

PERSPECTIVES ON SPIN GLASSES

Presenting and developing the theory of spin glasses as a prototype for complex systems, this book is a rigorous and up-to-date introduction to their properties.

The book combines a mathematical description with a physical insight of spin glass models. Topics covered include the physical origins of those models and their treatment with replica theory; mathematical properties such as correlation inequalities and their use in the thermodynamic limit theory; main exact solutions of the mean field models and their probabilistic structures; and the theory of the structural properties of the spin glass phase such as stochastic stability and the overlap identities. Finally, a detailed account is given of the recent numerical simulation results and properties, including overlap equivalence, ultrametricity, and decay of correlations. The book is ideal for mathematical physicists and probabilists working in disordered systems.

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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom
 One Liberty Plaza, 20th Floor, New York, NY 10006, USA
 477 Williamstown Road, Port Melbourne, VIC 3207, Australia
 314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
 103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

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 a department of the University of Cambridge.

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www.cambridge.org
 Information on this title: www.cambridge.org/9780521763349

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First published 2013

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data

Contucci, Pierluigi, 1964–

Perspectives on spin glasses / Pierluigi Contucci and Cristian Giardinà.

pages cm

Includes bibliographical references and index.

ISBN 978-0-521-76334-9

1. Spin glasses – Mathematical models. I. Giardinà, Cristian. II. Title.

QC176.8.S68C667 2013

530.4'12 – dc23 2012032860

ISBN 978-0-521-76334-9 Hardback

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Preface

Spin glasses are statistical mechanics systems with random interactions. The alternating sign of those interactions generates a complex physical behavior whose mathematical structure is still largely uncovered. The approach we follow in this book is that of mathematical physics, aiming at the rigorous derivation of their properties with the help of physical insight.

The book starts with the theoretical physics origins of the spin glass problem. The main models are introduced and a description of the replica approach is illustrated for the Sherrington–Kirkpatrick model.

Chapters 2 and 3 contain the starting points of the mathematical rigorous approach leading to the control of the thermodynamic limit for spin glass systems. Correlation inequalities are introduced and proved in various settings, including the Nishimori line. They are then used to prove the existence of the large-volume limit in both short-range and mean-field models.

Chapter 4 deals with exact results which belong to the mean-field case. The methods and techniques illustrated span from the Ruelle probability cascades to the Aizenman–Sims–Starr variational principle. In this framework the Guerra upper bound theorem for the pressure is presented and the Talagrand theorem is reported.

Chapter 5 deals with the structural identities characterizing the spin glass phase. These are obtained by an extension of the stochastic stability method, i.e. an invariance property of the system under small perturbations, together with the self-averaging property.

Chapter 6 features some problems which are still out of analytical reach and are investigated with numerical methods: the equivalence among different overlap structures, the hierarchical organization of the states, the decay of correlations, and the energy interface cost.

Needless to say there are innumerable important issues not covered by the book. Among them are the dynamical properties of spin glasses (see, for example, Sompolinsky and Zippelius (1982); Cugliandolo and Kurchan (1993); Bouchaud

(1992); Ben Arous *et al.* (2001, 2002); Bovier *et al.* (2001)). Another very large topic not covered is that of applications whose ideas originated within spin glass theory and successfully fertilized other areas.

It is a pleasure to thank the co-authors whose research work provided the foundations of this book: Michael Aizenman, Alessandra Bianchi, Mirko Degli Esposti, Claudio Giberti, Sandro Graffi, Andreas Knauf, Stefano Isola, Joel Lebowitz, Satoshi Morita, Hidetoshi Nishimori, Giorgio Parisi, Joe Pulé, Shannon Starr, Francesco Unguendoli, and Cecilia Vernia.

Useful conversations with many colleagues are acknowledged, particularly those with Louis-Pierre Arguin, Adriano Barra, Anton Bovier, Edouard Brézin, Aernout van Enter, Silvio Franz, Francesco Guerra, Frank den Hollander, Jorge Kurchan, Enzo Marinari, Marc Mezard, Chuck Newman, Dmitry Panchenko, Daniel Stein, and Michael Talagrand.

Last but not least we thank Claudio Giberti, Bernardo D’Auria, Aernout van Enter, and Cecilia Vernia for their careful reading of the manuscript.