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High-mass star formation by gravitational collapse of massive cores

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The current generation of millimeter interferometers have revealed a population of compact $(r \leq 0.1 \text{ pc})$, massive $(M \sim 100 M_{\odot})$ gas cores that are the likely progenitors of massive stars. I review models for the evolution of these objects from the observed massive-core phase through collapse and into massive-star formation, with particular attention to the least well-understood aspects of the problem: fragmentation during collapse, interactions of newborn stars with the gas outside their parent core, and the effects of radiation-pressure feedback. Through a combination of observation, analytic argument, and numerical simulation, I develop a model for massive-star formation by gravitational collapse in which massive cores collapse to produce single stars or (more commonly) small-multiple systems, and these stars do not gain significant mass from outside their parent core by accretion of either gas or other stars. Collapse is only very slightly inhibited by feedback from the massive star, thanks to beaming of the radiation by a combination of protostellar outflows and radiation-hydrodynamic instabilities. Based on these findings, I argue that many of the observed properties of young star clusters can be understood as direct translations of the properties of their gas-phase progenitors. Finally, I discuss unsolved problems in the theory of massive-star formation, and directions for future work on them.

1. Introduction

Massive-star formation occurs in the densest, darkest parts of molecular clouds. These clumps of gas have masses of thousands of M_{\odot} , radii $\lesssim 1$ pc, volume densities of $\sim 10^5$ H atoms cm⁻³, column densities of ~ 1 g cm⁻², visual extinctions of hundreds, and velocity dispersions of several km s⁻¹. Observations often reveal indicators of massive-star formation such as maser emission and infrared point sources within them, but the majority of their mass appears to be dark and cold. Due to their high extinctions and low temperatures, these regions are only accessible to observation through millimeter emission, either in molecular lines (e.g., Plume et al. 1997; Shirley et al. 2003; Yonekura et al. 2005; Pillai et al. 2006b) or dust continuum (e.g., Carey et al. 2000; Mueller et al. 2002), or through infrared absorption (e.g., Egan et al. 1998; Menten et al. 2005; Rathborne et al. 2005; Simon et al. 2006; Rathborne et al. 2006). They are likely the progenitors of the rich clusters in which massive stars form.

In the last few years, observations using millimeter interferometers to obtain still higher resolution have identified "cores" within these dense clumps, objects small enough that they approach the stellar-mass scale. Cores are distinguished by even higher volume densities than the massive clumps around them, 10^6 H cm⁻³ or more, and smaller radii, $r \leq 0.1$ pc. Some show mid-IR (MIR) point sources in their centers (e.g., Pillai et al. 2006a), while others show no MIR emission, or even MIR absorption, indicating that they are starless or contain only very low-mass stars (e.g., Sridharan et al. 2005). In some cases they show signs of molecular outflows, but not MIR emission, indicating that the extremely massive core contains a very low-mass protostar, and thus is near the onset of star formation (Beuther et al. 2005).

The characteristic mass, size, and density of massive cores make them appealing candidates to be the progenitors of massive stars (e.g., Garay 2005). Moreover, as I discuss

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in more detail in Section 2, the observed mass and spatial distribution of protostellar cores is quite similar to that of stars in young clusters. If massive cores are the direct precursors of massive stars, then one can explain many of the properties of newborn clusters directly from the observed properties of their gas-phase progenitors. The goal of this review is to construct a rough scenario based on this idea by following observed cores through collapse, fragmentation, accretion, and feedback, to the final formation of massive stars. In Section 2, I briefly review observations of the properties of massive cores to provide initial conditions for this scenario. In Sections 3–5, I discuss three major questions about how these cores turn into stars: Do they fragment into many stars, or only a handful? Do the stars they form accrete significant mass from outside the parent core? Does feedback significantly inhibit accretion? Finally, in Section 6, I discuss some outstanding problems in the modeling of massive-core evolution, and suggest directions for future work.

