Advanced Transport Phenomena

An integrated, modern approach to transport phenomena for graduate students, featuring traditional and contemporary examples to demonstrate the diverse practical applications of the theory. Written in an easy-to-follow style, the basic principles of transport phenomena and model building are recapped in Chapters 1 and 2 before progressing logically through more advanced topics including physicochemical principles behind transport models. Treatments of numerical, analytical, and computational solutions are presented side-by-side, often with sample code in MATLAB, to aid students' understanding and develop their confidence in using computational skills to solve real-world problems.

Learning objectives and mathematical prerequisites at the beginning of chapters orient students to what is required in the chapter, and summaries and over 400 end-of-chapter problems help them retain the key points and check their understanding. Online supplementary material including solutions to problems for instructors, supplementary reading material, sample computer codes, and case studies completes the package (available at www.cambridge.org/ramachandran).

P. A. Ramachandran is a Professor in the Department of Energy, Environment, and Chemical Engineering at Washington University, St. Louis. He has extensive teaching experience, mainly in transport phenomena, mathematical methods, and chemical reaction engineering, and he has also held many visiting appointments at various international institutions. He has written or co-written two previous books, as well as over 200 journal articles in which he has pioneered many new concepts and computational tools for the modeling of chemical reactors. He is a recipient of the Moulton Medal from the Institution of Chemical Engineers, UK, the NASA certificate of recognition, USA, and the NEERI award from the Institution of Chemical Engineers, India.

"Anyone who teaches transport phenomena will treasure this book because it provides an integrated approach to help students better understand the core theories through both traditional and contemporary examples of transport phenomena problems, along with side-by-side presentations of both analytical and numerical methods and sample MATLAB codes – the long-awaited, all-in-one solution."

Roger Lo
California State University

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ANALYSIS, MODELING, AND COMPUTATIONS

P. A. RAMACHANDRAN
Washington University, St. Louis

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Cover illustration: close-up of ink spreading in water mixed with droplets of paint. This image was created by dropping a small amount of oil- and xylene-based gold paint onto the surface of colored water. Different inks were then dropped onto the gold paint until the weight of the ink caused the gold paint to dip and allowed the inks to burst into the water. The complex colors, shapes, and patterns are a result of varying levels of flow rate, ink density, and surface tension. This creates eddies and vortices that affect the way light is reflected from the surface. Image © Perry Burge/Science Photo Library.

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PREFACE

The analysis, modeling, and computation of processes involving the transport of heat, mass, and momentum (transport phenomena) play a central role in engineering education and practice. The study of this subject originated in the field of chemical engineering but is now an integral part of most engineering curricula, for example, in biological, biomedical, chemical, environmental, mechanical, and metallurgical engineering both at undergraduate and at graduate level. There are many textbooks in this area, with varying levels of treatment from introductory to advanced, all of which are useful to students at various levels. However, my teaching experience over thirty years has convinced me that there is a need for a book that develops the subject of transport phenomena in an integrated manner with an easy-to-follow style of presentation. A book of this nature should ideally combine theory and problem formulation with mathematical and computational tools. It should illustrate the usefulness of the field with regard to practical problems and model development. This is the primary motivation for writing this book. This comprehensive textbook is intended mainly as a graduate-level text in a modern engineering curriculum, but parts of it are also useful for an advanced senior undergraduate class. Students studying this book will understand the methodology of modeling transport processes, along with the fundamentals and governing differential equations. They will develop an ability to think through a given physical problem and cast an appropriate model for the system. They will also become aware of the common analytical and numerical methods to solve these models, and develop a feel for the diverse technological areas where these concepts can be used.

Goals and outcome

The book is written with the objective that students finishing a first-year-level graduate course in this field should acquire the following skills and knowledge.

- **Fundamentals and basic understanding** of the phenomena and the governing differential equations. They should develop an ability to analyze a given physical problem and cast an appropriate model for the system. They should be exposed to the philosophy of the modeling process and appreciate the various levels at which models can be developed, and the interconnection and parameter requirements of various models.
- **Analytical and numerical skills** to solve these problems. They should develop the capability to solve some of the transport problems in a purely analytical setting and also expand their capability using numerical methods with some common software or programming tools. Often solving the same problem by both methods reinforces the physics and speeds up the learning process.
- **An understanding of technological areas** where transport models are useful. Students should develop an understanding of the diverse range of applications of this subject and

understand how the basic theory, models, and computations can be used in practical applications.

