1 Introduction to Theory of Gearing, Design, and Generation of Noncircular Gears

1.1 Historical Comments

Designers have tried for many years to apply noncircular gears in automatic machines and instruments. The obstacle was the lack of effective methods of generation of noncircular gears similar to those applied for the generation of circular gears. However, researchers have continued the investigation of application of noncircular gears – see the earlier works by Burmester (Burmester, 1888), Golber (Golber, 1939), Temperley (Temperley, 1948), or Boyd (Boyd, 1940) – and manufacturers have intensified their efforts for improvement of the generation of noncircular gears (Fellows, 1924; Bopp & Reuther G.m.b.H., 1938).

Due to the lack of exact methods of generation of noncircular gears, the efforts were first directed to the development of methods based on the meshing of generating tools with master gears. Figure 1.1.1 shows the Fellows' approach where the noncircular master gear 1 is in mesh with a master rack (Fellows, 1924). The rack cutter and gear being generated are denoted by 3 and 4, respectively.

Bopp and Reuther's approach (Fig. 1.1.2) is based on the simulation of meshing of a noncircular master worm gear c with a worm f that is identical to the hob d; a is the spur noncircular gear being generated; the cam b and the follower e form the cam mechanism designated for simulation of the required variable distance between c and f (Bopp & Reuther G.m.b.H., 1938). Weight g maintains the continuous contact between the cam and the follower. However, both approaches were difficult to apply due to the necessity of manufacturing noncircular master gears, which was expensive and time-consuming.

The breakthrough of generation of noncircular gears happened in years 1949–1951, wherein enveloping methods of generation of noncircular gears were developed based on generation by a rack cutter, hob, or a shaper (Litvin *et al.*, 1949 to 1951). Such methods were based on obtaining the gear tooth surface as the envelope to the family of tool surfaces and are based on the following ideas:

(a) The noncircular gears are generated by the same tools (rack cutters, hobs, and shapers) that are used for the manufacture of circular gears.

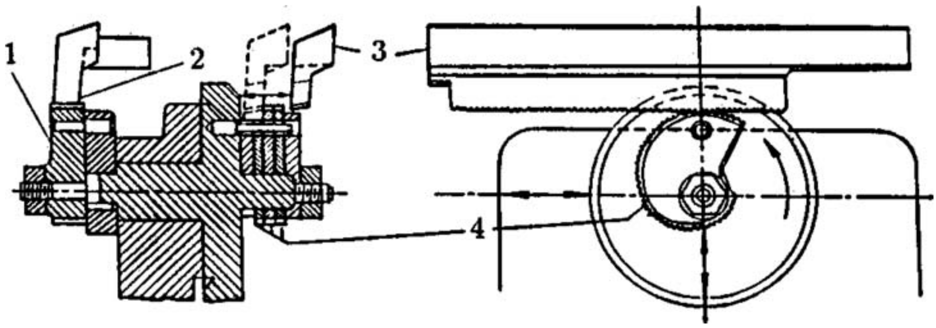

Figure 1.1.1. Generation of a noncircular gear by application of (i) a master rack 2; (ii) a master noncircular gear 1; (iii) a cutting rack cutter 3. The noncircular gear being generated is 4.

(b) Conjugated tooth shapes for noncircular gears are provided due to the imaginary rolling of the tool centrode over the given gear centrode.
(c) The imaginary rolling of the tool centrode over the centrode of the gear being generated is accomplished by proper relations between the motions of the tool and the gear in the process of cutting.