2. Massive cores: Initial conditions for massive-star formation

We know disappointingly little about massive cores, despite great observational effort. Due to their small sizes and large distances, massive cores are only marginally resolved, even in observations with the highest resolution telescopes available. Nonetheless, observations do allow us to determine some gross properties of individual massive cores, and of the massive-core population as a whole. Observations show that cores are centrally concentrated, although the exact density profile is difficult to determine with interferometer measurements, and fairly round, with aspect ratios of roughly 2:1 or less. They are cold, $T \approx 10\text{--}40$ K, except near stellar sources, so their observed velocity dispersions of ~1 km s⁻¹ imply the presence of highly supersonic motions (e.g., Reid & Wilson 2005; Beuther et al. 2005, 2006). At the characteristic density of ~10⁶ H cm⁻³ found in these cores, the free-fall time is only ~10⁵ yr, so the implied accretion rate when a massive core collapses is $10^{-4}\text{--}10^{-3} M_{\odot} \text{ yr}^{-1}$.

McKee & Tan (2003) propose a simple self-similar model of massive cores in which the core density and velocity dispersion are power-law functions of radius, such that at every radius, turbulent motions provide enough ram pressure to marginally support the core against collapse. The central idea is that, at the high pressures found in massive-star-forming regions, massive cores must be supported by internal turbulent motions. While a self-similar spherical model is obviously a significant simplification of a turbulently supported gas cloud, it provides a reasonably good fit to the available observations, and makes it possible to calculate quantities such as the timescale for star formation and the relationship between core mass, column density, pressure, and velocity dispersion. It also provides a good starting point for simulations and more detailed analytic work.

For the massive-core population as a whole, we know somewhat more, and the observations bolster the idea that massive cores might really be the progenitors of massive stars. Several authors, using different techniques and observing different regions, find that the mass distribution of massive cores matches the stellar initial mass function, shifted to higher mass by a factor of a few, with a Salpeter slope of $\Gamma \approx -2.3$ at high masses, and a flattening at lower masses (Beuther & Schilke 2004; Reid & Wilson 2005, 2006a,b). This extends earlier observational work indicating that in low- and intermediate-mass starforming regions the core-mass function resembles the stellar initial mass function (IMF) (e.g., Motte et al. 1998; Testi & Sargent 1998; Johnstone et al. 2001; Onishi et al. 2002), and suggests that the stellar IMF may simply be set by the mass distribution of prestellar cores, reduced by a factor of a few due to mass ejection by protostellar outflows (Matzner & McKee 2000). Simulations and analytic arguments, can, in turn, explain the core-mass Cambridge University Press & Assessment 978-0-521-76263-2 — Massive Stars Mario Livio, Eva Villaver Excerpt <u>More Information</u>

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distribution as arising naturally from the supersonic turbulence present in star-forming clumps (Padoan & Nordlund 2002; Tilley & Pudritz 2004; Li et al. 2004).

Clark & Bonnell (2006) argue that the mass distribution of *bound* cores in simulations does not have a Salpeter slope and thus is unlikely to be the origin of the stellar IMF. However, this misses a critical point: the Salpeter slope is only observed for stars significantly above the peak of the IMF. The full IMF is closer to a broken power law (Kroupa 2002) or a lognormal (Chabrier 2003), with the break or peak at ~0.5 M_{\odot} . This is roughly the Jeans mass in star-forming clumps, and indeed the simulations of Clark & Bonnell (2006) do show something like a lognormal distribution, with a peak at roughly the Jeans mass of their simulations. (The simulations are scale-free, since they include only hydrodynamics and gravity and use an isothermal equation of state.)

Furthermore, recent observations focusing on the spatial distribution of cores have shown that cores are mass segregated (Stanke et al. 2006) in much the same manner as stars in very young clusters: the core-mass function has the same lognormal or broken power-law form everywhere in clumps, with the exception that the most massive cores—those larger than a few M_{\odot} in size—are found only in the very center. The stellar population of the Orion Nebula Cluster (ONC) follows the same pattern (Hillenbrand & Hartmann 1998; Huff & Stahler 2006), indicating that the observed mass segregation in stars may simply be an imprinting of the prestellar-core spatial distribution. At least some of the mass segregation must be primordial rather than a result of dynamical evolution (Bonnell & Davies 1998), although recent evidence that cluster formation takes several crossing times (Tan et al. 2006) suggests that evolution may be important too. Nonetheless, it is quite suggestive that both the IMF and the spatial distribution of stars in a cluster seem to be explicable solely from the observed distribution of gas from which star clusters form. However, the origin of the mass segregation of cores is at present unknown.

