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978-0-521-76465-0 - Nonlife Actuarial Models: Theory, Methods and Evaluation

Yiu-Kuen Tse

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Nonlife Actuarial Models

Actuaries must pass exams, but more than that: they must put knowledge into practice. This coherent book gives complete syllabus coverage for Exam C of the Society of Actuaries (SOA) while emphasizing the concepts and practical application of nonlife actuarial models. Ideal for those approaching their professional exams, it is also a class-tested textbook for undergraduate university courses in actuarial science.

All the topics that students need to prepare for Exam C are here, including modeling of losses, risk, and ruin theories, credibility theory and applications, and empirical implementation of loss models. The book also covers more recent topics, such as risk measures and bootstrapping. Readers are assumed to have studied statistical inference and probability at the introductory undergraduate level.

Numerous examples and exercises are provided, with many exercises adapted from past Exam C questions. Computational notes on the use of Excel are included. Teaching slides are available for download.

International Series on Actuarial Science

Christopher Daykin, Independent Consultant and Actuary

Angus Macdonald, Heriot-Watt University

The *International Series on Actuarial Science*, published by Cambridge University Press in conjunction with the Institute of Actuaries and the Faculty of Actuaries, contains textbooks for students taking courses in or related to actuarial science, as well as more advanced works designed for continuing professional development or for describing and synthesizing research. The series is a vehicle for publishing books that reflect changes and developments in the curriculum, that encourage the introduction of courses on actuarial science in universities, and that show how actuarial science can be used in all areas where there is long-term financial risk.

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NONLIFE ACTUARIAL MODELS

Theory, Methods and Evaluation

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi
Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521764650

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First published 2009

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-76465-0 Hardback

Additional resources for this publication at
www.mysmu.edu/faculty/yktse/NAM/NAMbase.htm

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Preface

This book is on the theory, methods, and empirical implementation of nonlife actuarial models. It is intended for use as a textbook for senior undergraduates. Users are assumed to have done one or two one-semester courses on probability theory and statistical inference, including estimation and hypothesis testing. The coverage of this book includes all the topics found in Exam C of the Society of Actuaries (Exam 4 of the Casualty Actuarial Society) as per the 2007 Basic Education Catalog. In addition, it covers some topics (such as risk measures and ruin theory) beyond what is required by these exams, and may be used by actuarial students in general.

This book is divided into four parts: loss models, risk and ruin, credibility, and model construction and evaluation. An appendix on the review of statistics is provided for the benefit of students who require a quick summary. Students may read the appendix prior to the main text if they desire, or they may use the appendix as a reference when required. In order to be self contained, the appendix covers some of the topics developed in the main text.

Some features of this book should be mentioned. First, the concepts and theories introduced are illustrated by many practical examples. Some of these examples explain the theory through numerical applications, while others develop new results. Second, several chapters of the book include a section on numerical computation using Excel. Students are encouraged to use Excel to solve some of the numerical exercises. Third, each chapter includes some exercises for practice. Many of these exercises are adapted from past exam questions of the Society of Actuaries.

I would like to thank Tao Yang for painstakingly going through the manuscript and for providing many useful comments and suggestions. Diana Gillooly has professionally guided me through the publication process with admirable patience and efficiency. Clare Dennison has performed a superb job of

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coordinating the copy editing. I am also grateful to the Society of Actuaries for allowing me to use its past exam questions.

Resources are available at: www.mysmu.edu/faculty/yktse/NAM/NAMbase.htm. Slides in pdf format can be downloaded from this site, which will facilitate classroom teaching by instructors adopting this book. An errata file will be provided, and the solution manual for instructors is obtainable from the author on request.

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Notation and convention

- 1 Abbreviations are used in this book without periods. For example, “probability density function” is referred to as pdf (not p.d.f.) and “moment generating function” is referred to as mgf (not m.g.f.).
- 2 We do not make distinctions between a random variable and the distribution that describes the random variable. Thus, from time to time we make statements such as: “ X denotes the binomial distribution”.
- 3 We use calligraphic fonts to denote commonly used distributions. Discrete distributions are denoted with two alphabets and continuous distributions are denoted with one alphabet. For example, \mathcal{PN} stands for Poisson, \mathcal{BN} stands for binomial, \mathcal{N} stands for normal, and \mathcal{L} stands for lognormal.
- 4 The following conventions are generally used:
 - (a) Slanted upper case for random variables, e.g. X .
 - (b) Slanted lower case for fixed numbers, e.g. x .
 - (c) Slanted bold-faced upper case for vectors of random variables, e.g. \mathbf{X} .
 - (d) Slanted bold-faced lower case for vectors of fixed numbers (observations), e.g. \mathbf{x} .
 - (e) Upright bold-faced upper case for matrices of fixed numbers (observations), e.g. \mathbf{X} .
- 5 Natural logarithm is denoted by \log , not \ln .

Computation notes

- 1 In some chapters we include a section of Excel computation notes to discuss the use of Excel functions to facilitate computation. These functions require the Excel add-ins Analysis ToolPak and Solver Add-in.
- 2 Other computer languages for more advanced statistical analysis include R, C++, Splus, Gauss, and Matlab. All graphs in this book were produced using Matlab, and many of the computations were performed using Gauss.