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978-0-521-76394-3 - An Illustrated Guide to Relativity

Tatsu Takeuchi

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## An Illustrated Guide to Relativity

Aimed at both physics students and non-science majors, this unique book explains Einstein's Special Theory of Relativity pictorially, using diagrams rather than equations. The diagrams guide the reader, step-by-step, from the basics of relativity to advanced topics including the addition of velocities, Lorentz contraction, time dilation, twin paradox, Doppler shift, and Einstein's famous equation  $E = mc^2$ . The distinctive figures throughout the book enable the reader to visualize the theory in a way that cannot be fully conveyed through equations alone.

The illustrative explanations in this book maintain the logic and rigor necessary for physics students, yet are simple enough to be understood by non-scientists. The book also contains entertaining problems which challenge the reader's understanding of the materials covered.

TATSU TAKEUCHI is an Associate Professor in the Department of Physics at Virginia Tech. This book grew from the "Highlights of Contemporary Physics" course he taught for many years. Primarily aimed at non-physics majors, it has been highly popular among physics students as well.

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# An Illustrated Guide to Relativity

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CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,  
São Paulo, Delhi, Dubai, Tokyo, Mexico City  
Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521141000](http://www.cambridge.org/9780521141000)

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First published 2010

Printed in the United Kingdom at the University Press, Cambridge

*A catalog record for this publication is available from the British Library*

ISBN 978-0-521-76394-3 Hardback  
ISBN 978-0-521-14100-0 Paperback

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## Preface to English edition

This book explains Einstein's Special Theory of Relativity (SR) using diagrams only. Readers who are used to thinking of physics as a vast labyrinth of equations may feel somewhat uneasy about this unconventional approach and fear that it risks losing important information about SR that can only be conveyed via equations. However, this fear is not only unfounded but actually reversed: it is the equations that fail to convey the essence of SR that diagrams can easily display right in front of your eyes. After all, SR, and also the General Theory of Relativity (GR), are about the *geometry* of the spacetime that we inhabit, and what can best describe geometry if not diagrams? Equations are simply inadequate, to wit, one diagram is worth a thousand equations.

So if you are a reader for whom equations are anathema, rest assured that you will get much more out of this book than any physics student will get out of a textbook full of equations. If you are a physics student, this book will provide you with a deep enough understanding of SR that will enable you to reproduce any equation you may need from scratch, if such a need ever arises, and also prepare you for GR as well.

I would like to thank the readers of the Japanese [1] and Chinese [2] editions of this book who have provided precious feedback and encouragement through their reviews, blogs, and email, and have given me added confidence that the approach of this book is the right one. I would also like to thank my students at Virginia Tech whose constant desire to read this book in English motivated me to translate part III from Japanese. (I wrote parts I and II in English to begin with.)

Special thanks are due to my editors at Cambridge University Press: John Fowler for his enthusiastic support of this project, Lindsay Barnes for her meticulous attention to detail in making sure all the text and figures were in order, and Abigail Jones for guiding me through the production process. It was a great joy working with them all.

January, 2010  
Tatsu Takeuchi

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## Preface to Japanese edition

All physical theories, their mathematical expressions notwithstanding, ought to lend themselves to so simple a description that even a child could understand them.

Albert Einstein [3]

Einstein's celebrated Theory of Relativity is one of those scientific theories whose name is so famous that most people have heard of it, but very few people actually know what the theory says, or even what the theory is about. You, too, have probably heard the name, perhaps referred to in a science fiction novel or movie, even if you do not know much about it. And you may have received the impression that it is a very esoteric and difficult theory that could only be understood and appreciated by a select few.

The aim of this book is to show you that that impression is wrong. The Theory of Relativity comes in two flavors, the Special and the General, and if we limit our attention to the Special Theory of Relativity (SR), which is a theory of motion, it is not a particularly difficult theory at all and can be understood by anyone, perhaps "even a child." By "be understood" here, I do not mean that anyone can develop a vague idea of what the theory is saying, but that anyone can understand it in its full glory beginning from its basic tenets to all of its logical consequences. And furthermore, it can be understood without using ANY equations! In fact, *one can develop a deeper understanding of the theory by avoiding the use of equations altogether*. At least, this author thinks so.

Then, why is it that I do not declare that SR is an easy theory outright? The reason is that SR makes some statements about the concept of simultaneity which do not agree with our common sense based on everyday experience, and this is where a slight difficulty lies: we must listen to what SR is telling us with an open mind and not let our common sense obstruct our understanding.

This book is an attempt to explain Einstein's Special Theory of Relativity (SR) without using equations. Instead, we will use drawings called *spacetime diagrams* in a way that will let you "see" the essence of the theory. This book has three parts. Part I explains why SR was constructed, what it is telling us, and why SR had to be the way it is.



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*Preface to Japanese edition*

Part II is a collection of problems. The problems are designed so that by thinking about them you will confront the common sense that hampers our understanding of SR, and know where the pitfalls are that may mislead us to think that there must be something wrong with SR. Part III deals with the famous equation  $E = mc^2$ . Since  $E = mc^2$  itself is an equation, the appearance of some equations in part III could not be avoided. However, the basic argument proceeds via diagrams and can be followed without following the equations. The General Theory of Relativity, which is a theory of gravity, can, in principle, also be explained using drawings only. However, since it is a somewhat complicated theory, we will not be discussing it in this book.

Part I of this book is based on lecture notes I prepared for the course “Highlights of Contemporary Physics” which I have taught at Virginia Tech for many years. Part II consists of the problems I have used for the exams in this course. I thank all the students who have taken this course and have served as guinea pigs to see if the approach of this book works. I would also like to thank Djordje Minic, Joy Rosenthal, Simone Paterson, and Masako Saitō Koike for their critical reading of this manuscript and their many helpful suggestions.

Special thanks are due to Tōru Kawahara of Iwanami Shoten for his enthusiasm in bringing this book to publication. Without his help in digitizing my hand-drawn diagrams, and translating the characters’ words in the drawings into Japanese, this book would never have been completed.

August, 2005  
Tatsu Takeuchi