3. Fragmentation of massive cores

It is only possible to explain the properties of stars in terms of the properties of cores if there is a more or less direct mapping from core mass to star mass. Such a mapping exists only if cores do not fragment too strongly, i.e., if massive cores typically produce one or a few massive stars, rather than many low-mass stars. Fragmentation to a few objects does not present a problem, since observationally constructed mass functions are generally uncorrected for multiple systems, but fragmentation to many objects does.

One might expect massive cores to fragment because they contain many thermal Jeans masses of gas. At the densities of $\sim 10^6$ H cm⁻³ and temperatures of ~ 10 K typical of massive cores, the Jeans mass is only $\sim M_{\odot}$, so one might expect a 50 M_{\odot} core to form tens of stars. Dobbs et al. (2005) simulate the collapse and fragmentation of massive cores with initial conditions based on the McKee & Tan (2003) model, using a code that includes hydrodynamics and gravity. They try both isothermal and barotropic equations of state. (Barotropic here means that the gas is assumed to be isothermal at densities below some critical density, chosen to be 10^{-14} g cm⁻³ in the Dobbs et al. simulations, and adiabatic at higher densities). Dobbs et al. find that the cores fragment, forming anywhere from a few to several tens of objects, depending on the assumed initial conditions and equation of state. In no case do their simulations form a massive star.

However, the Dobbs et al. (2005) calculation omits the critical effect of radiation feedback from the forming star. The high densities in massive cores produce high accretion rates, so that the first protostar to condense within a core will immediately produce a large accretion luminosity as the gas that falls onto it radiates away its potential energy.

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FIGURE 1. Temperature (left panel) and Jeans mass (right panel) versus radius in a core with a mass of 50 M_{\odot} , a column density of 1 g cm⁻², and a density profile $\rho \propto r^{-1.5}$, taken from the models of Krumholz et al. (2006a). The lines show the result from a radiative transfer calculation when the central protostar is $0.05 M_{\odot}$ (thin solid line) and when it is $0.8 M_{\odot}$ (thick solid line), and from using the barotropic approximation (dashed line) or an isothermal equation of state (dotted line). The Jeans mass is computed using the density and temperature at each radius, and is defined as $M_J = \frac{4}{3}\pi^{5/2}[k_BT/(G\mu)]^{3/2}\rho^{-1/2}$, where ρ is the density, T is the temperature, and $\mu = 2.33m_H$ is the mass per particle for a gas of molecular hydrogen and helium in the standard interstellar abundance.

For a typical accretion rate of $10^{-4} M_{\odot} \text{ yr}^{-1}$ at massive core densities, a $0.1 M_{\odot}$, $1 R_{\odot}$ star releases approximately $300 L_{\odot}$ of accretion power. Because the core is very optically thick, the radiation is trapped within it and heats the gas as it diffuses out. As a result, the densest, inner parts of the core, where fragmentation is most likely to take place, are subject to rapid heating, which suppresses fragmentation. Isothermal or barotropic approximations completely ignore this effect.

Krumholz et al. (2006a) examines how feedback heating affects fragmentation in the context of a simple model of core accretion using a high-accuracy analytic radiative-transfer approximation (Chakrabarti & McKee 2005). Figure 1 shows a sample result, the radial temperature profile and Jeans mass versus radius for a McKee & Tan (2003) core with a mass of 50 M_{\odot} and a column density of 1 g cm⁻² accreting onto a protostar in its center. The figure compares the results using a radiative transfer approach to what one would find by neglecting radiative transfer and simply using a barotropic or isothermal equation of state. As the plot shows, both an isothermal equation of state and the barotropic approximation make order-of-magnitude errors in the temperature and Jeans mass.

One might worry whether this analytic treatment done in spherical symmetry applies to more realistic massive cores with complex density structures (e.g., Bonnell et al. 2007). The natural way to address this question is with radiation-hydrodynamic simulations of massive core evolution. Krumholz et al. (2007) simulate the collapse and fragmentation of massive cores to examine this effect. The simulations use an adaptive mesh-refinement radiation code to solve the Euler equations of gas dynamics coupled to gray radiation transport and radiation pressure force in the flux-limited diffusion approximation (Truelove et al. 1998; Klein 1999; Howell & Greenough 2003). They use the adaptive mesh capability to guarantee that the local Jeans length is always resolved by at least eight cells (Truelove et al. 1997), and that the radiation energy density changes by no more than 25% per cell, so radiation gradients are well resolved. The code uses Eulerian sink particles to represent stars (Krumholz et al. 2004), and the sink particles are, in turn, coupled to a simple protostellar evolution model (McKee & Tan 2003), which computes