We illustrate the developed approaches in Fig. 1.1.3, which shows that mating noncircular centrodes 1 and 2 are in mesh with a conventional rack cutter 3. The centrode of the rack cutter is a straight line $\overline{t-t}$ that is a common tangent to centrodes 1 and 2 and rolls over 1 and 2. Rolling of centrode 3 of the rack cutter is

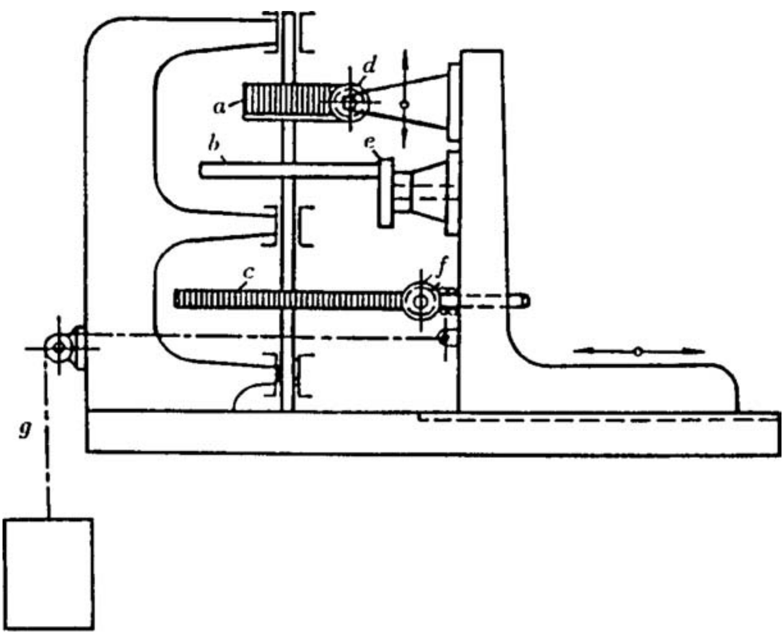

Figure 1.1.2. Generation of a noncircular gear by applying (i) a master worm gear c being in mesh with worm f; (ii) a cam b and follower e; (iii) a is the gear being generated; (iv) d is the hob.

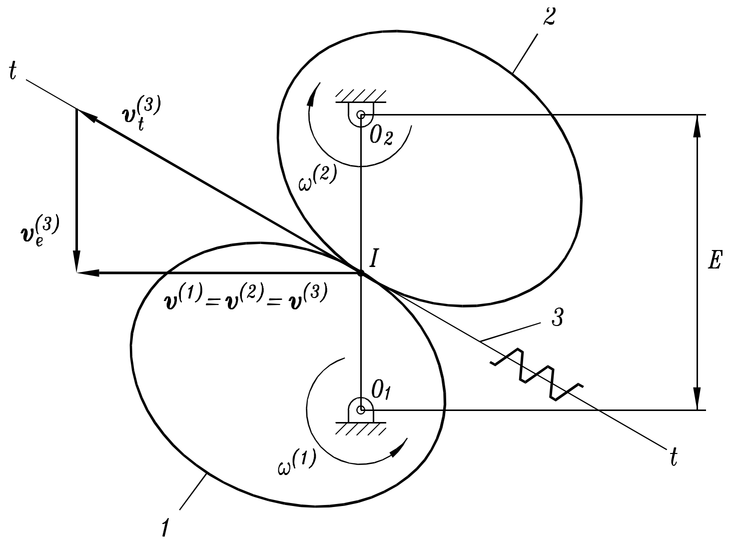

Figure 1.1.3. Illustration of generation of noncircular gears 1 and 2 by rack cutter 3.

achieved wherein the rack cutter translates along tangent $\overline{t-t}$ and is rotated about the instantaneous center of rotation I (Fig. 1.1.3). Tooth surfaces of gear 1, gear 2, and rack cutter 3 are in mesh simultaneously, and gear 1 and 2 are provided with conjugated surfaces.

Drawings of Fig. 1.1.4 show the related motions of a rack cutter being in mesh with one of the noncircular gears of the pair of noncircular gears shown in Fig. 1.1.3. Figure 1.1.4(a) shows that the rack cutter translates along $\overline{t-t}$, which is the common tangent to the rack cutter centrode 2 and centrode 1 of the noncircular gear. Centrode 2 is a straight line. Centrode 1 applied for generation is the same as that applied in the meshing of two mating noncircular gears, as shown in Fig. 1.1.3.

