1 Introduction

1.1 Background

The trend toward device miniaturization and large-scale integration, which has already revolutionized electronics, now promises a profound transformation of engineered mechanical systems, reducing their size by orders of magnitude while vastly increasing their capabilities. Microelectromechanical systems (MEMS) are now found in automotive airbag systems, computer projectors, digital cameras, gyroscopic sensors, and many other devices. Their small size invites a high degree of on-chip integration with essential drive, detection, and signal conditioning circuitry. These collective advances have spawned another new term, the *system-on-a-chip*, with its own inevitable acronym, SOC. Consider as an example the digital camera. Nowadays, even rather inexpensive models have MEMS chips installed to sense the camera's orientation with respect to gravity and to detect and compensate for the inadvertent jolts and motions of the picture taker. Such features would have been well beyond the expectations of the owner of even the most expensive SLR camera of 20 years ago. The capabilities mentioned above are made possible by mechanical devices with dimensions less than a millimeter or so, fabricated on a chip side by side with all the required control and drive electronics.

Additional evidence for the vitality of this new technology is that the microfabrication industry has appropriated the term *foundry* to describe their facilities. This word dates from sixteenth-century French. One of the authors (TBJ) has a vivid childhood recollection from the 1950s of the nightly spectacle of fumes and fire belching impressively from the venting chimneys atop a metal casting foundry in the small Midwestern city where he grew up. This plant produced manhole covers and other essential yet mundane components of the urban infrastructure. In this new century, however, the products manufactured in what we call a foundry range from heavy metal castings, with dimensions of meters, down to very intricate parts made of silicon and having dimensions of the order of microns.

As technology advances, etymological take-overs like this are sometimes accompanied by resurrections of words that fall out of favor but then find a path back to prominence. The term *electromechanics*, embedded in the acronym MEMS, provides a fine example of this phenomenon. Fifty years ago, an engineer specializing in electromechanics would certainly have anticipated a career devoted to relays, solenoids, motors, servos, or AC synchronous alternators. These critical components were inevitably magnetic devices and ranged in size from centimeters to meters. They still do. While the education of this engineer certainly would have involved perfunctory attention to capacitive devices, the greatest share of his time would have been spent on inductance, windings, large currents, and magnetic flux. A half-century ago, capacitive microphones and speakers were only starting to make inroads into established markets. There were no large, capacitively based electromechanical converters. There never will be, because the physics governing electromechanics unambiguously awards the advantage for power conversion applications to magnetic devices, at least in devices larger than approximately one centimeter.

Consider the modern microfabrication facility that calls itself a foundry but engages in the manufacture of tiny components with feature sizes as small as a few microns. The same laws of physical scaling that award the prize to magnetic devices of large dimensions strongly favor electrostatic, i.e., capacitive, devices for physical dimensions below approximately 10^2 microns. Refer to Section A.5 of Appendix A for a brief consideration of these scaling considerations. Indeed, the vast bulk of MEMS systems are capacitive. The flexibility of modern microfabrication technology in combination with designer ingenuity leads to MEMS actuator and sensor geometries that go far beyond the familiar parallel-plate capacitor to very novel structures, some of which could hardly have been manufactured at all 50 years ago. So, the twenty-first century still finds us making the components needed for the modern world in *foundries*, but this facility belches no smoke and fumes, the workers wear white suits, masks, and funny hats, the devices themselves are tiny, and they are apt to be actuated by electrostatic forces.

1.2 Some terminology

The growing interest in microsystems has reinvigorated *electromechanics* as a key engineering discipline. The term electromechanics is, in general, reserved to describe devices and systems that use electrical (or magnetic) forces to cause mechanical motion or sense motion and then induce measurable electrical signals. Electromechanics is subdivided into (i) electrical systems, where charge, voltage, capacitance, and small electrical current are important, and (ii) magnetic systems, where magnetic flux, induced voltage, inductance, and large currents are the critical quantities.

It is helpful to introduce some additional terminology to aid in categorizing intended applications of MEMS technology. We start with the word *transducer*, a surprisingly recent term dating from the 1920s. A transducer takes power in one form and converts it into another form. There are of course many types of transduction mechanisms. For example, a bimetallic strip converts thermal energy to mechanical form. Of more interest to us, an *electromechanical transducer* changes electrical power into mechanical form or mechanical to electrical.

