1

Radiometry, optics, statistics

1.1 Basics of radiometry

This book is about measuring photons. Therefore, we begin with a short description of how their production is treated in radiation physics. To compute the emission of an object, consider a surface element of it, dA, projected onto a plane perpendicular to the direction of observation (Figure 1.1). The projected area is $dA \cos\theta$, where θ is the angle between the direction of observation and the outward normal to dA. The specific intensity, L_ν, is the power (watts) leaving a unit projected area of the surface of the source (in m^2) into a unit solid angle (in ster-radians) and unit frequency interval (in Hz); the ν subscript lets us know this is a frequency-based quantity. It has SI units of $W\,m^{-2}\,Hz^{-1}\,ster^{-1}$. In physics, the same quantity is usually called the spectral radiance (in frequency units); we will use the common astronomical terminology here and refer the reader to Rieke (2003) for a table of terminology conversions. Similarly, we can define a specific intensity per wavelength interval, L_λ, where λ indicates a wavelength-based quantity. The intensity, L, is the specific intensity integrated over all frequencies or wavelengths, and the radiant exitance, M, is the integral of the intensity over solid angle. It is a measure of the total power emitted per unit surface area in units of $W\,m^{-2}$. For a blackbody,

$$M = \sigma T^4 = 5.669 \times 10^{-8}\,W\,m^{-2}\,K^{-4} T^4 \quad (1.1)$$

where σ is the Stefan–Boltzmann constant (physical and other constants are given in Appendix A) and T is the temperature in kelvin. The integral of the radiant exitance over the source gives the total output, that is, the *luminosity* of the source. The irradiance, E, is the power received from the source by a unit surface element a large distance away. It also has units of $W\,m^{-2}$, but is easily distinguished from radiant exitance if we think of a source so distant that it is entirely within the field of view of the detection system. In this case,

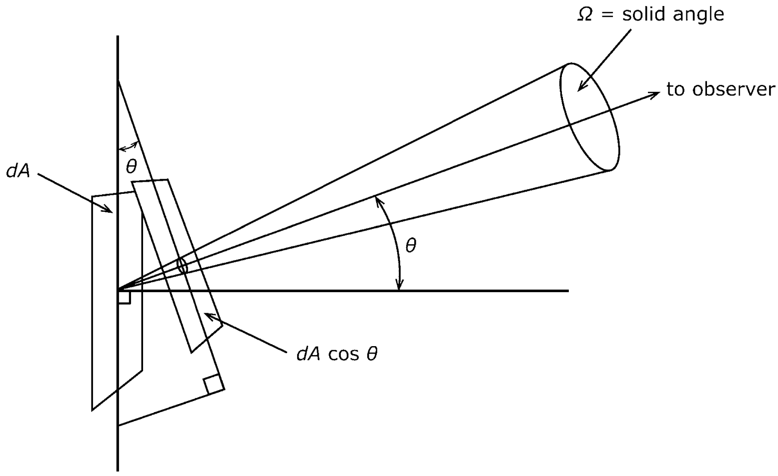

Figure 1.1. Emission geometry from a radiation source.

we might find it impossible to determine M, but E still has a straightforward meaning. The spectral flux density, that is, the irradiance per frequency or wavelength interval per unit of area, E_ν or E_λ, is very commonly used in astronomy; it has units of $\mathrm{W\,m^{-2}\,Hz^{-1}}$ (frequency units), or $\mathrm{W\,m^{-3}}$ (wavelength units). In fact, astronomers have their own unit, the Jansky (Jy), which is $10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$.

The energy of a photon is

$$\begin{aligned}\varepsilon_{\mathrm{ph}} = h\nu = \frac{hc}{\lambda} &= 6.626 \times 10^{-34}\,\mathrm{Js}\,\nu \\ &= 1.986 \times 10^{-25}\,\frac{\mathrm{Jm}}{\lambda}\end{aligned} \tag{1.2}$$

where $c = 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$ is the speed of light and $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$ is Planck's constant. Conversion to photons/second can be achieved by dividing the various measures of power output by equation (1.2).

