

## CONTENTS

Preface	<i>page</i> xvii
Topical outline	xxi
Notation	xxiii
<b>1 Introduction</b>	<b>1</b>
1.1 What, why, and how?	2
1.1.1 What?	2
1.1.2 Why?	3
1.1.3 How?	6
1.1.4 Conservation statement	6
1.1.5 The need for constitutive models	7
1.1.6 Common constitutive models	8
1.2 Typical transport property values	10
1.2.1 Viscosity: pure gases and vapors	10
1.2.2 Viscosity: liquids	11
1.2.3 Thermal conductivity	11
1.2.4 Diffusivity	12
1.3 The continuum assumption and the field variables	13
1.3.1 Continuum and pointwise representation	13
1.3.2 Continuum vs. molecular	16
1.3.3 Primary field variables	16
1.3.4 Auxiliary variables	16
1.4 Coordinate systems and representation of vectors	18
1.4.1 Cartesian coordinates	18
1.4.2 Cylindrical coordinates	19
1.4.3 Spherical coordinates	20
1.4.4 Gradient of a scalar field	20
1.5 Modeling at various levels	22
1.5.1 Levels based on control-volume size	22
1.5.2 Multiscale models	24
1.5.3 Multiscale modeling below the continuum level	25
1.6 Model building: general guidelines	25
1.7 An example application: pipe flow and tubular reactor	27
1.7.1 Pipe flow: momentum transport	28
1.7.2 Laminar or turbulent?	28
1.7.3 Use of dimensionless numbers	30
1.7.4 Pipe flow: heat transport	32
1.7.5 Pipe flow: mass exchanger	35
1.7.6 Pipe flow: chemical reactor	35
1.8 The link between transport properties and molecular models	36
1.8.1 Kinetic theory concepts	37
1.8.2 Liquids	42
1.8.3 Transport properties of solids	44

## Contents

1.9	Six decades of transport phenomena	45
1.10	Closure	48
	Summary	49
	Additional Reading	50
	Problems	50
<b>2</b>	<b>Examples of transport and system models</b>	<b>56</b>
2.1	Macroscopic mass balance	58
	2.1.1 Species balance equation	58
	2.1.2 Transient balance: tracer studies	63
	2.1.3 Overall mass balance	65
2.2	Compartmental models	68
	2.2.1 Model equations	68
	2.2.2 Matrix representation	69
	2.2.3 A numerical IVP solver in MATLAB	70
2.3	Macroscopic momentum balance	72
	2.3.1 Linear momentum	72
	2.3.2 Angular momentum	77
2.4	Macroscopic energy balances	79
	2.4.1 Single inlet and outlet	79
	2.4.2 The Bernoulli equation	81
	2.4.3 Sonic and subsonic flows	85
	2.4.4 Cooling of a solid: a lumped model	91
2.5	Examples of differential balances: Cartesian	97
	2.5.1 Heat transfer with nuclear fission in a slab	97
	2.5.2 Mass transfer with reaction in a porous catalyst	99
	2.5.3 Momentum transfer: unidirectional flow in a channel	101
2.6	Examples of differential models: cylindrical coordinates	102
	2.6.1 Heat transfer with generation	102
	2.6.2 Mass transfer with reaction	104
	2.6.3 Flow in a pipe	105
2.7	Spherical coordinates	106
2.8	Examples of mesoscopic models	108
	2.8.1 Tubular reactor with heat transfer	108
	2.8.2 Heat transfer in a pin fin	109
	2.8.3 Countercurrent heat exchanger	110
	2.8.4 Counterflow: matrix method	115
	Summary	116
	Problems	119
<b>3</b>	<b>Flow kinematics</b>	<b>126</b>
3.1	Eulerian description of velocity	128
3.2	Lagrangian description: the fluid particle	128
3.3	Acceleration of a fluid particle	130
3.4	The substantial derivative	130
3.5	Dilatation of a fluid particle	132
3.6	Mass continuity	134
3.7	The Reynolds transport theorem	135
3.8	Vorticity and rotation	136