Another important word, dating back to *c*. 1864, is *actuator*. This term is in fact derived from the verb *actuate*, which was first used in the seventeenth century and derives from Latin. An actuator is a mechanical device that creates or controls mechanical motion. Thus, an *electromechanical actuator* is a type of transducer that uses electrical input to create mechanical motion. Motors and electromechanical relays are traditional examples.

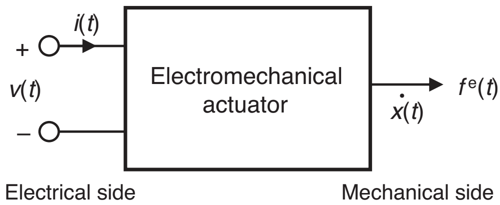


Figure 1.1 Basic electromechanical transducer with one electrical terminal pair and one mechanical degree of freedom. The system is assumed to be lossless.

A third term of importance to our subject is *sensor*. This word is also of early twentieth-century origin, though it springs from Latin. A sensor is any device that responds to some physical stimulus by emitting a signal. There are many, many examples of sensors. A particularly intriguing new one, coming from biochemistry, is the *chemical sensor*, where the output "signal" is actually a chemical response. Within our subject, MEMS, a microphone exemplifies an *electromechanical sensor* because it detects mechanical sound energy and converts it to voltage or current.

Another term may be introduced here, the *brake*. Braking action slows or dampens mechanical motion, and sometimes electromechanical systems are employed to achieve it. Such a brake may be thought of as a type of actuator.

1.3 Electromechanical systems

An electromechanical transducer is a device possessing mass and physical dimensions. It has electrical leads and at least one movable element. The heart of any such transducer is a mechanism for converting power between electrical and mechanical forms. Let us assume for expediency that the device has a single electrical terminal pair and a single mechanical degree of freedom. Although such restrictions will be abandoned later, we can learn much by considering this special case. We may represent this mechanism by the deceptively simple diagram of Fig. 1.1. For now, the electrical leads (terminals) are located on the left side and the mechanical system on the right. On the electrical side, there is a time-dependent voltage $v(t)$ and a time-dependent current $i(t)$. To represent the mechanical side, we need two more variables. For the present, it proves convenient to use the mechanical variable $x(t)$ along with the electrical force $f^{\mathrm{e}}(t)$ that the transducer exerts on the external mechanical system. Later, $x(t)$ will be replaced by velocity, that is, $\dot{x} = \mathrm{d}x/\mathrm{d}t$.

A useful physical interpretation of the nature of the force $f^{\mathrm{e}}(t)$ emerges from a consideration of power flow. The instantaneous electrical power input to the device is

$$p_{\mathrm{e}}(t) = v(t)\,i(t) \tag{1.1}$$

and the instantaneous mechanical power output is

$$p_{\mathrm{m}}(t) = f^{\mathrm{e}}(t)\,\dot{x}. \tag{1.2}$$

One might be tempted here to set $p_e(t) = p_m(t)$, but doing so is not justified because neither the nature of the electromechanical device itself nor what it is capable of doing with input electrical power has been specified. In particular, we must accommodate the possibility that the device can itself store energy.

To proceed, assume that the transducer is an electrically linear, capacitive device. Capacitance is defined as the coefficient of the *linear* relationship between electrical charge $q(t)$ and voltage $v(t)$,

$$q(t) = C[x(t)]\,v(t). \tag{1.3}$$

Equation (1.3) recognizes that the capacitance will be a function of time through its dependence on the mechanical variable $x(t)$. If charge conservation is now employed to express the current $i(t)$ using Eq. (1.3), then

$$i(t) = \frac{dq}{dt} = C(x)\frac{dv}{dt} + \frac{dC}{dx}\,v\,\frac{dx}{dt}. \tag{1.4}$$

This current expression contains the familiar *capacitive current*, $C(x)dv/dt$, plus another term that depends on the velocity, dx/dt. This new term, called the *motional current*, should be a tip-off that the device is more than a simple capacitor. Combining Eq. (1.4) with Eq. (1.1) yields

$$p_e(t) = C(x)\frac{dv}{dt}v + \frac{dC}{dx}\,v^2\frac{dx}{dt}, \tag{1.5}$$

which can be manipulated to obtain

$$p_e(t) = \frac{d}{dt}\left[\frac{C(x)}{2}v^2\right] + \frac{1}{2}\frac{dC}{dx}\,v^2\frac{dx}{dt}. \tag{1.6}$$