To achieve these goals the book focuses on analysis and model development of transport process in detail, starting from the very basics. It illustrates the solution methods by using the classical analytical tools as well as some common computational tools. The application of the theory is demonstrated with numerous illustrative problems; some sample numerical codes are provided for some problems to facilitate learning and the development of problem-solving abilities. References to many areas of application are provided, and some case-study problems are included.

Intended audience

The level and the sequence of presentation are such that the book is suitable for a first-level graduate course or a comprehensive advanced undergraduate course. In a modern graduate engineering curriculum, the entering students often have diverse backgrounds, and some graduate students might not have taken introductory undergraduate courses in transport phenomena. The introductory part of the book presented in the first two chapters is expected to bring these students up to speed.

Style and scope

The style of presentation is informal, and has more of a "classroom" conversational tone rather than being heavy scholarly writing. Each chapter starts off with clearly defined learning objectives and ends with a summary of "must-know" things that should have been mastered from that chapter. Computer simulations are also illustrated, together with analytical solutions. Often solutions to the same problem obtained by both analytical and numerical methods are shown. This helps the students to validate and benchmark their solutions, and to develop confidence in their computational skills. Also sample packages are included to accelerate the application of computer-aided problem solving in the classroom. These sample codes are presented in separate subsections or are boxed off for easier reading of the main text. Key equations are shown in boxes for easy reference. Case studies are given in several chapters, although the space limitation prohibits an extensive discussion of these applications. Additional material and computer codes will be posted on the accompanying website, which is being developed as supplementary material. This web-based material can be viewed as a living and evolving component of the book.

For instructors

Instructors will find the presentations novel and interesting and will be able to motivate the students to appreciate the beauty in the integrated structure of the field. They will also find

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the worked examples and exercise problems useful to amplify the class lectures and illustrate the theory. Also the mathematical prerequisites listed at the beginning of each chapter will help the instructor to adjust the lecture content according to the students' mathematical preparedness. Additional web-based material that will aid the teaching of these necessary mathematical tools in a concise manner is being planned.

The book has more material than can be covered in one semester, and it can be used in the following manner in teaching.

- For an integrated course for students entering a modern graduate program with **diverse** undergraduate background, Chapters 1–13 can be covered at a reasonable pace in a one-semester course with some reading materials assigned from the other chapters.
- For a course focused mainly on flow problems Chapters 3–6 followed by Chapters 14–17 will provide a nice one-semester textbook.
- For a course focused mainly on heat and mass transfer the course can start with Chapters 7–13 and end with Chapters 18–22.

Distinguishing features

The book provides an integrated approach to the field. Theory is illustrated with many worked examples and case-study problems are indicated. The book also discusses many important and practically relevant topics that are not adequately covered in many earlier books. Some novel topics and features of the current book are indicated below.

- Discussion on multiscale modeling, model reduction by averaging and "information" flow.
- Solution of illustrative problems by both numerical and analytical methods.
- Sample codes in MATLAB for help in the development of numerical problem-solving skills.
- Detailed analysis of coupled transport problems.
- Introduction to non-Newtonian flow, microfluid analysis, and magnetohydrodynamics.
- Introduction to perturbation, bifurcation, and stability analysis.
- Detailed discussion on analysis of transport with chemical reaction.
- Detailed analysis of multicomponent diffusion with many worked examples.
- A full chapter on electrochemical systems and ionic transport.
- Application examples drawn from a wide range of areas and some suggested case-study problems.

Acknowledgement

Washington University, St. Louis, provided me with an academic home, and I wish to express my gratitude. Many summers of being visiting professor at Kasetsart University,

Bangkok, helped me to teach and fine-tune many topics. I would like to mention my appreciation of my *alma mater*, ICT, Mumbai, formerly known as UDCT. In a significant manner, I have been beneficiary of the rigorous and often disciplinary system of education in India, starting from my elementary school and continuing all the way to UDCT. I would like to acknowledge my many mentors and colleagues, too numerous to thank individually, from whom I have benefited throughout my career. Most of all I would like to thank all my students. My real education started with them, and still continues.