To make calculations simple, we generally deal only with *Lambertian* sources. For them, the intensity is constant regardless of the direction from which the source is viewed. In the astronomical context, a Lambertian source has no limb darkening (or brightening). In fact, we usually deal only with the special case, a black or grey body (a grey body has the same spectrum as a blackbody but with an emission efficiency, or emissivity ε, less than 1). We then have for the specific intensities in frequency and wavelength units:

$$L_\nu = \frac{\varepsilon\,[2h\nu^3 n^2/c^2]}{e^{h\nu/kT} - 1} = \frac{1.474 \times 10^{-50}\,\varepsilon\, n^2 \nu^3}{e^{4.800\times10^{-11}\nu/T} - 1}\,\mathrm{W\,m^{-2}\,ster^{-1}\,Hz^{-1}}$$

$$L_\lambda = \frac{\varepsilon[2hc^2]}{\lambda^5 n^2 (e^{\frac{hc}{\lambda kT}} - 1)} = \frac{1.191 \times 10^{-16} \varepsilon}{\lambda^5 n^2 (e^{0.01438/\lambda T} - 1)} \, \mathrm{W\,m^{-3}\,ster^{-1}} \tag{1.3}$$

where $k = 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ is the Boltzmann constant and n is the refractive index (virtually always $n \cong 1$ for astronomical sources). We also have

$$\begin{aligned} E_\nu &= \frac{AL_\nu}{4\,r^2} \\ E_\lambda &= \frac{AL_\lambda}{4\,r^2} \end{aligned} \tag{1.4}$$

where r is the distance to the source and A is its surface area. It is convenient to convert from frequency to wavelength units by means of

$$E_\nu = \frac{\lambda^2}{c} E_\lambda \tag{1.5}$$

which can be derived by differentiating

$$\lambda = \frac{c}{\nu} \tag{1.6}$$

to obtain the relation between $d\lambda$ and $d\nu$. An easy way to remember the conversion is that multiplying the flux density in $\mathrm{W\,m^{-2}\,Hz^{-1}}$ by c/λ^2 with $c = 2.998 \times 10^{10}$ cm/s and λ in μm gives the flux density in $\mathrm{W\,cm^{-2}\,\mu m^{-1}}$.

1.2 Image formation and the wave nature of light

OK, some source has launched a stream of photons in our direction. How are we going to catch it and unlock its secrets? A photon is described in terms that are illustrated in Figure 1.2 (after Barbastathis 2004). In the figure, we imagine that time has been frozen, but the photon has been moving at the speed of light in the direction of the arrow. We often discuss the photon in terms of wavefronts, lines marking the surfaces of constant phase and hence separated by one wavelength.

As electromagnetic radiation, a photon has both electric and magnetic components, oscillating in phase perpendicular to each other and perpendicular to the direction of energy propagation. The amplitude of the electric field, its wavelength and phase, and the direction it is moving characterize the photon. The behavior of the electric field can be expressed as

$$E = E_0 \cos(\omega t + \varphi) \tag{1.7}$$

where E_0 is the amplitude, ω is the angular frequency, and φ is the phase.

Another formalism is to express the phase in complex notation:

$$E(t) = E_0 e^{-j\omega t\, t} = E_0 \cos \omega t - jE_0 \sin \omega t \tag{1.8}$$

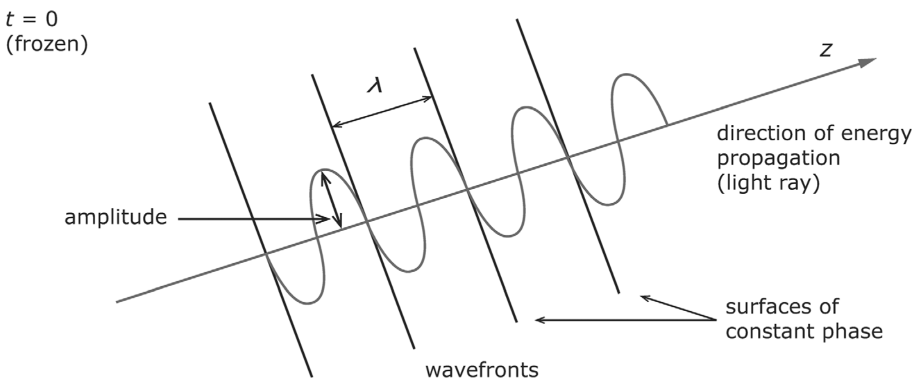

Figure 1.2. Terms describing the propagation of a photon, or a light ray. Redrawn with modifications from Barbastathis (2004).