The first term on the right-hand side of Eq. (1.6) is the time derivative of a familiar quantity, $Cv^2/2$, the electrostatic energy storage associated with the capacitance. Evidently, some of the electrical power flowing into the transducer can be intercepted and stored in electrostatic form within the device rather than being converted directly to mechanical power. So, if we assume that there is no loss inherent in the transduction mechanism and that the electromechanical coupling depicted in Fig. 1.1 has no other mechanism of energy storage, then it is permissible to equate the remaining term in Eq. (1.6) to the mechanical output power, $p_m(t)$, defined by Eq. (1.2). Thus,

$$p_e(t) = \frac{d}{dt}\left[\frac{C(x)}{2}v^2\right] + p_m(t). \tag{1.7}$$

Combining Eqs. (1.2), (1.6), and (1.7) yields an expression for the force f^e:

$$f^e = \frac{v^2}{2}\frac{dC}{dx}. \tag{1.8}$$

A more rigorous approach to evaluating the force, f^e, based on a thermodynamic argument, is detailed in Chapter 3. Nevertheless, Eq. (1.8) is a correct and very general result. While the exclusion of loss from the coupling mechanism precludes consideration of certain types of transducer, such as bimetallic devices and shape memory alloys, it presents no restriction concerning true electromechanical transducers. Further, we will find in a later chapter that restricting the energy storage to the capacitive terms facilitates a systematic approach to modeling real transducers with both electrical and mechanical constraints.

Note that the force f^e, henceforth to be referred to as the *force of electrical origin*, is proportional to the product of the square of the voltage and the first derivative of the capacitance. It takes its sign from the derivative $\mathrm{d}C/\mathrm{d}x$. Furthermore, the mechanical power flow will be positive (out) or negative (in), depending on the sign of the product: $(\mathrm{d}C/\mathrm{d}x)(\mathrm{d}x/\mathrm{d}t)$.

The sign of the electrical power can be investigated conveniently by considering the special case of constant voltage, that is, $\mathrm{d}v/\mathrm{d}t = 0$. Then, we have

$$p_\mathrm{e}(t)|_{v=\mathrm{constant}} = v^2 \frac{\mathrm{d}C}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}. \tag{1.9}$$

The sign-determining product, $(\mathrm{d}C/\mathrm{d}x)(\mathrm{d}x/\mathrm{d}t)$, reappears in Eq. (1.9). Furthermore, $p_\mathrm{e}(t)|_{v=\mathrm{constant}} = 2\, p_\mathrm{m}(t)$, showing that when $(\mathrm{d}C/\mathrm{d}x)(\mathrm{d}x/\mathrm{d}t) > 0$, half of the electrical input energy is converted directly to mechanical form and the other half is stored in the capacitor. On the other hand, if $(\mathrm{d}C/\mathrm{d}x)(\mathrm{d}x/\mathrm{d}t) < 0$, the system delivers electrical power output, half from the mechanical side and the other half provided by the capacitor.

Though derived for the special case of constant voltage, the conclusion that the transducer can convert energy from electrical to mechanical form or vice versa is general. Transducer operation is controlled by the external electrical and mechanical constraints, topics to be addressed in Chapter 3.

One systematic way to distinguish between the various operational modes of a transducer is to keep track of the signs of power flows, as defined for the lossless coupling shown in Fig. 1.1. First, consider time-average power, denoted by

$$\langle p(t)\rangle = \frac{1}{T}\int_0^T p(t)\,\mathrm{d}t, \tag{1.10}$$

where T is some appropriate time scale for averaging. Then, for an actuator operating in steady-state, the device delivers mechanical power output from electrical power input, that is, $\langle p_\mathrm{e}(t)\rangle > 0$ and $\langle p_\mathrm{m}(t)\rangle > 0$. A sensor operating in the steady-state is characterized by the conditions $\langle p_\mathrm{e}(t)\rangle < 0$ and $\langle p_\mathrm{m}(t)\rangle < 0$, converting mechanical to electrical power. Thinking of a sensor as an energy converter may seem strange but, after all, *all* measurements require conversion of energy from one form to another. In particular, the detection of a weak signal requires efficient conversion and then amplification.