I would like to express my appreciation of my immediate family in the USA, Nima, Josh, Gabe, and Maya, and my brothers, sisters, and sisters-in-law in India for all their support and encouragement. I would like to express my appreciation of my friends in University City, Missouri, and to thank Dawn, who stressed the importance of diet and nutrition when training for a marathon.

On the editorial side, many thanks are due to Cambridge editors and especially to Claire Eudall, who provided valuable advice on the style and structure of various chapters. Also I appreciate the help of Ramesh Prajapati for the preparation of many figures in the text.

TOPICAL OUTLINE

The topical organization of this book is as follows.

Chapter 1 is the basic introductory material which illustrates the richness of the subject, spanning applications to a wide range of problems in science and engineering. This chapter also provides the introduction to the basics of model building and shows the relationships among models of various levels of hierarchy. The basic vocabulary is introduced, and the physical properties needed in transport problems are discussed. The link between continuum and molecular models is indicated. The chapter concludes with a brief note on the historical development of the subject.

Chapter 2 illustrates the formulation of model equations for many common transport problems using a basic control-volume-balance type of approach. All three modes of transport are illustrated so that the student can grasp the similarities. Some "standard" problems are illustrated. This chapter is written assuming no significant earlier background knowledge in this field, and is therefore useful to bring such students up to speed.

The next few chapters, Chapters 3–6, provide the detailed framework for the analysis of momentum transport problems. The kinematics of flow are reviewed in Chapter 3, while the kinetics of flow are discussed in Chapter 4, leading to the derivation of the differential equations for the stress field and the velocity field in Chapter 5. Solutions to illustrative flow problems are then reviewed in Chapter 6, and here some "standard" flow problems shown in Chapters 1 and 2 are revisited in a more general setting, and solutions to some additional complex problems are reviewed. Flows involving non-Newtonian fluids and magnetohydrodynamics are also treated briefly, since they find extensive applications in practice and it is necessary to expose the student to these topics.

Chapters 7 and 8 deal with the differential equations for energy transport and the temperature field, with many illustrative heat-transfer problems in Chapter 8. Similarly, Chapters 9 and 10 deal with differential equations for mass transport and illustrative applications. These chapters bring out the close analogy and common problem-solving strategies for these two transport processes. In the heat-transfer context entropy balance is introduced in a simple manner and the relation to the second law is pointed out in a succinct manner. In the mass-transfer context several important topics such as gas–liquid reactions, membrane transport, and dispersion are presented in detail. Numerical methods involving MATLAB for both ODE and PDE are presented. Sample codes are provided as examples, and side-by-side comparisons with analytical solutions are provided for many problems, so that the students can benchmark their results. The transient problems for both heat and mass are then analysed in Chapter 11 in a unified setting, while some convective transport problems are studied in Chapter 12.

Chapter 13 provides an analysis of a number of coupled problems, for example natural convection, simultaneous heat and mass transfer, condensation, fog formation, and temperature effects in porous catalysts.

Chapter 14 develops some tools to analyze transport problems in further detail. The dimensionless analysis is revisited using novel matrix-algebra-based methods. The concept of scaling and pertubation methods is introduced together with many applications. The scaling tools also provide the necessary background to the boundary-layer flows discussed in Chapter 15. Chapter 15 also discusses additional topics in fluid mechanics such as low-Reynolds-number flow and irrotational flows. Chapter 16 deals with bifurcation and stability analysis. Chapter 17 provides an introductory treatment of turbulent flows.

Chapters 18 and 19 deal with additional topics in heat transfer, including convection in turbulent flow, boiling, condensation, and radiation heat transfer (Chapter 19). The final three chapters (Chapters 20–22) discuss some topics in mass transfer, including more discussion on convective transport and axial dispersion (Chapter 20), multicomponent systems (Chapter 21), and transport of charged species (Chapter 22).