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FIGURE 2. Column density of the entire core (upper panel) and zoomed in on the forming stars (lower panel) for a simulation of a $100 M_{\odot}$, 0.1 pc McKee & Tan (2003) core at a time of 2.0×10^4 yr. The positions of the stars are indicated by the diamonds. Their masses are, from left to right, $0.31 M_{\odot}$, $5.33 M_{\odot}$, and $0.16 M_{\odot}$.

the instantaneous stellar luminosity, including the effects of accretion, Kelvin-Helmholtz contraction, deuterium burning, and hydrogen burning. This luminosity becomes a source term in the radiation equation. Further details on the code are given in Krumholz et al. (2005a).

The simulations begin with cores following the model of McKee & Tan (2003). The initial density profile is chosen with $\rho \propto r^{-1.5}$, to a maximum density of $\rho = 10^{-14}$ g cm⁻³, corresponding roughly to the density of the inner, thermally supported zone of McKee & Tan cores. The temperature is 20 K throughout the core. There are initial turbulent velocities chosen from a Gaussian random distribution (Dubinski et al. 1995) with a

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FIGURE 3. Temperature distribution in a 2D slice through a simulation of a 100 M_{\odot} , 0.1 pc McKee & Tan (2003) core at a time of 1.6×10^4 yr. The image is centered on the $3.83 M_{\odot}$ star, indicated by the diamond, the most massive present in the simulation at that time. The outermost contour corresponds to a temperature of 40 K, and each subsequent contour represents a factor of two increase in the temperature.

power spectrum $P(k) \propto k^{-4} d^3 k$ over wavelengths ranging from the size of the core to the size of the inner thermal zone, subject to the constraint that the initial velocity field be divergence free. The magnitude of the velocity field is normalized to give approximate hydrostatic balance on the largest scale (equation 6 of McKee & Tan 2003). The simulations reach a maximum resolution of 10 AU.

Figure 2 shows the column density distribution in a simulation of a core with an initial mass of $100 M_{\odot}$ and radius of 0.1 pc, $2.0 \times 10^4 \text{ yr}$ (0.37 mean-density free-fall times) after the start of a simulation. The core is not forming many stars—it is forming a triple system. Moreover, it is a highly unequal triple: the masses of the three stars are $5.33 M_{\odot}$, $0.31 M_{\odot}$, and $0.16 M_{\odot}$, so the vast majority of the mass has gone into the most massive object, the one at the center of the large disk. There are no apparent signs of further fragmentation, so unless feedback disrupts this system, it seems destined to form a massive star incorporating a significant fraction of the initial core mass, rather than dozens of small stars. The weak fragmentation we find from simulations provides strong support for the idea that the core-mass function directly sets the stellar-mass function.

To understand the origin of the weak fragmentation, it is helpful to examine the temperature distribution in the core. Figure 3 shows the temperature distribution in the simulation at $t = 1.6 \times 10^4$ yr, when the central star is only $3.83 M_{\odot}$. At this point, the star has not yet begun hydrogen burning, and the luminosity of a few thousand L_{\odot} is entirely due to accretion. This accretion power has doubled the initial temperature of the gas out to more than 2000 AU from the central star, and increased the temperature to more than 100 K over a radius of many hundreds of AU. This heating strongly suppresses fragmentation in the densest gas, where it is most likely to occur. Of the two stars that do form in addition to the most massive, one does so at an initial separation of several thousand AU, far enough that it can condense, and the other does so inside the protostellar disk, where the high column density provides shielding against the stellar

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FIGURE 4. Ratio of the temperature distribution shown in Figure 3 to the temperature one would infer using the barotropic equation of state of Dobbs et al. (2005). Note that the color scale starts at a ratio of 1.0, so any color other than black indicates that the true temperature is higher than the barotropic temperature. The outermost contour corresponds to a ratio of 0.4 dex, increasing by 0.2 dex with each subsequent contour.

radiation and produces a lower temperature. In examining a movie of the simulation, one clearly sees many overdense clumps that look like they might collapse, but do not do so because they are bathed in the radiation field of the central star. Rather than forming stars of their own, they fall onto the central star and accrete.