To consider other modes, it is better not to restrict attention to time-average power. In one form of *dynamic braking*, the conditions $p_\mathrm{e}(t) > 0$ and $p_\mathrm{m}(t) < 0$ are maintained for a transient interval of time until some moving element has stopped. Both electrical and mechanical power are supplied to the transducer and this energy is stored in the device.

A rather different example of device operation is the *force-balance* sensor, which employs feedback to balance mechanical and electrical forces so that the mechanical element does not move at all. Sensor information is extracted by monitoring the feedback system. Because $p_\mathrm{m}(t) \approx 0$, there is no conversion of energy. One final case might be hypothesized, where both electrical and mechanical power are extracted from the transducer, that is, $p_e(t) < 0$ and $p_m(t) > 0$. This situation can exist only during a transient period as energy stored in the transducer is drained away on the electrical and mechanical sides until depleted.

Example 1.1 **An interdigitated MEMS transducer**

Figure 1.2 shows a basic version of the interdigitated electrode geometry of the well-known *comb-drive* capacitive transducer. This device is found in automotive airbag sensors, gyroscopes, and many other commercially successful devices. Usually, one electrode is fixed and the other is constrained to move only in one direction, here x. In a real comb drive, the constraint to x-directed motion is only approximately achieved and then only over a limited frequency range. The device capacitance is

$$C(x) = 2N\varepsilon_0 \frac{h(L_{\text{eff}} + x)}{g} + C_s.$$

Here, $2N$ = number of air gaps, L_{eff} = overlap between the fixed and the moving electrodes when $x = 0$, g = air gap between the electrodes, h = height, and $\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m, the permittivity of free space. C_s, the parasitic capacitance, is likely to be large compared with the x-dependent term if the electrodes are connected on the substrate via long conductive traces. This problem is common to most MEMS devices.

This capacitive transducer exemplifies *variable-area* capacitive transducers, which will be treated in subsequent chapters. For the device shown, the moving electrode bank has $N = 6$ active movable plates. Using $g = 2$ μm, $h = 10$ μm, and $L_{\text{eff}} = 15$ μm, the air capacitance is calculated to be $C(x = 0) = C_0 \approx 8 \cdot 10^{-15}$ F = 8 fF, an exceedingly small value. The capacitance gradient is

$$\frac{dC}{dx} = 2N\varepsilon_0 \frac{h}{g} \approx 0.5 \text{ fF}/\mu\text{m}.$$

This parameter can be increased by adding more electrode pairs or by increasing the electrode height-to-gap ratio. Note that the electrode length, L_{eff}, does not affect dC/dx. To conserve chip area, L_{eff} should be as small as possible, but large enough for sufficient overlap to accommodate the expected operational range of motion.

The electrostatic force in a capacitive transducer can only be attractive, pulling the electrodes together. The necessary restoring force in the negative x direction is achieved with a spring.

Assume a square pulse voltage excitation of amplitude 5 V with a 50% duty cycle. If T is the period of the periodic excitation, then, from Eq. (1.8),

$$f^e = \frac{v^2}{2}\frac{dC(x)}{dx} = \begin{cases} 6.25 \cdot 10^{-9}\, N, & 0 \le t \le T/2, \\ 0, & T/2 < t < T, \end{cases}$$

the time average of which is

$$\langle f^e \rangle = \int_0^T f^e(t)\, dt \approx 3.12 \cdot 10^{-9} \text{ N}.$$

To appreciate the scale of this force, consider terrestrial gravity acting on a 1 cm length of human hair. Average human hair has a mass per unit length of ~30 μg/cm, so in a gravitational field of 9.81 m/s^2, this hair experiences a force of ~290 nN, almost 100 times larger than the calculated f^e. It is also instructive to compute the electrostatic energy stored in the capacitor at $v = 5$ V and $x = 0$ m:

$$\frac{1}{2} C_0 v^2 = \frac{1}{2}(8 \cdot 10^{-15}) \times 25 \approx 10^{-13} \text{ J}.$$

For comparison, the gravitational potential energy of this 1 cm length of hair lifted 1.8 m above the floor is $290 \cdot 10^{-9} \times 1.8 \approx 5.2 \cdot 10^{-7}$ J, a value many orders of magnitude larger than the capacitive energy!

