BAYESIAN DECISION ANALYSIS

Bayesian decision analysis supports principled decision making in complex but structured domains. The focus of this textbook is on the faithful representation and conjugate analyses of discrete decision problems. It takes the reader from a formal analysis of simple decision problems to a careful analysis of the sometimes very complex and data rich structures confronted by practitioners. The book contains basic material on subjective probability theory and multiattribute utility theory, event and decision trees, Bayesian networks, influence diagrams and causal Bayesian networks. The author demonstrates when and how the theory can be successfully applied to a given decision problem, how data can be sampled and expert judgements elicited to support this analysis, and when and how an effective Bayesian decision analysis can be implemented.

Evolving from a third-year undergraduate course taught by the author over many years, all of the material in this book will be accessible to a student who has completed introductory courses in probability and mathematical statistics.

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Principles and Practice

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Preface

This book introduces the principles of Bayesian Decision Analysis and describes how this theory can be applied to a wide range of decision problems. It is written in two parts. The first presents what I consider to be the most important principles and good practice in mostly simple settings. The second part shows how the established methodology can be extended so that it can address the sometimes very complex and data-rich structures a decision maker might face. It will serve as a course book for a 30-lecture course on Bayesian decision modelling given to final-year undergraduates with a mathematical core to their degree programme and statistics Master's students at Warwick University. Complementary material given in two parallel courses, one on Bayesian numerical methods and the other on Bayesian Time Series given subsequently at Warwick, is largely omitted although these subjects are motivated within the text. This book contains foundational material on the subjective probability theory and multiattribute utility theory – with a detailed discussion of efficacy of various assumptions underlying these constructs – and quite an extensive treatment of frameworks such as event and decision trees, Bayesian Networks, as well as Influence Diagrams and Causal Bayesian Networks. These graphical methods help draw different aspects of a decision problem together into a coherent whole and provide frameworks where data can be used to support a Bayesian decision analysis.

This is not just a text book; it also provides additional material to help the reader develop a more profound understanding of this fascinating and highly cross-disciplinary subject. First, it includes many more worked examples than can be given in a such a short programme. Second, I have supplemented this material with extensive practical tips gleaned from my own experiences which I hope will help equip the budding decision analyst. Third, there are supplementary technical discussions about when and why a Bayesian decision analysis is appropriate. Most of this supplementary material is drawn from various postgraduate and industrial training courses I have taught. However all the material in the book should be accessible and of interest to a final-year maths undergraduate student. I hope the addition of this supplementary material will make the book interesting to practitioners who have reasonable skills in mathematics and help them hone their decision analytic skills. An asterisk denotes that a section contains more advanced material and can be skipped without loss of continuity to the rest of the text.

The book contains an unusually large number of running examples which are drawn – albeit in a simplified form – from my experiences as an applied Bayesian modeller and used to illustrate theoretical and methodological issues presented in its core. There are many exercises throughout the book that enable the student to test her understanding. As far as possible I have tried to keep technical mathematical details in the background whilst respecting the intrinsic rigour behind the arguments I use. So the text does not require an advanced course in stochastic processes, measure theory or probability theory as a prerequisite.

Many of the illustrations are based on simple finite discrete decision problems. I hope in this way to have made the book accessible to a wider audience. Moreover, despite keeping the core of the text as nontechnical as possible, I have tried to leave enough hooks in the text so that the advanced mathematician can make these connections through pertinent references to more technical material. Over the last 20 years many excellent books have appeared about Bayesian Methodology and Decision Analysis. This has allowed me to move quickly over certain more technical material and concentrate more on how and when these techniques can be drawn together. Of course some important topics have been less fully addressed in these texts. When this has happened I have filled these gaps here.

Obviously many people have influenced the content of the book and I am able here only to thank a few. I learned much of this material from conversations with Jeff Harrison, Tom Leonard, Tony O'Hagan, Chris Zeeman, Dennis Lindley, Larry Phillips, Bob Oliver, Morris De Groot, Jay Kadane, Howard Raiffa, Phil Dawid, Michael Goldstein, Mike West, Simon French, Saul Jacka, Steffen Lauritzen and more recently with Roger Cooke, Tim Bedford, Joe Eaton, Glen Shafer, Milan Studeny, Henry Wynn, Eva Riccomagno, David Cox, Nanny Wermuth, Thomas Richardson, Michael Pearlman, Lorraine Dodd, Elke Thonnes, Mark Steel, Gareth Roberts, Jon Warren, Jim Griffin, Fabio Rigat and Bob Cowell. Postdoctoral fellows who were instrumental in jointly developing many of the techniques described in this book include Alvaro Faria, Raffaella Settimi, Nadia Papamichail, David Ranyard, Roberto Puch, Jon Croft, Paul Anderson and Peter Thwaites. Of course my university colleagues and especially my PhD students, Dick Gathercole, Simon Young, Duncan Atwell, Catriona Queen, Crispin Allard, Nick Bisson, Gwen Tanner, Ali Gargoum, Antonio Santos, Lilliana Figueroa, Ana Mari Madrigal, Ali Daneshkhah, John Arthur, Siliva Liverani, Guy Freeman and Piotr Zwirnick have all helped inform and hone this material. My thanks go out to these researchers and the countless others who have helped me directly and indirectly.