Part of the reason suppression of fragmentation is effective is because, despite the complex density structure shown in Figure 2, the temperature distribution is relatively round and smooth. The only significant deviation from sphericity is in the protostellar disk. Thus, the gas around the protostar is heated quite uniformly, and outside the disk, where shear suppresses most fragmentation, there are no cold spots favorable to fragmentation. This is to be expected: the entire core is very optically thick, so radiation diffuses outward rather than free streaming. As a result, there is very little shadowing, and clumps that are only starting to collapse are not sufficiently overdense to exclude the radiation field and remain cooler than their surroundings. As I discuss in Section 5, only when gas reaches the densities typical of accretion disks, or when optically thick structures begin to form, can there can be significant temperature anisotropies due to collimation.

The weak fragmentation shown in simulations with radiation is strikingly different from what one obtained without it, where the number of fragments generally approaches the number of thermal Jeans masses in the initial cloud. Figure 4 shows why: the barotropic equation of state severely underestimates the temperature, making fragmentation far easier than it should be. The magnitude of the underestimate ranges from factors of a few at distances of thousands of AU to orders of magnitude in the central hundreds of AU. Since the Jeans mass depends on temperature to the 1.5 power, the error in the critical mass for fragment growth is larger still. It is easy to intuitively understand why the barotropic approximation fails so badly: the physical assumption underlying the barotropic approximation is that above some critical density the gas cannot radiate

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efficiently, and all its gravitational potential energy is converted into heat. However, 3D simulations of star formation cannot resolve stellar surfaces, so any gas that falls into sink particles of radius ~10 AU disappears from the simulation, taking its gravitational potential energy with it. However, since potential energy varies as r^{-1} , the vast majority of the energy is released in the final plunge from ~10 AU to the stellar surface. In the barotropic approximation one simply ignores this energy, which is the dominant source of heating until nuclear burning begins.

4. Competitive accretion

Weak fragmentation means that a significant fraction of the mass in a massive core will end up in a single star or a few stars. However, for cores to be the direct progenitors of massive stars, any additional mass a star accretes from outside its parent core must be small compared to the stellar mass. The idea that most of a star's mass comes not from a parent core, but from gas in the cluster-forming clump not originally bound to that star, is called competitive accretion (Bonnell et al. 2007, and references therein). Several authors have made simple theoretical models based on numerical simulations that appear to show exactly this process. In these models, all stars are born from cores at roughly equal masses, with the initial mass ranging from brown dwarf masses (Bate & Bonnell 2005) to as much as ~0.5 M_{\odot} —the peak of the IMF—in the most recent models (Bonnell & Bate 2006). In these models, most of the seeds do not accrete much mass in addition to that in their parent core, but the special location of a few stars allows them to undergo rapid accretion, reaching high masses. This process determines the IMF above the peak.

4.1. Under what conditions does competitive accretion occur?

Competitive accretion definitely occurs in some simulations, and there is no reason to doubt those simulations produce the correct result for the physics they include. However, in order to determine whether the properties of real star-forming clumps are accurately reflected, it is necessary to investigate the physics behind the competitive accretion process. Krumholz et al. (2005d) defines the fractional mass change $f_M \equiv \dot{M}_* t_{\rm dyn}/M_*$ as the fractional mass change that a star of mass M_* undergoes per dynamical (crossing) time $t_{\rm dyn}$ of its parent clump, where \dot{M}_* is understood to refer to the accretion rate after the star has consumed its initial bound core. Competitive accretion models require $f_M \gg 1$. Accretion of gas that is not initially bound to a star can occur in one of two forms: either the star may capture other gravitationally bound cores and then accrete them, as proposed for example by Stahler et al. (2000), or it may accrete gas that is not organized into bound structures.