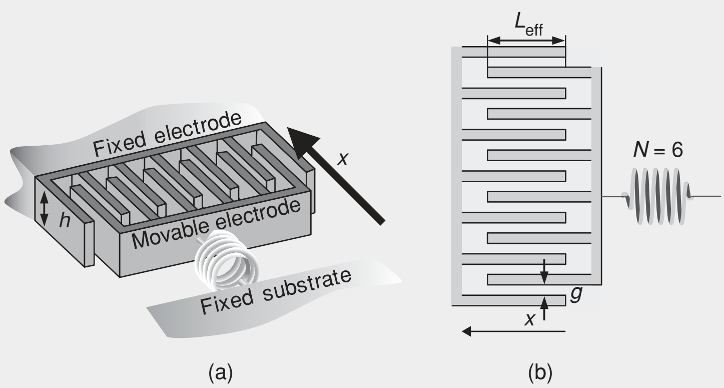


Figure 1.2 Interdigitated electrode geometry of a basic comb-drive capacitive transducer consisting of fixed and movable electrodes. The movable electrode bank is attached to a mechanical restoring spring, which constrains it to move only in the x direction. (a) Perspective view of the structure. (b) Top view with definitions for the principal geometric dimensions.

1.4 Conclusion

The values calculated in Example 1.1 for capacitance, force, and electrostatic energy of the comb-drive structure are tiny. In this book, we will show that, despite such seemingly small numbers, these and other MEMS devices can be put to work effectively in broad classes of engineered microsystems.

Problems

1.1 For mechanical variable $x < x_0$, the capacitance of a MEMS sensor is $C(x) = C_0\,(1 - x/x_0)$. Find the current $i(t)$ for these sets of conditions:

(a) $v(t) = V_0$ and $x(t) = 0$.
(b) $v(t) = V_0$ and $x(t) = u_0 t$.

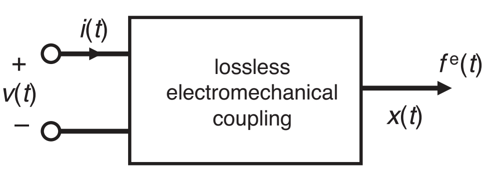
(c) $v(t) = V_0\,[1 - \exp(-\alpha t)]$ and $x(t) = 0.5x_0$.
(d) $v(t) = V_0\cos(\omega t)$ and $x(t) = u_0 t$

1.2 For the capacitor described in Problem 1.1 and the conditions (a) through (d), find time-dependent expressions for the force of electrical origin $f^{\mathrm{e}}(t)$.

1.3 Assume that the capacitor described Problem 1.1 is charged to voltage V_0 with $x = 0$, then open-circuited to fix the electric charge. Under this condition with the voltage v no longer constrained externally, what is the algebraic relationship between v and the mechanical variable x?

1.4 The objective of this problem is to develop an appreciation for the practical units appropriate to MEMS devices and the conversions needed to use them. A MEMS transducer has a capacitance that depends upon a mechanical variable x as follows: $C(x) = 100 + 10\,x$, where C is expressed in femtofarads and x is expressed in microns.
(a) Assuming that the voltage is $v = 5$ V-DC and the mechanical position is $x = 5$ μm, what is the static force of electrical origin f^{e} expressed in newtons?
(b) If the voltage is unchanged from (a) but now $x(t) = 5\cos(2\pi \cdot 10^4 t)$, find predictive expressions for the electric current $i(t)$ in nanoamps.

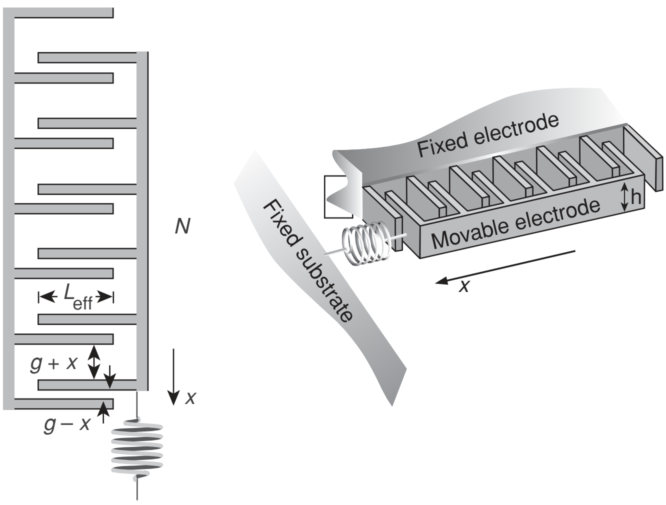
1.5 At some instant of time, an electromechanical sensor device (see Figure below), biased at a constant voltage of $v = 10$ V-DC, is converting mechanical energy to electrical form at a steady rate of $p_{\mathrm{e}}(t) = -20$ pW. Assume that the instantaneous velocity of the moving element in this device is a constant 1.0 mm/s and the acceleration is zero.