NOTATION

a_w	activity of water or solute indicated in the subscript in Section 10.7
A	area of cross-section for flow
A	Arrhenius pre-exponential factor in Section 8.3.2
A	amplitude of surface temperature oscillation in Section 11.10, K
A_1, A_2	usually integration constants
A_p	projected area of solid in the direction of flow
Ar	Archimedes number
B	dimensionless parameter defined as $(L/R)Pe$ in Section 12.4
Bi_G	Biot number in gas–liquid mass transfer, $k_G H_A/k_L$
Bi_h	Biot number for heat transfer, hL_{ref}/k_{solid}
Bi_m	Biot number for mass transfer, $k_m L_{ref}/D$
Bo	Bond number
Br	Brinkman number for viscous production of heat, Eq. (13.23)
C	total molar concentration of a multicomponent mixture, mol/m^3
Ca	capillary number, $\mu v_{ref}/\sigma$
C_A	local concentration of species indicated in the subscript (A here), mol/m^3
C_A^*	concentration of A in liquid if in equilibrium with the bulk gas (Section 10.5)
$\langle C_A \rangle$	cross-sectionally averaged concentration
C_{Ab}	concentration of species A indicated in the bulk phase, mol/m^3
C_{Ab}	cup mixed average concentration of species A, Section 12.4
C_{Ai}	concentration of species A at the interface, mol/m^3
$C_{A,i}$	inlet concentration of species A for flow reactor, Chapter 2, mol/m^3
C_{As}	concentration of species A at a solid surface, mol/m^3
C_{AG}	Concentration of species indicated in the subscript in the bulk gas, mol/m^3
C_{AL}	Concentration of species indicated in the subscript in the bulk liquid, mol/m^3
C_{AL}^*	hypothetical concentration of A if in equilibrium with the bulk gas, mol/m^3
$C_{A,e}$	concentration of species indicated in the subscript exit
C_b	cup mixed (flow) average concentration of species A, Section 20.5
C_{BL}	concentration of liquid-phase reactant in bulk liquid in Section 10.5
c	molecular speed in Chapter 1 (kinetic theory)
$\bar{c}$	average molecular speed in Chapter 1 (kinetic theory)
$\bar{c^2}$	average of the squares of the molecular speed in Chapter 1 (kinetic theory)
c	speed of sound in Chapter 2
c	speed of light in radiation heat transfer in Chapter 19
c_A	dimensionless concentration of species indicated in the subscript (A here), C_A/C_{ref}
$\bar{c}$	average speed of molecules in Section 1.8.1
C_D	drag coefficient
C_L	lift coefficient
c_p	specific heat of a species, mass basis, under constant-pressure conditions, J/kg · K

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C_p	specific heat of a species, mole basis, J/mol · K
c_v	specific heat of a species, mass basis, under constant-volume conditions, J/kg · K
d	diameter of the molecules treated as rigid spheres in Section 1.8.1
d, d_t	diameter of a tube or pipe
d_I	impeller or pump diameter, Sections 14.1.5 and 14.1.6
d_P	particle or solid diameter
D_e	effective diffusivity of a species in a heterogeneous medium
D_i	molecular diffusivity of species i
D_K	Knudsen diffusion coefficient for small pores
D_t	turbulent mass diffusivity, m^2/s
Da	Damköhler number Vk/Q
e	charge on an electron in Chapter 22
e	pipe roughness parameter in Sections 5.5 and 17.6.1
e	total energy content per unit mass
$\boldsymbol{e}_x$	unit vector in the x-direction
$\boldsymbol{E}$	electric field
E^2	operator defined by Eq. (3.53) or Eq. (3.55)
E^4	Stokes operator defined as E^2E^2
E	emissive power of a gray body
E_b	emissive power of a black body, W/m^2
E_{bk}	emissive power of a black body from surface k, W/m^2
$E_{b\lambda}$	spectral emissive power, W/m^2 nm
$\tilde{E}$	rate-of-strain tensor
f	dimensionless streamfunction in boundary-layer flow
f	Fanning friction factor
F_{ik}	radiation view factor, surface i to k
F	Faraday constant = 96 485 C/mol
$\boldsymbol{F}$	force acting on a control volume
$\mathcal{F}, \mathcal{F}_m$	correction factor for unidirectional mass transfer, Sections 10.1 and 20.2.1
$\mathcal{F}_h$	augmentation factor for heat transfer due to blowing
g	acceleration due to gravity
g_s	rate of production of entropy per unit volume, $W/K \cdot m^3$
G	pressure-drop parameter defined as $-dP/dx$
$\dot{G}$	superficial gas velocity, $kg/m^2 \cdot s$
Gr	Grashof number
$\hat{h}$	enthalpy per unit mass
h	heat transfer coefficient (usually from solid to fluid), $W/m^2 \cdot k$
h	elevation or height from a datum plane for flow problems
h	Planck's constant in radiation chapter, 6.6208×10^{-34} J · s
h_f	head loss due to friction
h_G	heat transfer coefficient in the gas film
h_L	heat transfer coefficient in the liquid film
$\hat{h}_{gl}$	heat released on condensation of a species, J/kg
$\hat{h}_{lg}$	heat of vaporization, J/kg
$\hat{h}_{sl}$	heat needed for melting a solid, J/kg
H_A	Henry's-law constant for solubility of A defined by $P_A = H_A C_A$, $Pa\,m^3/mol$