where j is the imaginary square root of –1. In this case, the quantity is envisioned as a vector on a two-dimensional diagram with the real component along the usual x-axis and the imaginary one along the usual y-axis. The angle of this vector represents the phase. The flux (energy flow per unit area per second) is given by the Poynting vector, or in simplified form:

$$S = \varepsilon_0 \, c \, E^2 = \varepsilon_0 \, c \, E_0^2 \cos^2(\omega t + \varphi) \tag{1.9}$$

where ε_0 is the permittivity of free space, 8.854×10^{-12} F m^{-1}. The intensity of the light is the average of S over time.

For efficient detection of this light, we use optics to squeeze it into the smallest possible space. The definition of "the smallest possible space" is a bit complex. We want to map the range of directions toward the target onto a physical surface where we put an instrument or a detector. This goal implies that we form an image, and since light is a wave, images are formed as a product of interference. That is, the collector must bend the rays of light from a distant object to bring them to a focus at the image plane (see Figure 1.3). There, they must interfere constructively, requiring that they arrive at the image in phase, having traversed paths of identical length from the object. This requirement is summarized in Fermat's Principle, which is expressed in a number of ways including:

"The optical path from a point on the object through the optical system to the corresponding point on the image must be the same length for all neighboring rays."

The optical path is defined as the integral of the physical path times the index of refraction of the material that path traverses.

$$s = \int n(x) \, dx \tag{1.10}$$

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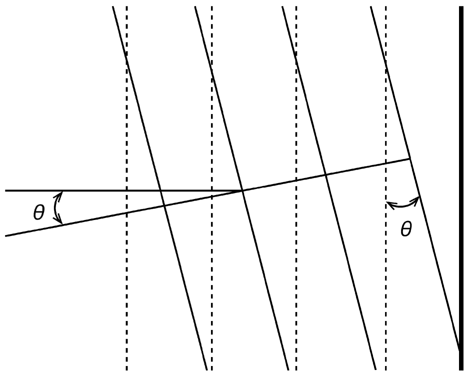

Figure 1.3. Approach of light wavefronts to an image plane.

What tolerance is there on the lengths of the optical paths? As shown in Figure 1.3, if the light waves impinge on the focal plane over too wide a range of angle, then the range of phases across the area where we detect the light can be large enough to result in a significant level of destructive interference. From the figure, this condition can be expressed as

$$l\tan(\theta_{\max}) = \frac{\lambda}{2} \approx l\,\theta_{\max} \tag{1.11}$$

Here, l is some characteristic distance over which we collect the light – say the size of a detector at the focus of a telescope, and $\theta_{\max}$ is the largest angle over which generally constructive interference occurs. Equation (1.11) seems to imply that the size of the detector enters into the fundamental performance of a telescope. However, there is another requirement on the optical system. The beam of light entering the telescope is characterized by an area at its base, A, normally the area of the telescope primary mirror, and a solid angle, $\Omega \approx \pi\theta^2$ where θ is the half-angle of the convergence of the beam onto the telescope (i.e., the angular diameter of the beam is 2θ). The product of A and Ω (and the refractive index n) can never be reduced as a beam passes through an optical system, by the laws of thermodynamics. This product

$$n\ A\ \Omega = C \tag{1.12}$$

is called the etendue (French for extent or space). Since astronomical optics generally operate in air or vacuum with $n \cong 1$, we will drop the refractive index term in future discussions of the etendue. Assuming a round telescope and detector and a circular beam (for convenience – with more mathematics we could express the result generally), we find that an incoming beam with

$$\Theta = \frac{\lambda}{D} \tag{1.13}$$

(where Θ is the full width at half maximum (FWHM) of the beam in radians, λ is the wavelength of the light, and D is the diameter of the telescope aperture) is just at the limit expressed in equations (1.11) and (1.12) regardless of detector size. Equation (1.13) should be familiar as the diffraction limit of the telescope (although perhaps not as familiar as it should be because often the FWHM of the diffraction-limited image is erroneously given as $1.22\lambda/D$). The destructive interference in Figure 1.3 is the fundamental image-forming mechanism. The smaller the telescope the broader the range of angles on the sky before destructive interference at the focal plane starts to reduce the signal, corresponding to an inverse relation between the resolution limit of a telescope and its aperture (assuming it is diffraction limited).