The noncircular gear 1 being in mesh with rack cutter 2 performs two related motions during the process of generation (Fig. 1.1.4(a)): (a) rotation about O_1 with angular velocity $\boldsymbol{\omega}^{(1)}$, and (b) translational motion with linear velocity $\mathbf{v}_{tr}^{(1)}$ in a direction that is perpendicular to $\overline{t-t}$. Rolling of the rack cutter 2 about centrode 1 is provided by observation of the vector equation

$$\mathbf{v}^{(2)} = \mathbf{v}_{rot}^{(1)} + \mathbf{v}_{tr}^{(1)} \tag{1.1.1}$$

We may consider that the translation of the rack cutter is performed with a constant velocity $\mathbf{v}^{(2)}$, and the motions of the noncircular gear are provided by observation of the nonlinear function $\boldsymbol{\omega}^{(1)}(\mathbf{v}^{(2)})$ and $\mathbf{v}_{tr}^{(1)}(\mathbf{v}^{(2)})$.

Three coordinate systems are applied for generation: (i) movable ones, S_2 and S_1, rigidly connected to the rack cutter 2 and noncircular gear 1 (Fig. 1.1.4(b)), and (ii) fixed coordinate system S_f, in which we consider the motions of S_2 and S_1 (Fig. 1.1.4(b)). Symbols $x_f^{(O_2)}$ and $y_f^{(O_1)}$ denote the displacements of the rack cutter and noncircular gear, respectively. Angle ϕ_1 denotes the rotation of the gear.

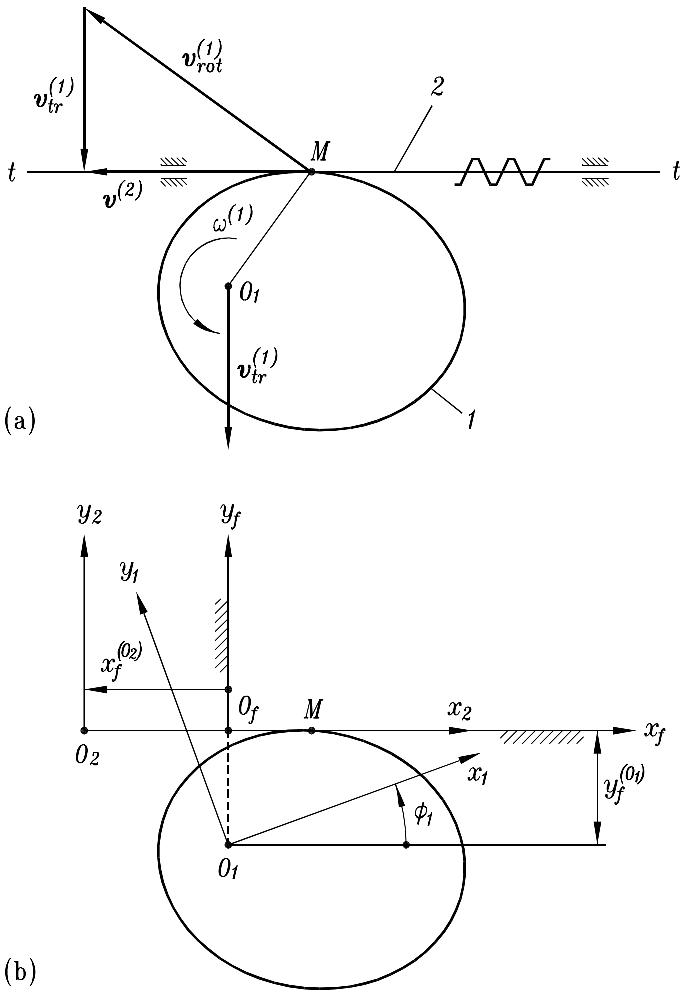

Figure 1.1.4. Toward derivation of related motions of rack cutter 2 and noncircular gear 1.