Ha	Hartmann number
Ha	Hatta number for gas–liquid reactions
i	current density in Chapter 22, $\mathrm{A/m^2}$
i	square root of -1 in Section 11.11
I	intensity of radiation, $\mathrm{W/m^2}$
j_A	mass diffusion flux of A (mass reference), $\mathrm{kg/m^2 \cdot s}$
J_A	molar diffusion flux of A (mole reference), $\mathrm{mol/m^2 \cdot s}$
J_k	radiosity of a surface in radiation, $\mathrm{W/m^2}$
k_G	mass transfer coefficient from gas to interface (partial pressure driving force), $\mathrm{mol/Pa \cdot m^2 \cdot s}$
k_L	mass transfer coefficient from an interface to bulk liquid (concentration driving force), m/s
k	thermal conductivity of a species, subscript l for liquid, g for gas, s for solid, $\mathrm{W/m \cdot K}$
k	turbulent kinetic energy per unit mass, $\mathrm{m^2/s^2}$
k	rate constant for reaction, general
k_B	Boltzmann constant, 1.38×10^{-23} J/K
k_0	rate constant for a zeroth-order reaction, $\mathrm{mol/m^3 \cdot s}$
k_1	rate constant for a first-order reaction, 1/s
k_2	rate constant for a second-order reaction, $\mathrm{m^3/mol \cdot s}$
k_m	mass transfer coefficient from a solid to fluid (concentration driving force), m/s
k_m°	mass transfer coefficient under low-mass-flux conditions, m/s
$\tilde{K}$	diffusivity matrix in Section 21.4
$\tilde{K}$	matrix of multicomponent diffusion coefficient in Section 21.4
K_G	overall mass transfer coefficient from a bulk gas to a bulk liquid (gas phase partial pressure driving force), $\mathrm{mol/m^2 \cdot s \cdot Pa}$
K_L	overall mass transfer coefficient from a bulk gas to a bulk liquid (liquid concentration driving force), m/s
L	length of the plate or tube or catalyst slab, m
M	local Mach number, v/c
m	mass of a molecule in Section 1.8.1
$\dot{m}$	mass flow rate, kg/s
$m_\mathrm{A,tot}$	total mass of A in an unit or control volue, kg
$\dot{m}_\mathrm{Ai}$	mass flow rate of A entering a unit, kg/s
$\dot{m}_\mathrm{Ae}$	mass flow rate of A exiting a unit, kg/s
$\dot{m}_\mathrm{AW,tot}$	total mass of A transferred to walls from an unit or procss, kg/s
$\bar{M}$	average molecular weight of a mixture, kg/g.mol
$\boldsymbol{M}$	momentum flow rate vector, N
M_A	molecular weight of species indicated in the subscript, kg/g.mol
M_w	molecular weight in general
$\mathcal{M}$	total moles present in a control volume, g-mol
$\dot{\mathcal{M}}$	moles per second entering/leaving the unit, i = inlet, e = exit
$\mathcal{M}_\mathrm{A}$	moles of A in the system or control volume
Nu	Nusselt number, usually defined as hd_t/k or hx/k
N_Av	Avogadro number = 6.23×10^{23} molecules/g-mol
n	number density of molecules in Section 1.8.1