3.8.1	Curl in other coordinate systems	137
3.8.2	Circulation along a closed curve	139
3.9	Vector potential representation	140
3.10	Streamfunctions	141
3.10.1	Two-dimensional flows: Cartesian	141
3.10.2	Two-dimensional flows: polar	143
3.10.3	Streamfunctions in axisymmetric flows	143
3.10.4	The relation to vorticity: the $E^2$ operator	144
3.11	The gradient of velocity	145
3.12	Deformation and rate of strain	146
3.12.1	The physical meaning of the rate of strain	148
3.12.2	Rate of strain: cylindrical	151
3.12.3	Rate of strain: spherical	151
3.12.4	Invariants of a tensor	152
3.13	Index notation for vectors and tensors	152
	Summary	154
	Problems	155
<b>4</b>	<b>Forces and their representations</b>	<b>159</b>
4.1	Forces on fluids and their representation	160
4.1.1	Pressure forces	161
4.1.2	Viscous forces	163
4.1.3	The divergence of a tensor	167
4.2	The equation of hydrostatics	169
4.2.1	Archimedes' principle	169
4.2.2	The force on a submerged surface: no curvature	170
4.2.3	Force on a curved surface	171
4.3	Hydrostatics at interfaces	172
4.3.1	The nature of interfacial forces	172
4.3.2	Contact angle and capillarity	174
4.3.3	The Laplace–Young equation	175
4.4	Drag and lift forces	177
	Summary	180
	Problems	181
<b>5</b>	<b>Equations of motion and the Navier–Stokes equation</b>	<b>184</b>
5.1	Equation of motion: the stress form	185
5.1.1	The Lagrangian point particle	185
5.1.2	The Lagrangian control volume	186
5.1.3	The Eulerian control volume	187
5.2	Types of fluid behavior	189
5.2.1	Types and classification of fluid behavior	189
5.2.2	Stress relations for a Newtonian fluid	191
5.3	The Navier–Stokes equation	191
5.3.1	The Laplacian of velocity	192
5.3.2	Common boundary conditions for flow problems	193
5.4	The dimensionless form of the flow equation	195
5.4.1	Key dimensionless groups	195
5.4.2	The Stokes equation: slow flow or viscous flow	196
5.4.3	The Euler equation	197

## Contents

5.5	Use of similarity for scaleup	197
5.6	Alternative representations for the Navier–Stokes equations	201
5.6.1	Plane flow: the vorticity–streamfunction form	201
5.6.2	Plane flow: the streamfunction representation	201
5.6.3	Inviscid and potential flow	202
5.6.4	The velocity–vorticity formulation	202
5.6.5	Slow flow in terms of vorticity	202
5.6.6	The pressure Poisson equation	203
5.7	Constitutive models for non-Newtonian fluids	203
	Summary	205
	Problems	206
<b>6</b>	<b>Illustrative flow problems</b>	<b>208</b>
6.1	Introduction	210
6.1.1	Summary of equations	210
6.1.2	Simplifications	211
6.1.3	Solution methods	211
6.2	Channel flow	212
6.2.1	Entry-region flow in channels or pipes	212
6.2.2	General solution	214
6.2.3	Pressure-driven flow	215
6.2.4	Shear-driven flow	215
6.2.5	Gravity-driven flow	216
6.3	Axial flow in cylindrical geometry	218
6.3.1	Circular pipe	219
6.3.2	Annular pipe: pressure-driven	219
6.3.3	Annular pipe: shear-driven	220
6.4	Torsional flow	220
6.5	Radial flow	222
6.6	Flow in a spherical gap	223
6.7	Non-circular channels	224
6.8	The lubrication approximation	227
6.8.1	Flow between two inclined plates	227
6.8.2	Flow in a tapered pipe	228
6.9	External flow	230
6.10	Non-Newtonian viscoelastic fluids	233
6.10.1	A power-law model	233
6.10.2	Flow of a Bingham fluid in a pipe	234
6.10.3	The Rabinowitsch equation	236
6.11	The effect of fluid elasticity	237
6.12	A simple magnetohydrodynamic problem	240
	Summary	244
	Additional Reading	246
	Problems	246
<b>7</b>	<b>The energy balance equation</b>	<b>251</b>
7.1	Application of the first law of thermodynamics to a moving control volume	252
7.2	The working rate of the forces	253