The former process is reasonably easy to understand, since it is simply an extension of standard calculations of collision rates in stellar dynamics. The only significant complication is that collisions between stars and cores that occur at too large a relative velocity do not result in capture, since the star will simply plough through the core without dissipating enough energy for the two to become bound. Even with this complication, the calculation is relatively straightforward, and Krumholz et al. (2005d) show that the fractional mass change due to captures of cores with mass comparable to the stellar mass M_* in a star-forming clump of mass M is

$$f_{M-\text{cap}} \approx 0.4\phi_{\text{co}} \left[4 + 2u^2 - (4 + 7.32u^2) \exp(-1.33u^2) \right]$$
, (4.1)

where $\phi_{\rm co}$ is the fraction of the clump mass that is in bound cores, $u \approx 10 \alpha_{\rm vir}^{-1} (M_*/M)^{1/2}$ is the ratio of the escape velocity from the surface of a core to the velocity dispersion in the clump, and $\alpha_{\rm vir}$ is the virial parameter for the core, roughly its ratio of turbulent

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FIGURE 5. Slices through a simulation of Bondi-Hoyle accretion in a turbulent medium by Krumholz et al. (2006b), showing density in the entire simulation box (upper panel) and in a small region (indicated by the black box) around one of the accreting particles (lower panel). The position of the particle is indicated by the small white diamond. The density and length are in dimensionless units where the mean density in the box is unity, and the box extends from -1 to 1. The maximum resolution of the simulation is 8192^3 .

kinetic energy to its gravitational potential energy. The significant thing to notice about this expression is that it does not approach unity unless u is quite large, which in turn only happens for virial parameters $\alpha_{\rm vir} \ll 1$, i.e., for clumps where the turbulent velocity dispersion is small compared to that needed to prevent collapse.

Accretion of unbound gas is somewhat more complex, since to determine the accretion rate one requires a theoretical model for Bondi-Hoyle accretion in a turbulent medium. Krumholz et al. (2005b, 2006b) have developed such a theory and shown that it reproduces the results of simulations quite well. Figure 5 shows a sample of an adaptive mesh-refinement simulation in which a grid of 64 Eulerian sink particles (Krumholz et al.

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FIGURE 6. Cumulative (left panel) and differential (right panel) distribution of accretion rates measured for particles in the simulation shown in Figure 5. Accretion rates are normalized to $\dot{M}_0 \equiv 4\pi \overline{\rho} (GM_*)^2 / (\sqrt{3}\sigma)^3$, where $\overline{\rho}$ is the mean density, M_* is the stellar mass, and σ is the 1D velocity dispersion in the simulation box. The histogram shows the simulation results, the solid line shows the Krumholz et al. (2006b) model, and the dashed and dotted lines show alternative models.

2004) are placed into a turbulent medium and allowed to accrete until the mean accretion rate reaches equilibrium. Figure 6 compares the model prediction for the probability distribution of accretion rates to the simulation results. The model predicts, and the simulation confirms, that the mean accretion rate for a star of mass M_* accreting from a medium of mean density $\overline{\rho}$ and 1D velocity dispersion σ is

$$\dot{M}_* 4\pi \phi_{\rm BH} \approx \overline{\rho} \frac{(GM_*)^2}{(\sqrt{3}\sigma)^3} \quad , \tag{4.2}$$

where the quantity $\phi_{\rm BH}$ is a function of the Mach number and size scale of the turbulent region, an approximate analytic form which is given in Krumholz et al. (2006b). For the properties of observed star-forming regions, it is generally ≤ 5 . From this result, one can compute f_M due to accretion of unbound gas in a star-forming clump of mass M:

$$f_{M-\rm BH} \approx 10\phi_{\rm BH}\alpha_{\rm vir}^{-2}(M_*/M) \quad . \tag{4.3}$$

Again, for cluster-clumps hundreds to thousands of M_{\odot} in mass, f_{M-BH} can be of order unity only for $\alpha_{\rm vir} \ll 1$.

Combining the two potential sources of mass, one can derive an approximate criterion that a star-forming gas clump must satisfy in order for competitive accretion to occur within it. For seed stars of mass $0.5 M_{\odot}$, this condition is

$$\alpha_{\rm vir}^2 M \lesssim 50 \, M_\odot \quad . \tag{4.4}$$

Straightforward application to observed star-forming clumps shows that they are nowhere near meeting this condition, since their typical masses are many hundreds to thousands of M_{\odot} , and their observed virial parameters generally are near unity. From this, Krumholz et al. (2005d) conclude that competitive accretion does not occur in real clumps. It occurs in simulations only because there either was very little turbulence present in the initial conditions (e.g., Klessen & Burkert 2000, 2001), or the initial turbulence has decayed away (e.g., Bonnell et al. 2003), leaving the clumps sub-virial.