$\boldsymbol{n}$	normal vector outward from a control surface
$\boldsymbol{n}_{\mathrm{A}}$	mass flux vector of species A, stationary frame, kg-A/m$^2 \cdot$ s
$n_{\mathrm{A}x}$	component of mass flux vector of A in the x-direction, kg-A/m$^2 \cdot$ s
N_{tu}	number of transfer of unit parameter
p	fluid pressure; equal to the average normal stress, Pa
p_{vap}	vapor pressure of a species, Pa
P	thermodynamic pressure used in equation of state, Pa
p	the concentration gradient or temperature gradient in the p-substitution method
p^*	dimensionless pressure, $p/\rho v_{\mathrm{ref}}^2$
p^{**}	dimensionless pressure, $p^* Re$
P	power input for agitated vessels, W
P_{c}	critical pressure of a species, Pa
$\mathcal{P}$	modified pressure defined as $p + \rho g h$
p	temperature gradient in Example 8.3 and concentration gradient in Section 10.4.6
Pe	Péclet number, $d_{\mathrm{t}}\langle v\rangle/\alpha$
Pe_R	Péclet number based on pipe radius, $d_{\mathrm{t}}\langle v\rangle/D$
Pe^*	dispersion Péclet number in Section 12.5, $\langle v\rangle L/D_E$
Po	power number as $p/(\rho N_i^3 d_i^5)$ in Section 14.15
Pr	Prandtl number, $c_p\mu/k$
q	dimensionless stoichiometric ratio defined by Eq. (10.44) in Section 10.5
Q	volumetric flow rate in a pipe, m^3/s
$(\dot{Q})_{\mathrm{v}}$	internal heat generation rate, W/m^3
$\boldsymbol{q}$	heat flux vector (molecular) W/m^2
$\boldsymbol{q}^{(\mathrm{m})}$	heat flux vector (molecular), same as $\boldsymbol{q}$, W/m^2
q_{s}	heat flux from a surface or wall to a flowing fluid
$\boldsymbol{q}^{(\mathrm{t})}$	heat flux vector due to turbulence, W/m^2
q_x	component of the heat flux vector in the x-direction
q_y	component of the heat flux vector in the y-direction
q_{w}	heat flux to the wall of a pipe from a fluid
$\dot{Q}$	rate at which heat is added to the control volume; unit volume basis, W/m^3
$\dot{Q}_{\mathrm{V}}$	rate at which heat is generated within control volume per unit volume, W/m^3
q_z	component of the heat flux vector in the z-direction
r	radial coordinate in cylindrical and spherical system
R	radius of cylinder or catalyst particle
r_{A}	local rate of mass production of A by reaction per unit volume, mass units, kg/m$^3 \cdot$ s
R_{A}	local rate of mole production of A by reaction per unit volume, mole units, mol/m$^3 \cdot$ s
R^*	gas constant defined as $R_{\mathrm{G}}/M_{\mathrm{w}}$
R_{A}	rate of production of a species A by reaction
Re	Reynolds number, $L_{\mathrm{ref}} v_{\mathrm{ref}}\rho/\mu$
R_{G}	gas constant, 8.314 Pa m^3/mol $\cdot$ K
$\hat{s}$	entropy energy per unit mass of fluid, J/K $\cdot$ kg
$\boldsymbol{s}$	entropy flux vector, W/K $\cdot$ m^2
s	shape parameter for conduction or diffusion, 1 for slab, 2 for long cylinder, 3 for sphere