7.3	Kinetic energy and internal energy equations	256
7.4	The enthalpy form	257
7.5	The temperature equation	257
7.6	Common boundary conditions	259
7.7	The dimensionless form of the heat equation	261
7.8	From differential to macroscopic	262
7.9	Entropy balance and the second law of thermodynamics	263
	7.9.1 Some definitions from thermodynamics	263
	Summary	267
	Problems	268
<b>8</b>	<b>Illustrative heat transport problems</b>	<b>269</b>
8.1	Steady heat conduction and no generation	270
	8.1.1 Constant conductivity	270
	8.1.2 Variable thermal conductivity	273
	8.1.3 Two-dimensional heat conduction problems	274
8.2	Heat conduction with generation: the Poisson equation	276
	8.2.1 The constant-generation case	276
8.3	Conduction with temperature-dependent generation	277
	8.3.1 Linear variation with temperature	277
	8.3.2 Non-linear variation with temperature	279
	8.3.3 Two-dimensional Poisson problems	281
8.4	Convection effects	282
	8.4.1 Transpiration cooling	282
	8.4.2 Convection in boundary layers	285
8.5	Mesosopic models	286
	8.5.1 Heat transfer from a fin	286
	8.5.2 A single-stream heat exchanger	288
8.6	Volume averaging or lumping	290
	8.6.1 Cooling of a sphere in a liquid	290
	8.6.2 An improved lumped model	291
	Summary	292
	Problems	293
<b>9</b>	<b>Equations of mass transfer</b>	<b>296</b>
9.1	Preliminaries	298
9.2	Concentration jumps at interfaces	300
9.3	The frame of reference and Fick's law	302
9.4	Equations of mass transfer	307
	9.4.1 Mass basis	308
	9.4.2 Mole basis	310
	9.4.3 Boundary conditions	311
9.5	From differential to macroscopic	312
9.6	Complexities in diffusion	313
	Summary	316
	Problems	317
<b>10</b>	<b>Illustrative mass transfer problems</b>	<b>321</b>
10.1	Steady-state diffusion: no reaction	322
	10.1.1 Summary of equations	322

**Contents**

10.2	The film concept in mass-transfer analysis	328
10.2.1	Fluid–solid interfaces	328
10.2.2	Gas–liquid interfaces: the two-film model	331
10.3	Mass transfer with surface reaction	333
10.3.1	Heterogeneous reactions: the film model	333
10.4	Mass transfer with homogeneous reactions	334
10.4.1	Diffusion in porous media	334
10.4.2	Diffusion and reaction in a porous catalyst	335
10.4.3	First-order reaction	335
10.4.4	Zeroth-order reaction	339
10.4.5	Transport in tissues: the Krogh model	340
10.4.6	<i>m</i> th-order reaction	342
10.5	Models for gas–liquid reaction	343
10.5.1	Analysis for the pseudo-first-order case	346
10.5.2	Analysis for instantaneous asymptote	347
10.5.3	The second-order case: an approximate solution	347
10.5.4	The instantaneous case: the effect of gas film resistance	348
10.6	Transport across membranes	350
10.6.1	Gas transport: permeability	350
10.6.2	Complexities in membrane transport	352
10.6.3	Liquid-separation membranes	353
10.7	Transport in semi-permeable membranes	354
10.7.1	Reverse osmosis	355
10.7.2	Concentration-polarization effects	356
10.7.3	The Kedem–Katchalsky model	358
10.7.4	Transport in biological membranes	360
10.8	Reactive membranes and facilitated transport	360
10.8.1	Reactive membrane: facilitated transport	360
10.8.2	Co- and counter-transport	363
10.9	A boundary-value solver in MATLAB	364
10.9.1	Code-usage procedure	364
10.9.2	BVP4C example: the selectivity of a catalyst	364
	Summary	367
	Additional Reading	370
	Problems	370
<b>11</b>	<b>Analysis and solution of transient transport processes</b>	<b>377</b>
11.1	Transient conduction problems in one dimension	378
11.2	Separation of variables: the slab with Dirichlet conditions	380
11.2.1	Slab: temperature profiles	383
11.2.2	Slab: heat flux	384
11.2.3	Average temperature	384
11.3	Solutions for Robin conditions: slab geometry	385
11.4	Robin case: solutions for cylinder and sphere	387
11.5	Two-dimensional problems: method of product solution	388
11.6	Transient non-homogeneous problems	389
11.6.1	Subtracting the steady-state solution	390
11.6.2	Use of asymptotic solution	391