The derivations of the nonlinear functions $\phi_1(x_f^{(O_2)})$ and $y_f^{(O_1)}(x_f^{(O_2)})$ are presented in Chapter 5. Initially, observation of functions mentioned previously has been accomplished by the remodeling of existing equipment and using cam mechanisms for the generation of the required functions. Figure 1.1.5 shows the first cutting machine for noncircular gears applied in 1951. At present, observation of functions $\phi_1(x_f^{(O_2)})$ and $y_f^{(O_1)}(x_f^{(O_2)})$ is obtained by computer controlled machines (Smith, 1995).

By using enveloping methods of generation of noncircular gears, various new types of noncircular gears have been developed with closed and non-closed centrodes. An example of a pair of noncircular gears with non-closed centrodes applied for generation of a given function $y(x)$ represented in the closed interval $[x_1, x_2]$, if $y(x_1) \neq y(x_2)$ (see Chapter 10), is shown in Fig. 1.1.6.

Figure 1.1.7 shows a 3D model of a helical elliptical gear. It has found good application for the driving of a crank-slider mechanism in heavy-press machines.

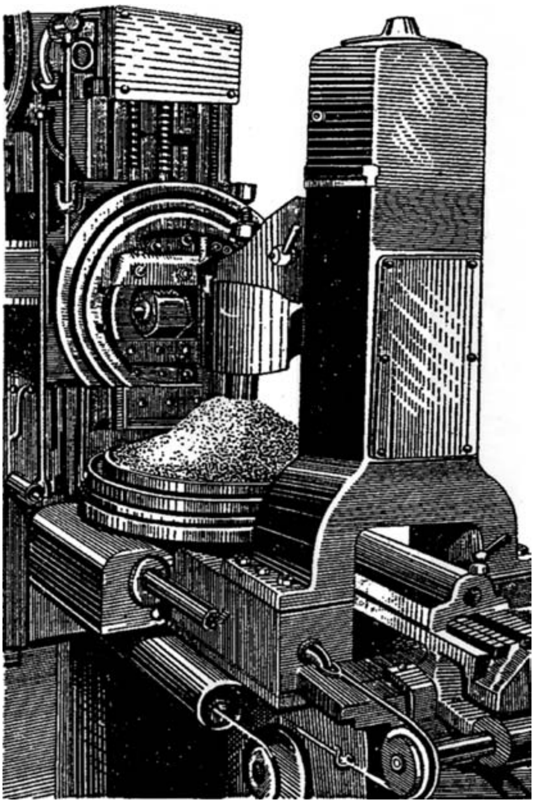

Figure 1.1.5. Remodeled cutting machine for generation of noncircular gears (1951).