Sc	Schmidt number, $\mu/(\rho D)$
Sh	Sherwood number, $k_m x/D$
St	Stanton number, $Nu/(RePr)$ or $Sh/(ReSc)$
t	time variable
t_E	exposure time for a gas–liquid interface
T	local temperature in the medium
T_a	temperature of the surroundings
$\langle T \rangle$	cross-sectionally averaged temperature
T_b	cup mixing (flow-averaged) temperature
T_c	critical temperature of a species
T_f	temperature of the surrounding fluid in contact with a solid
T_i	temperature of a gas–liquid interface
T_w	temperature of a wall or tube
T_∞	temperature of the approaching fluid
$\hat{u}$	internal energy unit mass of fluid, J/kg
$\hat{U}$	internal energy per unit mole of fluid, J/mol
U	overall heat transfer coefficient from hot fluid to cold fluid, $W/m^2 \cdot K$
$\hat{v}$	specific volume, $1/\rho, m^3/kg$.
$\boldsymbol{v}$	velocity vector; also mass-fraction-averaged velocity in a multicomponent mixture, m/s
$\boldsymbol{v}'$	fluctuating velocity vector in turbulent flow
$\bar{\boldsymbol{v}}$	time-averaged velocity vector in turbulent flow
$\boldsymbol{v}^*$	mole-fraction-averaged velocity in a multicomponent mixture, m/s
v_x	x-component of the velocity; v_y and v_z defined similarly
v_z	axial (z-) component of velocity in cylindrical coordinates
v_θ	velocity component in the tangential (θ) direction
$\boldsymbol{v}_A$	velocity of species A in a multicomponent mixture, stationary frame, m/s
v_e	velocity component in the fluid outside the boundary layer, m/s
V	total control volume
$\hat{V}$	molar volume, m^3/mol
V	speed of a moving solid in shear flow in flow direction, m/s
v_b	molecular volume at boiling point of solvent
V_f	friction velocity defined as $\sqrt{\tau_f/\rho}$ used in turbulent flow, m/s
$\dot{W}$	rate at which work is done on the control volume, W/m^3
$\dot{W}_s$	rate at which work is done by a moving part on the control volume, W/m^3
$\dot{W}_f$	rate at which heat energy is produced by friction, W/m^3
x	distance variable in the x-direction, y and x defined similarly.
x_i	mole fraction of species indicated by the subscript (usually in the liquid phase)
y	distance variable in the y-direction
y_i	mole fraction of species indicated by the subscript (usually in the gas phase)
y^+	dimensionless length used in turbulence analysis near a wall
y_B(l.m)	log-mean mole fraction of the non-diffusing component
z	axial distance variable in cylindrical coordinates
z^*	dimensionless axial distance variable in cylindrical coordinates, z/R
z_i	number of charges on an ionic species
Z	frequency of molecular collisions in Section 1.8.1

Greek letters and other symbols

α	thermal diffusivity of a solid, m^2/s
α	absorptivity of a surface in radiation
α_t	turbulent heat diffusivity, m^2/s
ϵ_H	turbulent heat diffusivity, m^2/s
β	bulk modulus of elasticity, N/m^2
$\boldsymbol{\beta}$	angular velocity vector
γ	dimensionless activation energy in Section 13.7 and Example 16.1
γ	ratio of specific-heat values, c_p/c_v
∇	gradient operator
∇_*	gradient operator in dimensionless coordinates
∇^2	Laplacian operator defined by Eqs. (1.56)–(1.58) for scalars
∇^2	Laplacian operator defined in Sections 5.3.1 and 5.3.2 for vectors
∇^4	biharmonic operator defined by Eq. (5.31)
Δ	difference operator, out – in,
Δ	ratio of boundary-layer thickness, heat/mass to momentum
ΔH	heat of reaction, J/mol
ΔH_v	heat of vaporization, mole basis, J/mol
$\Delta\pi$	osmotic pressure diffference in Section 10.7, Pa
δ	parameter in Frank-Kamenetskii model
δ	thickness of momentum boundary layer in general
δ_f	film thickness for mass transfer, abbreviated as δ in Chapter 10
δ_m	thickness of mass-transfer boundary layer
δ_t	thickness of thermal boundary layer
ϵ	dielectric permittivity of a medium in Chapter 22
ϵ	emissivity of the medium
ϵ	energy dissipation rate in turbulent flow analysis
ϵ	a parameter in Lennard-Jones model in Chapter 1
η	effectiveness factor of a porous catalyst in Chapter 10
ζ	dimensionless axial distance, z^*/Pe
η	similarity variable defined by Eq. (11.30) in Chapter 11 for heat conduction
η	similarity variable defined in Chapter 12.2 for convective heat transfer
κ	circulation (line integration of tangential velocity) in Section 15.4.3
κ	conductivity of an ionized liquid in Section 22.1.3
κ	ratio of radius values, R_c/R_o, in Chapter 6
κ	Boltzmann constant, also denoted as k_B
λ	Debye length in Sections 22.5 and 22.6
λ	mean free path in Section 1.8.1
Λ	consistency index parameter for power law fluids
θ	angular direction in polar coordinates