**Contents**

11.7	Semi-infinite-slab analysis	391
11.7.1	Constant surface temperature	392
11.7.2	Constant flux and other boundary conditions	393
11.8	The integral method of solution	394
11.9	Transient mass diffusion	396
11.9.1	Constant diffusivity model	396
11.9.2	The penetration theory of mass transfer	399
11.9.3	The effect of chemical reaction	399
11.9.4	Variable diffusivity	403
11.10	Periodic processes	404
11.10.1	Analysis for a semi-infinite slab	405
11.10.2	Analysis for a finite slab	407
11.11	Transient flow problems	408
11.11.1	Start-up of channel flow	409
11.11.2	Transient flow in a semi-infinite mass of fluid	409
11.11.3	Flow caused by an oscillating plate	409
11.11.4	Start-up of Poiseuille flow	411
11.11.5	Pulsatile flow in a pipe	412
11.12	A PDE solver in MATLAB	413
11.12.1	Code usage	413
11.12.2	Example general code for 1D transient conduction	415
	Summary	417
	Additional Reading	418
	Problems	419
<b>12</b>	<b>Convective heat and mass transfer</b>	<b>425</b>
12.1	Heat transfer in laminar flow	427
12.1.1	Preliminaries and the model equations	427
12.1.2	The constant-wall-temperature case: the Graetz problem	430
12.1.3	The constant-flux case	434
12.2	Entry-region analysis	435
12.2.1	The constant-wall-temperature case	435
12.2.2	The constant-flux case	437
12.3	Mass transfer in film flow	437
12.3.1	Solid dissolution at a wall in film flow	438
12.3.2	Gas absorption from interfaces in film flow	439
12.4	Laminar-flow reactors	440
12.4.1	A 2D model and key dimensionless groups	440
12.4.2	The pure convection model	443
12.5	Laminar-flow reactor: a mesoscopic model	444
12.5.1	Averaging and the concept of dispersion	444
12.5.2	Non-linear reactions	446
12.6	Numerical study examples with PDEPE	446
12.6.1	The Graetz problem	446
	Summary	449
	Problems	450
<b>13</b>	<b>Coupled transport problems</b>	<b>453</b>
13.1	Modes of coupling	454
13.1.1	One-way coupling	454

## Contents

13.1.2	Two-way coupling	455
13.2	Natural convection problems	455
13.2.1	Natural convection between two vertical plates	455
13.2.2	Natural convection over a vertical plate	459
13.2.3	Natural convection: concentration effects	460
13.3	Heat transfer due to viscous dissipation	460
13.3.1	Viscous dissipation in plane Couette flow	460
13.3.2	Laminar heat transfer with dissipation: the Brinkman problem	461
13.4	Laminar heat transfer: the effect of viscosity variations	463
13.5	Simultaneous heat and mass transfer: evaporation	465
13.5.1	Dry- and wet-bulb temperatures	465
13.5.2	Evaporative or sweat cooling	468
13.6	Simultaneous heat and mass transfer: condensation	468
13.6.1	Condensation of a vapor in the presence of a non-condensable gas	468
13.6.2	Fog formation	472
13.6.3	Condensation of a binary gas mixture	472
13.7	Temperature effects in a porous catalyst	476
	Summary	480
	Additional Reading	481
	Problems	481
<b>14</b>	<b>Scaling and perturbation analysis</b>	<b>484</b>
14.1	Dimensionless analysis revisited	485
14.1.1	The method of matrix transformation	486
14.1.2	Momentum problems	486
14.1.3	Energy transfer problems	489
14.1.4	Mass transfer problems	491
14.1.5	Example: scaleup of agitated vessels	492
14.1.6	Example: pump performance correlation	493
14.2	Scaling analysis	495
14.2.1	Transient diffusion in a semi-infinite region	495
14.2.2	Example: gas absorption with reaction	496
14.2.3	Kolmogorov scales for turbulence: an example of scaling	496
14.2.4	Scaling analysis of flow in a boundary layer	497
14.2.5	Flow over a rotating disk	501
14.3	Perturbation methods	503
14.3.1	Regular perturbation	503
14.3.2	The singular perturbation method	506
14.3.3	Example: catalyst with spatially varying activity	507
14.3.4	Example: gas absorption with reversible reaction	508
14.3.5	Stokes flow past a sphere: the Whitehead paradox	511
14.4	Domain perturbation methods	513
	Summary	515
	Additional Reading	516
	Problems	516
<b>15</b>	<b>More flow analysis</b>	<b>523</b>
15.1	Low-Reynolds-number (Stokes) flows	525
15.1.1	Properties of Stokes flow	525
15.2	The mathematics of Stokes flow	527