1.3 Power received by an optical system

1.3.1 Basic geometry

An optical system will receive a portion of the source power that is determined by a number of geometric factors (plus efficiencies) – see Figure 1.4. If you refer to this diagram and follow it carefully, you will (almost) never get confused on how to do radiometry calculations in astronomy. As Figure 1.4 shows, the system accepts radiation only from a limited range of directions, called its field of view (FOV). The area of the source that is effective in

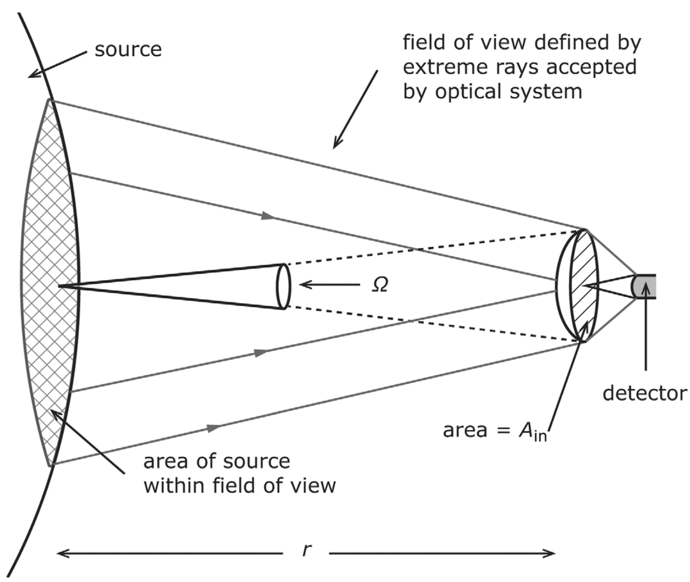

Figure 1.4. Geometry for detected signals.

producing a signal is determined by the FOV and the distance between the optical system and the source (or by the size of the source if it all lies within the FOV). This area emits with some angular dependence (e.g., for a Lambertian source, the dependence just compensates for the foreshortening and off-axis emission direction). The range of emission directions that is intercepted by the system is determined by the solid angle, Ω, that the entrance aperture of the system subtends as viewed from the source:

$$\Omega = \frac{A_{\text{in}}}{r^2} \quad (1.14),$$

where A_{in} is the area of the entrance aperture of the system and r is the distance from it to the source (we assume the entrance aperture is perpendicular to the line from its center to the center of the source). If the aperture is circular,

$$\Omega = 4\,\pi \sin^2\left(\frac{\theta}{2}\right) \quad (1.15)$$

where θ is the half-angle of the circular cone with its vertex on a point on the surface of the source and its base across the entrance aperture of the optical system.

If none of the emitted power is lost (by absorption or scattering) on the way to the optical system, then the power it receives is the intensity the source emits in its direction multiplied by (1) the source area within the system FOV times, (2) the solid angle subtended by the optical system as viewed from the source. We often get to make this situation even simpler, because for us many sources fall entirely within the FOV of the system. In this case, the spectral irradiance (that is, the flux density (equations (1.4)) is a convenient unit of measure.

1.3.2 How much gets to the detector?

Once the signal reaches the optical system, a number of things happen to it. Before reaching the detector, part of the signal is lost due to diffraction, optical imperfections, absorption, and scattering within the instrument. Once it reaches the detector, only a fraction of the signal is absorbed and utilized to produce the signal; this fraction is called the *quantum efficiency*. All of these efficiencies (i.e., quantum efficiency and transmittances < 1) must be multiplied together to get the net efficiency of conversion into the output signal.

For groundbased telescopes, there is another important component to the transmission term, the absorption by the atmosphere of the earth. The overall absorption is shown in Figure 1.5 – the "windows" through which

