PERSPECTIVES ON SPIN GLASSES

Presenting and developing the theory of spin glasses as a prototype for complex systems, this book is a rigorous and up-to-date introduction to their properties.

The book combines a mathematical description with a physical insight of spin glass models. Topics covered include the physical origins of those models and their treatment with replica theory; mathematical properties such as correlation inequalities and their use in the thermodynamic limit theory; main exact solutions of the mean field models and their probabilistic structures; and the theory of the structural properties of the spin glass phase such as stochastic stability and the overlap identities. Finally, a detailed account is given of the recent numerical simulation results and properties, including overlap equivalence, ultrametricity, and decay of correlations. The book is ideal for mathematical physicists and probabilists working in disordered systems.

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Preface

Spin glasses are statistical mechanics systems with random interactions. The alternating sign of those interactions generates a complex physical behavior whose mathematical structure is still largely uncovered. The approach we follow in this book is that of mathematical physics, aiming at the rigorous derivation of their properties with the help of physical insight.

The book starts with the theoretical physics origins of the spin glass problem. The main models are introduced and a description of the replica approach is illustrated for the Sherrington–Kirkpatrick model.

Chapters 2 and 3 contain the starting points of the mathematical rigorous approach leading to the control of the thermodynamic limit for spin glass systems. Correlation inequalities are introduced and proved in various settings, including the Nishimori line. They are then used to prove the existence of the large-volume limit in both short-range and mean-field models.

Chapter 4 deals with exact results which belong to the mean-field case. The methods and techniques illustrated span from the Ruelle probability cascades to the Aizenman–Sims–Starr variational principle. In this framework the Guerra upper bound theorem for the pressure is presented and the Talagrand theorem is reported.

Chapter 5 deals with the structural identities characterizing the spin glass phase. These are obtained by an extension of the stochastic stability method, i.e. an invariance property of the system under small perturbations, together with the self-averaging property.

Chapter 6 features some problems which are still out of analytical reach and are investigated with numerical methods: the equivalence among different overlap structures, the hierarchical organization of the states, the decay of correlations, and the energy interface cost.

Needless to say there are innumerable important issues not covered by the book. Among them are the dynamical properties of spin glasses (see, for example, Sompolinsky and Zippelius (1982); Cugliandolo and Kurchan (1993); Bouchaud

(1992); Ben Arous *et al*. (2001, 2002); Bovier *et al*. (2001)). Another very large topic not covered is that of applications whose ideas originated within spin glass theory and successfully fertilized other areas.

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