## Contents

15.2.1	General solutions: spherical coordinates	527
15.2.2	Flow past a sphere: use of the general solution	528
15.2.3	Bubbles and drops	531
15.2.4	Oseen's improvement	533
15.2.5	Viscosity of suspensions	534
15.2.6	Nanoparticles: molecular effects	535
15.3	Inviscid and irrotational flow	536
15.3.1	Properties of irrotational flow	536
15.3.2	The Bernoulli equation revisited	537
15.4	Numerics of irrotational flow	539
15.4.1	Boundary conditions	539
15.4.2	Solutions using harmonic functions	540
15.4.3	Solution using singularities	542
15.5	Flow in boundary layers	546
15.5.1	Relation to the vorticity transport equation	547
15.5.2	Flat plate: integral balance	548
15.5.3	The integral method: the von Kármán method	549
15.5.4	The average value of drag	550
15.5.5	Non-flat systems: the effect of a pressure gradient	550
15.6	Use of similarity variables	551
15.6.1	A simple computational scheme	553
15.6.2	Wedge flow: the Falkner–Skan equation	554
15.6.3	Blasius flow	554
15.6.4	Stagnation-point (Hiemenz) flow	555
15.7	Flow over a rotating disk	556
	Summary: Stokes flow	557
	Summary: potential flow	558
	Summary: boundary-layer theory	558
	Additional Reading	559
	Problems	559
<b>16</b>	<b>Bifurcation and stability analysis</b>	<b>566</b>
16.1	Introduction to dynamical systems	567
16.1.1	Arc-length continuation: a single-equation example	571
16.1.2	The arc-length method: multiple equations	572
16.2	Bifurcation and multiplicity of DPSS	576
16.2.1	A bifurcation example: the Frank-Kamenetskii equation	576
16.2.2	Bifurcation: porous catalyst	577
16.3	Flow-stability analysis	578
16.3.1	Evolution equations and linearized form	578
16.3.2	Normal-mode analysis	580
16.4	Stability of shear flows	581
16.4.1	The Orr–Sommerfeld equation	581
16.4.2	Stability of shear layers: the role of viscosity	583
16.4.3	The Rayleigh equation	583
16.4.4	Computational methods	584
16.5	More examples of flow instability	585
16.5.1	Kelvin–Helmholtz instability	585
16.5.2	Rayleigh–Taylor instability	586
16.5.3	Thermal instability: the Bénard problem	587