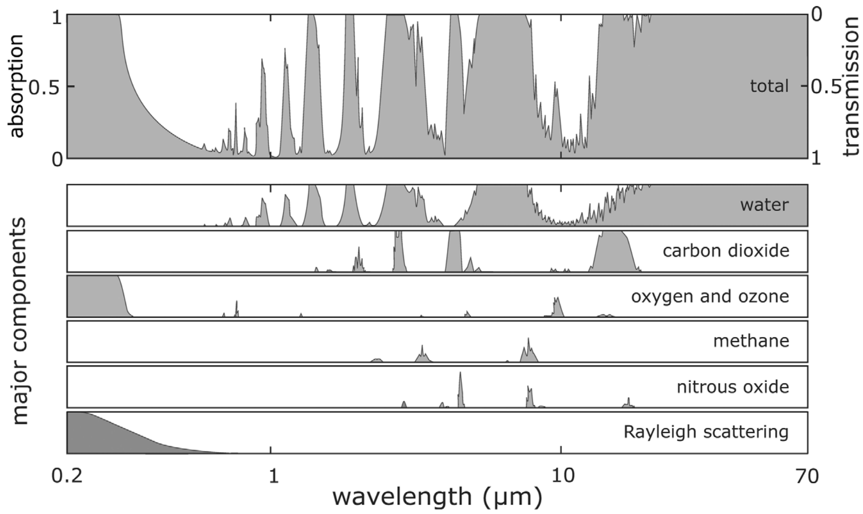

Figure 1.5. Atmospheric absorption and scattering. Redrawn with modifications from R. Rohde (n.d.).

astronomers can peer are obtained by turning the figure at the top upside down. The atmosphere is opaque at all wavelengths shorter than 0.3 microns and between 40 and 300 μm. Further details between 1 and 25 μm are given in Chapter 5. Figure 1.6 shows the transmission in the submillimeter and millimeter-wave. For still longer wavelengths (i.e., beyond 1 cm) the atmosphere is transparent until about 100 m, where absorption by the layer of free electrons in the ionosphere becomes strong.

Many of the long-lived atmospheric constituents are well-mixed; that is, the composition of the air is not a strong function of altitude. The pressure can be characterized by an exponential with a scale height (the distance over which the pressure is reduced by a factor of $1/e$) of 8 km. However, water is in equilibrium with processes on the ground and has an exponential scale height of about 2 km. In addition, the amount of water in the air is reduced rapidly with decreasing temperature. Typical high-quality observing sites have overhead water vapor levels equivalent to about 2 mm; Figure 1.6 shows the improvement in the submillimeter for a reduction by a factor of 4 in water vapor. Therefore, there is a premium in placing telescopes for water-vapor-sensitive wavelengths on high mountains, and even in the Antarctic.

The atmosphere can also contribute unwanted foreground emission that raises the detection limit for faint astronomical objects. There are many components (Leinert *et al*. 1998), but most of them play a relatively minor role. Two major ones are scattered moonlight, which can be significant from 0.3 to 1 μm, and the airglow lines (mostly from OH with a contribution

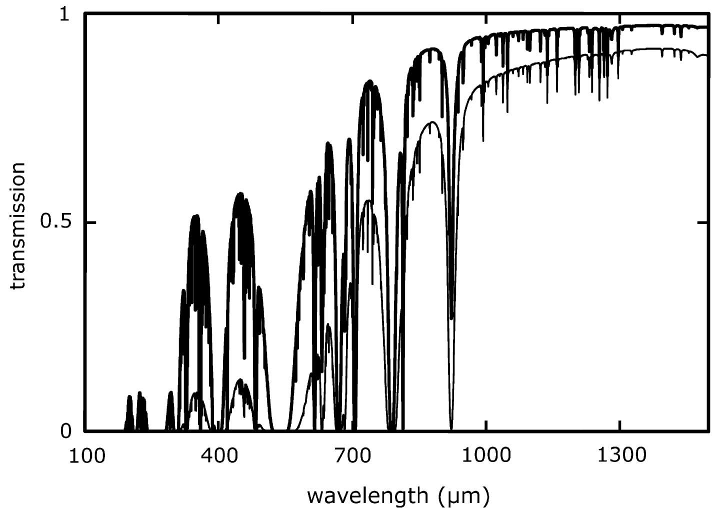

Figure 1.6. Atmospheric transmission in the submillimeter, for precipitable water vapor levels of 0.5 (heavy line) and 2 mm (light line). Prepared using APEX transmission calculator.