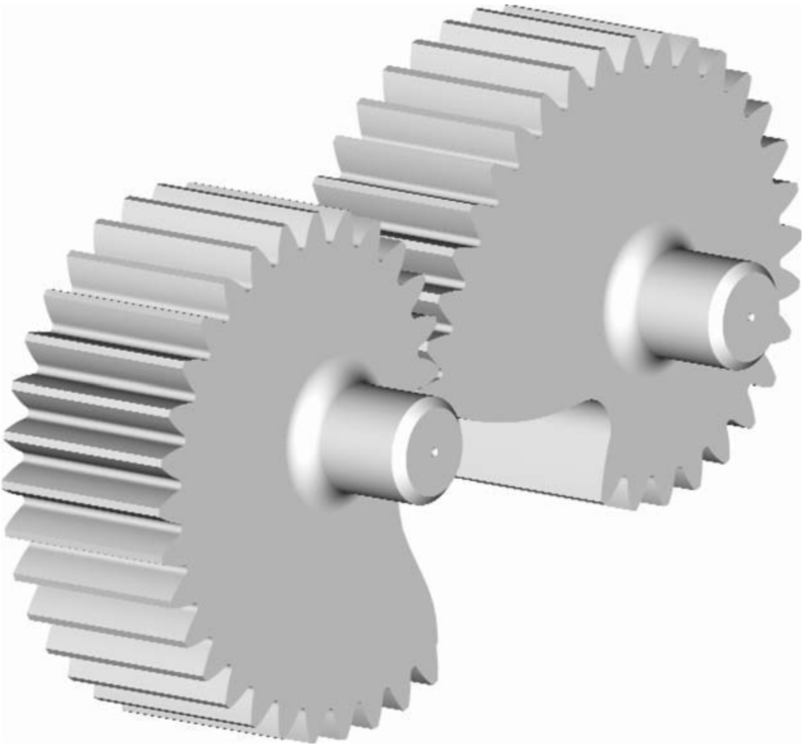

Figure 1.1.6. Illustration of noncircular gear drive with non-closed centrodes.

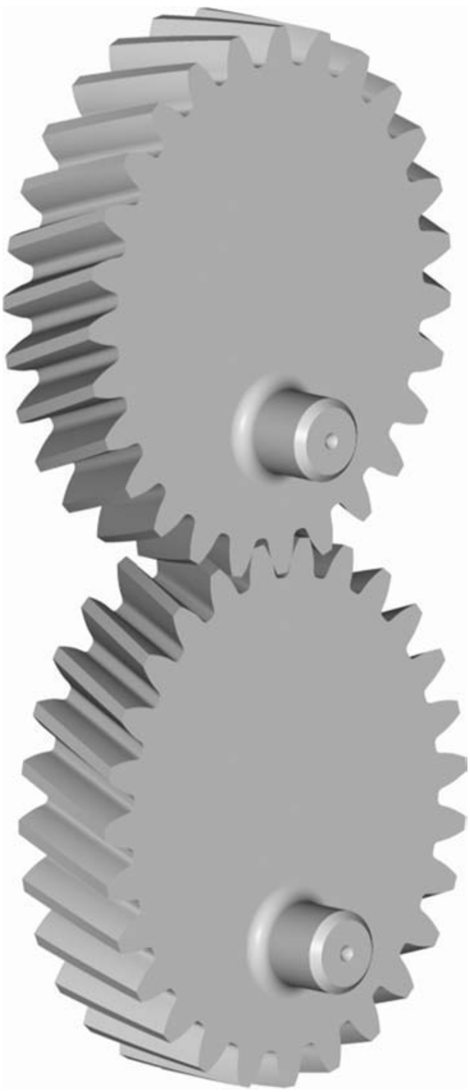

Figure 1.1.7. Illustration of helical elliptical gear drive.

Figure 1.1.8 shows a gear drive formed by an eccentric involute pinion and a conjugated noncircular gear. Application of this gear drive has been found in rice planting machines. It can be used as well in tandem design of the Scotch-Yoke mechanism coupled with eccentric involute gears to obtain an improved function of the output velocity.

1.2 Toward Design and Application of Noncircular Gears

1.2.1 Examples of Previous Designs

Noncircular gears have found application in the industry for (i) variation of the output speed (for instance, in presses, conveyers, rice planting machines, etc.) and