## Contents

16.5.4	Marangoni instability	588
16.5.5	Non-Newtonian fluids	588
	Summary	589
	Additional Reading	589
	Problems	589
<b>17</b>	<b>Turbulent-flow analysis</b>	<b>592</b>
17.1	Flow transition and properties of turbulent flow	593
17.2	Time averaging	594
17.3	Turbulent heat and mass transfer	597
17.4	Closure models	598
17.5	Flow between two parallel plates	599
17.6	Pipe flow	603
17.6.1	The effect of roughness	605
17.7	Turbulent boundary layers	606
17.8	Other closure models	607
17.8.1	The two-equation model: the $k-\epsilon$ model	608
17.8.2	Reynolds-stress models	609
17.8.3	Large-eddy simulation	610
17.8.4	Direct numerical simulation	610
17.9	Isotropy, correlation functions, and the energy spectrum	610
17.10	Kolmogorov's energy cascade	613
17.10.1	Correlation in the spectral scale	614
	Summary	615
	Additional Reading	616
	Problems	616
<b>18</b>	<b>More convective heat transfer</b>	<b>619</b>
18.1	Heat transport in laminar boundary layers	620
18.1.1	Problem statement and the differential equation	620
18.1.2	The thermal boundary layer: scaling analysis	621
18.1.3	The heat integral equation	624
18.1.4	Thermal boundary layers: similarity solution	627
18.2	Turbulent heat transfer in channels and pipes	628
18.2.1	Pipe flow: the Stanton number	633
18.3	Heat transfer in complex geometries	635
18.4	Natural convection on a vertical plate	636
18.4.1	Natural convection: computations	640
18.5	Boiling systems	641
18.5.1	Pool boiling	641
18.5.2	Nucleate boiling	641
18.6	Condensation problems	645
18.7	Phase-change problems	647
	Summary	650
	Additional reading	651
	Problems	651

<b>19</b>	<b>Radiation heat transfer</b>	<b>656</b>
19.1	Properties of radiation	657
19.2	Absorption, emission, and the black body	657
19.3	Interaction between black surfaces	661
19.4	Gray surfaces: radiosity	664
19.5	Calculations of heat loss from gray surfaces	666
19.6	Radiation in absorbing media	670
	Summary	674
	Additional Reading	675
	Problems	675
<b>20</b>	<b>More convective mass transfer</b>	<b>678</b>
20.1	Mass transfer in laminar boundary layers	679
20.1.1	The low-flux assumption	679
20.1.2	Dimensional analysis	680
20.1.3	Scaling analysis	681
20.1.4	The low-flux case: integral analysis	682
20.1.5	The low-flux case: exact analysis	685
20.2	Mass transfer: the high-flux case	686
20.2.1	The film model revisited	686
20.2.2	The high-flux case: the integral-balance model	688
20.2.3	The high-flux case: the similarity-solution method	689
20.3	Mass transfer in turbulent boundary layers	689
20.4	Mass transfer at gas–liquid interfaces	691
20.4.1	Turbulent films	691
20.4.2	Single bubbles	692
20.4.3	Bubble swarms	693
20.5	Taylor dispersion	693
	Summary	696
	Additional Reading	696
	Problems	697
<b>21</b>	<b>Mass transfer: multicomponent systems</b>	<b>700</b>
21.1	A constitutive model for multicomponent transport	701
21.1.1	Stefan–Maxwell models	701
21.1.2	Generalization	702
21.2	Non-reacting systems and heterogeneous reactions	703
21.2.1	Evaporation in a ternary mixture	703
21.2.2	Evaporation of a binary liquid mixture	704
21.2.3	Ternary systems with heterogeneous reactions	707
21.3	Application to homogeneous reactions	709
21.3.1	Multicomponent diffusion in a porous catalyst	709
21.3.2	MATLAB implementation	710
21.4	Diffusion-matrix-based methods	713
21.5	An example of pressure diffusion	717
21.6	An example of thermal diffusion	719

## Contents

Summary	720
Additional Reading	721
Problems	721
<b>22 Mass transport in charged systems</b>	<b>725</b>
22.1 Transport of charged species: preliminaries	726
22.1.1 Mobility and diffusivity	726
22.1.2 The Nernst–Planck equation	727
22.1.3 Potential field and charge neutrality	728
22.2 Electrolyte transport across uncharged membranes	732
22.3 Electrolyte transport in charged membranes	734
22.4 Transport effects in electrodialysis	735
22.5 Departure from electroneutrality	738
22.6 Electro-osmosis	741
22.7 The streaming potential	744
22.8 The sedimentation potential	746
22.9 Electrophoresis	747
22.10 Transport in ionized gases	748
Summary	750
Additional Reading	751
Problems	751
Closure	757
References	758
Index	766