θ	latitude direction in spherical coordinates
θ	dimensionless temperature in heat transfer examples
μ	coefficient of viscosity, Pa · s
μ_i	mobility of charged species i in Chapter 22
μ_{w}	chemical potential of water in Section 10.7
ν	coefficient of kinematic viscosity, μ/ρ, m^2/s
ν_{t}	turbulent kinematic viscosity, μ_t/ρ, m^2/s
ν_{T}^{+}	dimensionless total (molecular + turbulent) kinematic viscosity
ρ	density of the medium or the fluid, $\mathrm{kg/m}^3$
ρ_c	electric charge density in Chapter 22
ρ_{A}	density of A in a multicomponent mixture, $\mathrm{kg/m}^3$
σ	surface tension, N/m
σ_{xx}	total stress (viscous and pressure) in the x-direction
σ	Staverman constant in Section 10.7.1
σ_{yx}	same as τ_{yx} since shear stress has no pressure contribution
σ	Stefan–Boltzmann constant
τ	dimensionless time in Chapter 11, t/t_{ref}
$\lvert\tau_{\mathrm{w}}\rvert$	stress exerted by the wall opposite to the flow direction in response to $-\tau_{\mathrm{w}}$
τ_{w}	stress exerted by the solid on the fluid in pipe flow, $\mu\, dv_z/dr$ at $r = R$, usually negative in the flow direction
τ_{f}	stress exerted by the fluid on the solid, $\mu\, dv_x/dy$ at $y = 0$
τ_0	yield stress for Bingham flow
τ_{xx}	viscous stress in the x-direction on a plane whose unit normal is in the x-direction
τ_{yx}	viscous stress in the x-direction on a plane whose unit normal is in the y-direction; other components are defined similarly
ϕ	blowing parameter in Section 13.6.1
ϕ	electric potential in Chapter 22
ϕ	longitude in the spherical coordinate system
ϕ	velocity potential defined by Eq. (3.49) in Section 3.10
ϕ	Thiele parameter for a first-order reaction
ϕ_0	Thiele parameter for a zeroth-order reaction defined by Eq. (10.34)
Φ_{v}	rate of heat production by viscosity per unit volume, Eq. (7.12), $\mathrm{W/m}^3$
ψ	streamfunction defined by Eq. (3.39) or Eq. (3.40)
ω	frequency of oscillation in periodic flow, s^{-1}
ω^*	dimensionless frequency of oscillation in periodic flow, ωt_{ref}
ω	vorticity for a plane flow defined as ω_z
$\boldsymbol{\omega}$	vorticity vector for a general 3D flow, $\nabla \times \boldsymbol{V}$
ω	specific energy-dissipation rate in turbulent flow
ω_{A}	mass fraction of species indicated by the subscript, kg-A/kg-total
ξ	dimensionless radial position, r/R or x/L
Ω	angular velocity, rotational speed
Ω_i	speed of rotation or agitation in Section 14.1.5 and 14.1.6, r.p.s.

Common subscripts

b	bulk conditions
g, G	gas-phase properties
e	exit values (Chapter 2)
i	inlet values (Chapter 2)
i	interface conditions (Chapters 9 and 10)
l, L	liquid-phase properties
s	conditions at a surface of a solid or catalyst