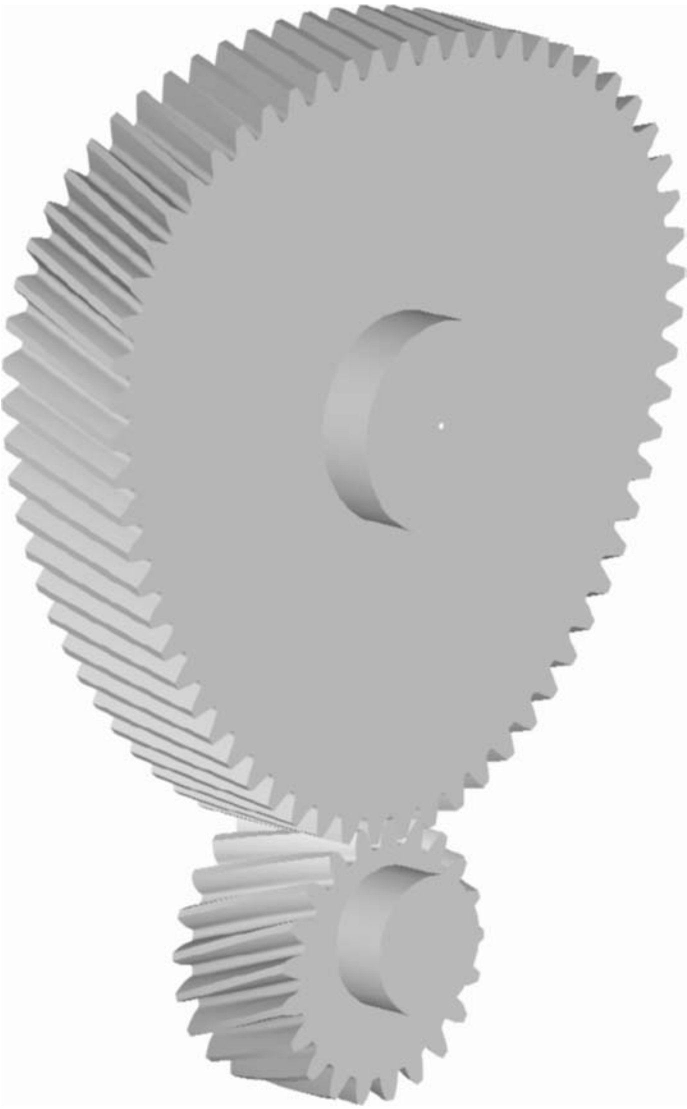

Figure 1.1.8. Illustration of eccentric involute pinion and conjugated noncircular gear.

(ii) generation of a given function (by a single pair of centrodes or a multigear drive).

The contents of this section cover only a small number of examples of the previous design of mechanisms with noncircular gears. Figure 1.2.1 shows application of a gear drive with elliptical gears applied for driving a Maltese cross mechanism. The Maltese cross mechanism, also known as the Geneva mechanism, is used to convert a continuous rotary motion into an intermittent rotary motion, and it is applied in many instruments or in other applications where an intermittent rotary motion is required. The purpose of the design shown in Fig. 1.2.1 is obtaining lower and higher speeds for the working and free-running parts of the cycle.

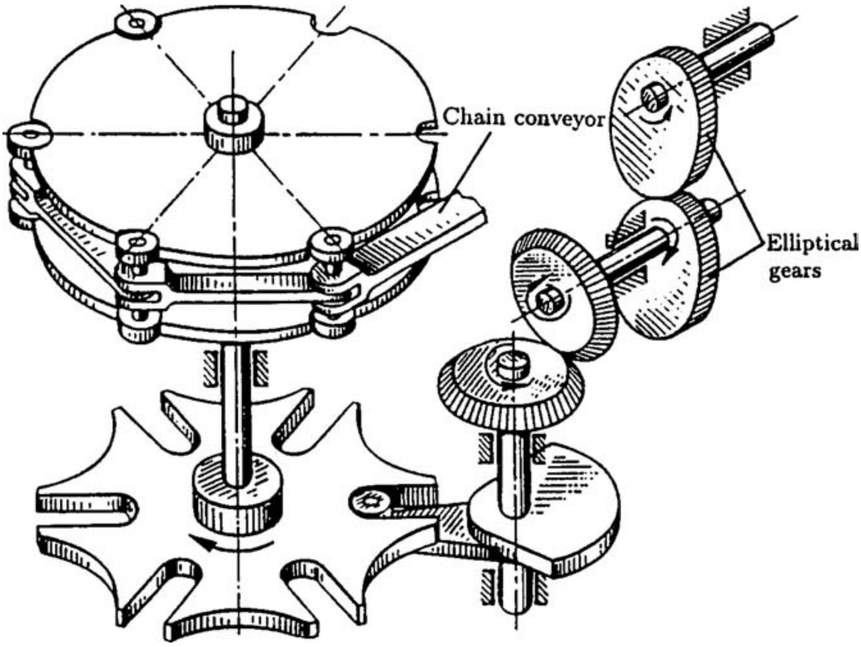

Figure 1.2.1. Application of elliptical gears in combination with a Maltese cross mechanism.