(a) Find the value of the derivative of the capacitance $\mathrm{d}C/\mathrm{d}x$ for this device in pF/μm.
(b) What are the values of the supplied instantaneous mechanical power $p_{\mathrm{m}}(t)$ in picowatts?
(c) What is the rate of change of the stored energy in picowatts?

1.6 An initially uncharged capacitive MEMS device is charged to $v = 5$ V-DC. With the voltage fixed at this value by a battery, the capacitance is then changed at a uniform rate over a time interval of $\Delta t = 1$ ms from $C = 5$ fF to 10 fF.
(a) Account for all energy flows occurring in this process, quantifying and identifying where power is coming from and where it is flowing to.
(b) With the device now mechanically fixed so that the capacitance $C = 10$ fF, it is connected to a resistor $R = 10$ kΩ starting at $t = 0$ and discharged. Determine the transient decay of voltage and then account quantitatively for all energy flows.

1.7 Example 1.1 described an interdigitated capacitive transducer with fixed electrode gaps and variable overlap. The same interdigitated geometry can be configured as a variable-gap device, as shown below. Obtain expressions for the capacitance $C(x)$ and the force of electrical origin f^{e} of this device, assuming it has $2N$ variable gaps and fixed overlap of the electrodes.



1.8 The capacitance of a MEMS transducer depends upon a single mechanical variable x; this dependence is $C(x) = C_0 \exp(-\alpha x)$. Assume that the device is initially charged up to a DC voltage V_0 with the movable element held fixed at $x = x_0$, and then open-circuited so that the charge is constant. Then, at time $t = 0$, the movable element is released and the voltage is monitored with an oscilloscope as a function of time. It is found that the voltage is $v(t) = V_0 \exp(-\beta t)$ for $t \geq 0$. Obtain an expression for the time dependence of the mechanical variable, $x(t)$.

Reference

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2 Circuit-based modeling

2.1 Fundamentals of circuit theory

This chapter summarizes basic circuit theory concepts, shows how to model capacitive MEMS actuator structures as simple circuit devices, and introduces elements of multiport network theory for later use in modeling electromechanical systems. The approach is a very practical one, based on reasonable approximations and easy-to-use inspection methodologies. Students seeking more background on the theoretical underpinnings of circuit modeling should refer to Appendix A, which summarizes the electroquasistatic and magnetoquasistatic approximations, reveals the origins of the models for capacitors and inductors, and relates them to basic circuit theory.

Our emphasis on circuit-based representations for MEMS devices is motivated by the fact that they can be embedded directly into the electronic system models for the control and sensing circuitry. With this groundwork, we will later investigate conventional implementations of capacitive sensors and actuators, including inverting operational-amplifier circuits, two-plate and three-plate topologies, and the half-bridge differential scheme. For sufficiently complex systems, software tools such as PSPICE or CADENCE might be used, but in this text on fundamentals, we restrict the focus to systems that can be treated analytically.

2.1.1 Motivation

Electromechanical energy transduction occurs in consequence of the interaction of electric fields with charge, or magnetic fields with electric current. For this reason, reliable prediction of the performance of microelectromechanical devices would seem to hinge upon detailed knowledge of the electric field (or magnetic field) inside the device. Such an impression can be intimidating when one starts to consider the huge variety and complexity of MEMS geometries. “Where to begin?” the student might reasonably ask. Solutions for the electric or magnetic fields in these geometries often necessitate the use of numerical computation tools. Fortunately, the situation for the newcomer to MEMS is not nearly so bad. One can uncover and then study virtually all the important behavior of capacitive MEMS devices – including frequency-dependent response, sensitivity, dynamic response, and stability – from knowing the capacitance $C(x)$ and its first two derivatives.