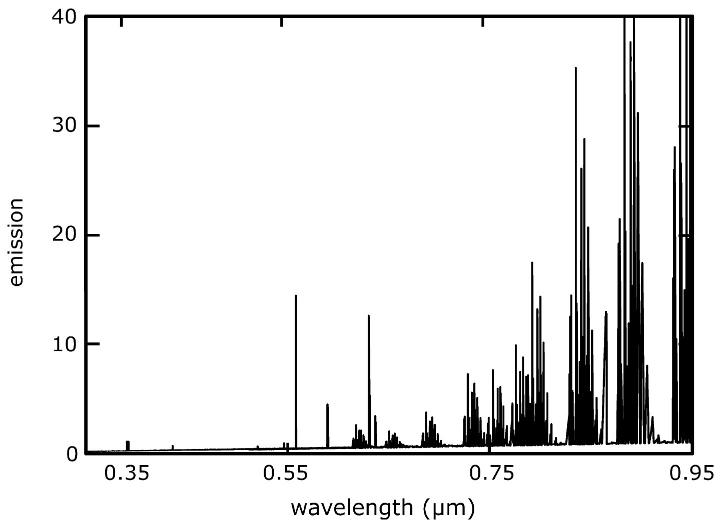

Figure 1.7. Sky brightness in the visible and near-infrared. Based on Rousselot *et al.* (2000).

from O_2) that start in the 0.7–1 μm range (Figure 1.7) and become totally dominant in the near-infrared bands (Figure 1.8; Rousselot *et al.* 2000). In the visible (0.56 μm), a typical dark sky provides about 200 photons s^{-1} m^{-2} $arcsec^{-2}$ μm^{-1} but with the full moon above the horizon this emission is increased by about a factor of 5 (the increase is much greater in the blue

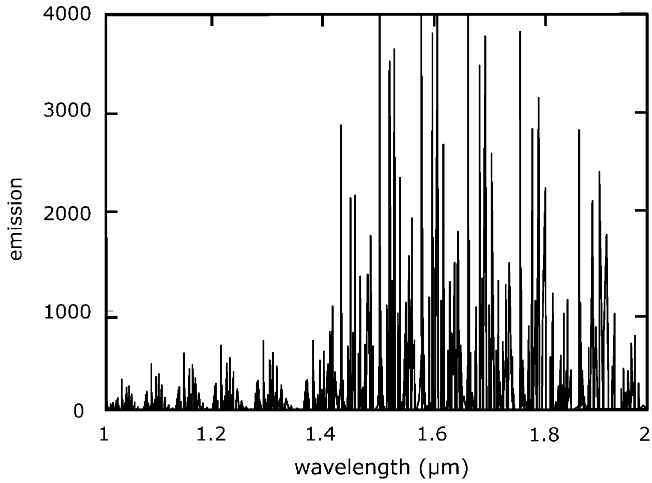

Figure 1.8. Sky brightness from 1 to 2 μm. The vertical scale has been made 100 times larger than in Figure 1.7. Based on Rousselot *et al.* (2000).

and ultraviolet). Where they are strongest, between 1.5 and 1.8 μm, the airglow lines typically provide an integrated flux of about 3×10^4 photons s^{-1} m^{-2} $arcsec^{-2}$ μm^{-1}, with rapid variations (hourly timescales) by factors of 2 around this value (e.g., Maihara *et al.* 1993). Another major source of interference occurs in the mid-infrared, beyond 2 μm. Here, the thermal emission of the atmosphere and telescope dominates all other foreground sources. In the clearest regions of atmospheric transmission, telescopes can be built with effective emissivities approaching 5%, that is, the foreground is about 5% of that expected from a blackbody cavity at the temperature of the telescope. Assuming this optimistic value, the integrated photon flux at 10 μm is about 5×10^8 photons s^{-1} m^{-2} $arcsec^{-2}$ μm^{-1}. This emission dominates over the zodiacal and galactic radiation at these wavelengths (which is the fundamental foreground for our vicinity in space) by a factor of more than a million.

1.3.3 Radiometry example

We now illustrate some of the points just discussed. A 1000 K spherical blackbody source of radius 1 m is viewed in air by a detector system from a distance of 1000 m. The entrance aperture of the system has a radius of 5 cm; the optical system is 50% efficient and has a field of view half-angle of 0.1°. The detector operates at a wavelength of 1 μm and the light passes through a filter with a spectral bandpass of 0.1 μm. Compute the specific intensities in both frequency and wavelength units. Calculate the corresponding flux densities at the system entrance aperture and the power received by the detector.