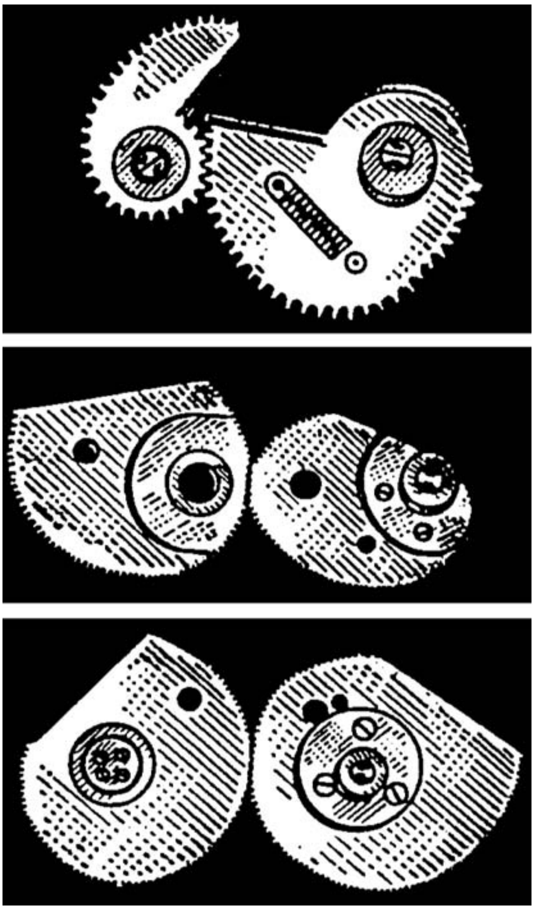

Figure 1.2.2. Illustration of noncircular gears applied in fine mechanics.

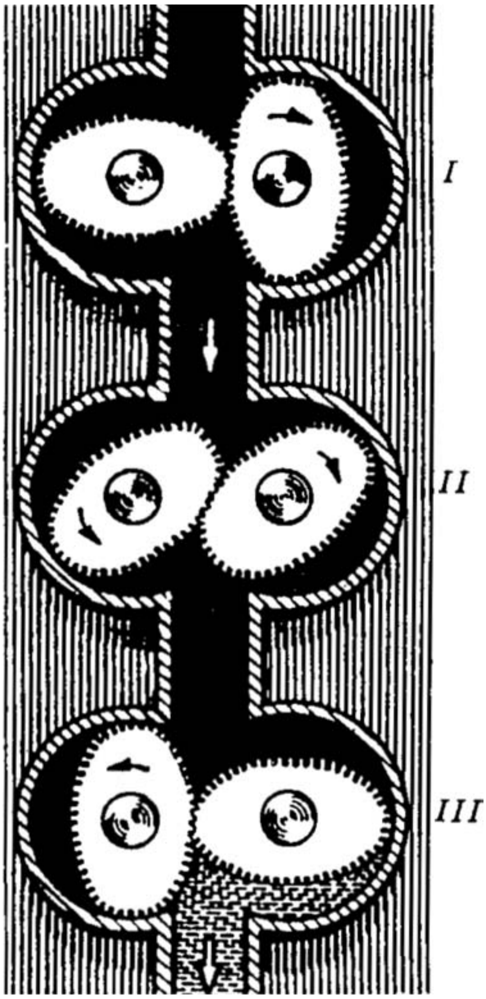

Figure 1.2.3. Illustration of transportation of the flow of liquid: I, II, and III positions of the same pair of oval gears.

Figure 1.2.2 shows examples of noncircular gears applied in the past in fine mechanics for the generation of functions of one variable. The development of electronic ways of generation of functions has eliminated such an approach. Figure 1.2.3 shows a comparatively modern example of the application of oval gears in flowmeters, proposed by Bopp and Reuthers (Bopp & Reuther G.m.b.H., 1938). The oval shape of the centrodes is obtained by modification of a conventional ellipse (see Section 4.3.5).

