Game Theory

Game theory is concerned with strategic interaction among several decision-makers. In such strategic encounters, all players are aware of the fact that their actions affect the other players. Game theory analyzes how these strategic, interactive considerations may affect the players' decisions and influence the final outcome. This textbook focuses on applications of complete-information games in economics and management, as well as in other fields such as political science, law, and biology. It guides students through the fundamentals of game theory by letting examples lead the way to the concepts needed to solve them. It provides opportunities for self-study and self-testing through an extensive pedagogical apparatus of examples, questions, and answers. The book also includes more advanced material suitable as a basis for seminar papers or elective topics, including rationalizability, stability of equilibria (with discrete-time dynamics), games and evolution, equilibrium selection, and global games.

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Game Theory

Interactive Strategies in Economics and Management

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In loving memory of my father
Gutman Heifetz, 1933–2010

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FOREWORD

Game theory is concerned with strategic interaction among several decision-makers. In a strategic encounter of this kind, each player is aware of the fact that her actions affect the well-being of the other players, just as their actions affect hers. Game theory analyzes how these mutual influences channel the players' decisions and lead to the ultimate outcome. Since the basic theoretical foundations of game theory were laid in the mid twentieth-century, numerous applications have been found for it in economics and management, as well as in political science, anthropology, sociology, biology and computer science.

It is important to realize that game-theoretic tools do not provide off-the-shelf solutions for predicting players' behavior in a complex, real-life situation. In the social sciences, *y compris* in game theory, every model distils only a small number of critical aspects out of the vast plethora of dimensions characterizing a given situation. And it is exclusively in light of those aspects that it proceeds to analyze the situation, using highly stylized hypotheses regarding a whole host of other aspects. The purpose of the model is to provide a framework for *systematic thinking* about the complicated situation. Insights gained while analyzing the model may enable one to think more intelligently and profoundly about an actual, realistic situation. In this way, game theoretic models have shed light on the *modus operandi* of many economic and political mechanisms; and insights gained from such models have made a substantial contribution towards a more intelligent design of such mechanisms – incentives for workers in firms, financial markets and auctions for a large variety of assets, policies for diminishing air pollution, voting and election systems, and numerous other types of mechanisms, institutions and organizations.

Acknowledging the importance and salience of game theory, the Nobel Prize for Economics was awarded in 1994 to three of the founding fathers of game theory – John Nash, John Harsanyi, and Reinhard Selten, and in 2005 to another two of these founding fathers, Robert (Yisrael) Aumann and Thomas Schelling. In 2007, the Nobel Prize in Economics was dedicated to the domain of mechanism design with tools from information economics and game theory, and the prize was awarded to Leonid Hurwicz, Roger Myerson and Eric Maskin.

This textbook sets forth key notions in the analysis of non-cooperative games, with an emphasis on its applications for analyzing and designing economic

institutions and markets. Special emphasis is placed on realistic examples and the results of laboratory experiments of games, which are conducted on a progressively larger scale in recent years. Especially interesting are those instances in which the results of the experiments do not align perfectly with the theoretical predictions. In such instances, the experiment results may provide a pointer to new directions for enhancing and improving the model. I have tried to indicate such innovative directions insofar as possible.

The textbook is suited for a one-semester elective course for Economics or Management undergraduates, as well as for MBA students. The book assumes minimal mathematical pre-requisites – mainly acquaintance with very basic concepts in probability and the technique for finding the maximum of a function. In fact, the book elaborates the requisite mathematical tools while making use of them. It is written in a style allowing for self study, with detailed solutions to many of the problems and questions that appear in the text.

Most chapters deal with classical themes of non-cooperative game theory: normal-form games, dominant and dominated strategies, Nash equilibrium, pure and mixed strategies, the minimax theorem, extensive-form games, subgame perfect equilibrium, and repeated games.

A small number of chapters address more advanced topics that can serve as enrichment and as the basis for seminar papers. Chapter 14 deals with rationalizable strategies. Chapter 15 addresses the stability of equilibria – a topic of particular importance in analyzing real-life problems. The chapter provides a definition of a discrete-time dynamical system; and for a number of games, it offers an analysis of the stability of their equilibria under the best-reply dynamics. This chapter prepares readers for the consecutive chapter on games and evolution. The chapter starts with a discussion of the discrete-time replicator dynamics, modeling directly a process of natural selection in which the survivors are the fittest types of players. The second part of Chapter 15 presents the static notion of an evolutionary stable strategy (ESS), and analyzes the link between ESS and the notion of stability under the replicator dynamics.

Chapter 17 delves into global games, with the focus on an example of currency exchange-rates attack. The discussion opens with a case in which there is a finite number of investor types, and only later proceeds to analyze the continuous case. The fact that the game between investor types is solvable by dominance considerations makes it possible to analyze the game, which is, in fact, a game with asymmetric information, using the tools provided in the book. Chapter 19, too, brings an example of a game with asymmetric information, addressing the issue of buyer types' screening by an uninformed seller. This example allows for the analysis of basic concepts in information economics, even though a general presentation of games with asymmetric information and an analysis of their solution concepts are beyond the scope of this book.

The book was originally published in Hebrew by the Open University of Israel. Since its publication in 2008 it served as a textbook for hundreds of students. I'm grateful to Chen Cohen who has vigorously coordinated the course at the Open University since its inception. I'm also grateful to colleagues in other Israeli universities, from whom I was delighted to hear comments based on their use of the book. Special warm thanks go to Zvika Neeman, Shmuel Nitzan, Motty Perry and Ella Segev, who took part in reviewing the book when it was still in draft form, and provided so many invaluable suggestions.

I'm deeply indebted to Judith Yalon Fortus, who translated the book into English with remarkable lucidity and rigor. Working along with Judith was a particularly pleasant experience. During much of the preparation of the English edition I was on a two-year visit to the MEDS department at Kellogg School of Management, Northwestern University. I'm grateful to my colleagues there for the very warm hospitality and the energetic spirit which made the visit so enjoyable. Burkhard Schipper from UC Davis deserves special thanks for reviewing the final draft of the English edition and providing additional very helpful suggestions.

I'm grateful for the excellent support I have received from the entire team at Cambridge University Press. I'm particularly indebted to the social sciences publishing director, Chris Harrison, with whom it was a splendid experience to work right from the very start. Always encouraging and attentive, this project would not have materialized without his genuine engagement. My gratitude goes also to the senior production editor, Daniel Dunlavey, who was so helpful with skillfully overcoming last-stage formatting issues and finalizing the project.

I was very happy that Noa Dar permitted us to use for the cover the photo from her choreography "Children's Games," inspired by Pieter Breughel's painting. May this be a tribute to Noa Dar's thrilling creations and to the vibrant Israeli dance scene at large.

And, of course, warm hugs to the ladies with whom I'm blessed to share my life – my wife Dana and our daughters Inbal and Maayan – for being so supportive along the entire journey of writing this book, and for making life playful as it should be.

Aviad Heifetz
The Open University of Israel
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