are tabulated.

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the field have relied on this volume as a standard reference for the design of MEMS systems.


For mechanical modeling of MEMS devices, John A. Pelesko and David H. Berstein’s *Modeling MEMS and NEMS* (Chapman and Hall/CRC, 2003) chiefly concerns
modeling of beams, diaphragms, and other continua on the microscale. A particularly valuable feature is its coverage of numerical analysis methods relevant to MEMS devices.

On the subject of piezoelectric MEMS devices, *Micromechatronics* (Marcel Dekker, 2003), written by Kenji Uchino and Jayne R. Giniewicz, is the best modern reference available on ferroelectric phenomena. It provides treatment of the relevant solid mechanics and examples of piezoelectric devices.


Another general MEMS reference, *Comprehensive Microsystems* by Y. B. Gianchandani, O. Tabata, and H. Zappe, in three volumes (Elsevier, 2008), is exhaustively complete and up-to-date.

Finally, V. Kaajakari’s *Practical MEMS* (Small Gear Publishing, 2009) is a new textbook featuring coverage of many areas of MEMS with excellent practical examples distributed throughout the text.

Special acknowledgments

To provide the student with concrete examples of working MEMS devices, we have incorporated images of MEMS devices throughout the text. These inclusions were made possible through permissions granted by the engineers, students, and faculty researchers who created the images. We are humbly grateful for this generosity. Further, Weiqiang Wang obtained for us the SEM of the pyramidal etched pit shown that appears in Fig. C.13. We acknowledge James Moon, who thoroughly reviewed Appendix C, and Erica MacArthur, who helped us by preparing some of the SEM images. Additional assistance from Scott Adams, Zeljko Ignjatovic, Kelly Lee, Christopher Keimel, and Paul H. Jones is gratefully acknowledged.

Final note

The sources and inspirations for this text are many, and we can rightly claim full credit only for the errors. More than anything else, it was excellent undergraduate-level teaching that fostered our appreciation of electromechanics. The lead author (TBJ) was introduced to the subject in the Fall Semester of 1968 at MIT by Herman Schneider, who delivered crystal-clear, virtually error-free lectures without resort to any notes. A few years later, though not quite having mastered the ability to lecture without notes, TBJ got the chance to teach this same course. Anyone teaching the class in those days relied upon a thick, unwieldy binder of mimeographed notes, which were destined to become the textbook entitled *Electromechanical Interactions* and written by H. H. Woodson and J. R. Melcher.
In the first year of his electrical engineering undergraduate studies at the University of Novi Sad in Serbia, the second author (NN) was confronted by the requirement to take a course titled Introduction to Mechanics. Any doubts harbored about the value of this course were rapidly dispelled by the inspiring lectures of Dorde Đukić and Teodor Atanacković. In 1996, during the first year of his graduate studies at the University of Rochester, NN enrolled in a course entitled Transducers and Actuators. This course revealed MEMS technology to be a tightly woven fabric of mechanics, electricity and magnetism, circuit theory, electronics, and beam mechanics. The lecture notes and problems prepared for this course by TBJ served as the foundation of the present text.