Figure 1.2.4 illustrates the possibility of application of twisted centrodes, which allow an increasing of the interval of the function to be generated. The gears with such centrodes may perform during the process of meshing an angle of rotation $\phi > 2\pi$ but, in addition to rotation, the gears have to perform an axial displacement. Figure 1.2.5 shows the centrodes of gears for generation of function $y(x) = 1/x$ for $1 \leq x \leq 3$ and $\phi_{1_{max}} = 5\pi$.

Figure 1.2.6 is the sketch of a heavy press machine designed as a combination of a crank-slider linkage coupled with elliptical gears. Such a design provides two

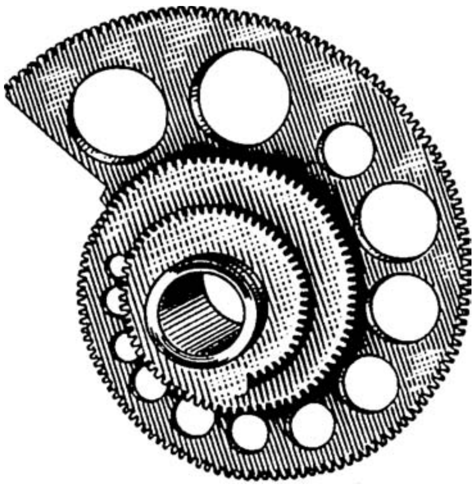

Figure 1.2.4. Illustration of a twisted noncircular gear.

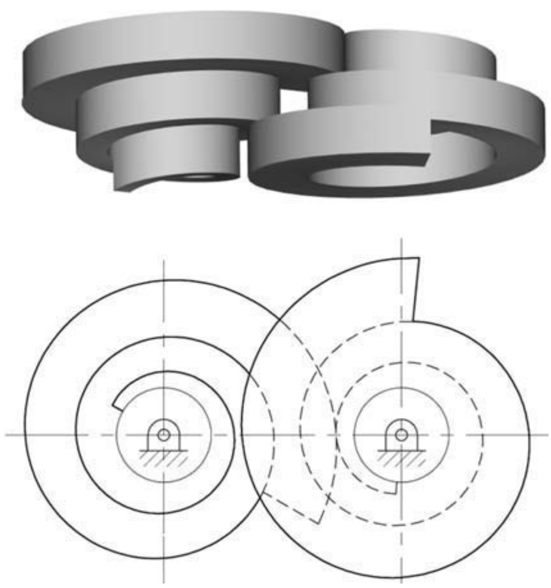

Figure 1.2.5. Centrodes of gears for generation of the function $y(x) = 1/x$ for $1 \leq x \leq 3$ and $\phi_{1_{max}} = 5\pi$.

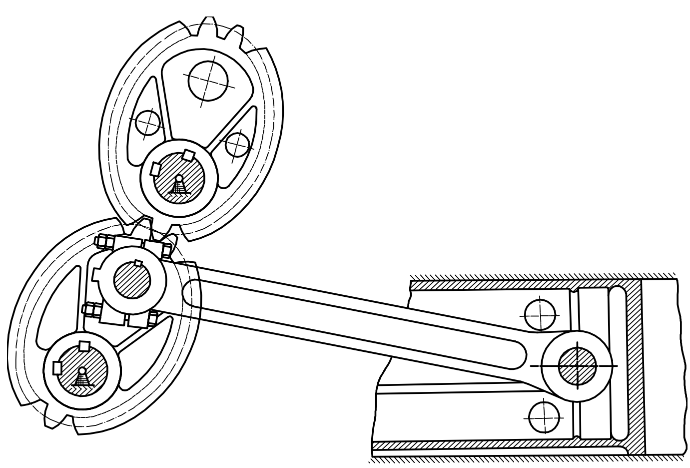

Figure 1.2.6. Application of elliptical gears in combination with a crank-linkage mechanism for a heavy press machine.