Cultural Development of Mathematical Ideas

Drawing upon field studies conducted in 1978, 1980, and 2001 with the Oksapmin, a remote Papua New Guinea group, Geoffrey B. Saxe traces the emergence of new forms of numerical representations and ideas in the social history of the community. In traditional life, the Oksapmin used a counting system that makes use of 27 parts of the body; there is no evidence that the group used arithmetic in prehistory. As practices of economic exchange and schooling have shifted, children and adults unwittingly reproduced and altered the system in order to solve new kinds of numerical and arithmetical problems, a process that has led to new forms of collective representations in the community. While Dr. Saxe's focus is on the Oksapmin, the insights and general framework he provides are useful for understanding shifting representational forms and emerging cognitive functions in any human community.

Dr. Geoffrey B. Saxe has conducted research on mathematical cognition and culture in a variety of settings, including remote parts of Papua New Guinea, urban and rural areas of northeastern Brazil, and elementary and middle school classrooms in the United States. His prior books include *Culture and Cognitive Development: Studies in Mathematical Understanding* (1991) and *Social Processes in Early Number Development* (with S. Guberman and M. Gearhart, 1987). He is currently a professor at the Graduate School of Education, University of California, Berkeley.

LEARNING IN DOING: SOCIAL, COGNITIVE AND COMPUTATIONAL PERSPECTIVES

Cultural Development of Mathematical Ideas

Papua New Guinea Studies

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with
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In memory of my parents, Carl and May

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Series Foreword

This series for Cambridge University Press is widely known as an international forum for studies of situated learning and cognition. Innovative contributions are being made by anthropology; by cognitive, developmental, and cultural psychology; by computer science; by education; and by social theory. These contributions are providing the basis for new ways of understanding the social, historical, and contextual nature of learning, thinking, and practice that emerges from human activity. The empirical settings of these research inquiries range from the classroom to the workplace to the high-technology office, and on to learning in the streets and in other communities of practice. The situated nature of learning and remembering through activity is a central fact. It may appear obvious that human minds develop in social situations and extend their sphere of activity and communicative competencies. But cognitive theories of knowledge representation and learning alone have not provided sufficient insight into these relationships. This series was born of the conviction that new, exciting interdisciplinary syntheses are under way as scholars and practitioners from diverse fields seek to develop theory and empirical investigations adequate for characterizing the complex relations of social and mental life, and for understanding successful learning wherever it occurs. The series invites contributions that advance our understanding of these seminal issues.

Roy Pea
Christian Heath
Lucy A. Suchman

Preface

My aim in this book is to contribute to our understanding of relations between culture and thought. The context for key ideas and argumentation is my research in the Oksapmin region of Papua New Guinea, an area that encompasses two small valleys in the central highlands of the New Guinea island. The circumstances of the Oksapmin people have provided me with an extraordinary opportunity for research and reflection. The Oksapmin have historically used a 27-body-part counting system, and the change in their numerical practices since first contact by westerners in 1938 has been remarkable. In the pages that follow, I report my investigation of the history of quantification practices among the Oksapmin, with a particular focus on shifts that occurred over my three stints of fieldwork in 1978, 1980, and 2001. Having this historical perspective brings culture-cognition relations into unusual relief.

A Trajectory of Research – Tensions and Partial Resolutions

In a 2005 commentary on some of my Oksapmin research (Saxe & Esmonde, 2005), Anna Sfard identified a conceptual shift in my work over a period of 23 years (Sfard, 2005), which includes, at one end, my first visit to Oksapmin in 1978 and, at the other, my last visit in 2001 and the publication of the manuscript in 2005. She pointed to a movement away from a structural developmental perspective, building on the work of Piaget, to a different approach to problems of culture-cognition relations, one that focused on collective practices and the reproduction and alteration of mathematical ideas in communities.

My first reaction was to disagree with Anna. I saw my own development as having had no radical disjunctions, but rather as a continuous process that was firmly grounded in my early training. In preparing this

book, however, I came to understand that what I had regarded as minor alterations constituted a marked conceptual shift in analytic approach. This preface provides a brief history of my disciplinary background and the trajectory of my work, providing an orientation to the perspectives that have shaped differently the waves of the Oksapmin fieldwork. I begin the short narrative with my undergraduate studies in psychology, and I conclude with the current treatment, which locates the reproduction and alteration of representations and ideas in community life as core conceptual and empirical problems that this volume will address. I believe knowing about my shifting orientation may be useful in understanding some nuanced methodological differences across the time frames.

Moral Reasoning in a Yu'pik Community

In 1969, as an undergraduate psychology major at U.C. Berkeley, I lived for a summer in an Eskimo village and conducted independent field research on moral development. My goal was to extend the structural-developmental treatment of moral reasoning elaborated by Lawrence Kohlberg (Kohlberg, 1969) to an investigation of the reasoning of Eskimo villagers. The methods that I used reflected a prominent paradigm of the time: A researcher, in garnering supportive evidence that a posited stage sequence was universal, sampled divergent communities and interviewed samples of people at different age levels, coding interviews by means of preestablished schemes. Following this paradigm, I adapted Kohlberg's notion of moral dilemmas into versions that I thought would be relevant to village life, and I interviewed village children and adults sampled from preplanned age groups. Although it was not my intention, I also became a participant observer in the community, simply by living among the Eskimos. I bathed daily with Yu'pik elders in an earthen sauna, a traditional practice; I attended ceremonies that blended indigenous and Russian Orthodox practices; I was an observer of interpersonal disputes inflamed by home brew and firearms; and I attempted (with only partial success) to thwart the rumors that I was a Russian spy. By the end of the summer, I left the region unclear whether I belonged in the Eskimo village as an odd-looking Yu'pik or in Berkeley as a student.

Back in Berkeley, I found myself struggling to make sense of the fieldwork. I learned Kohlberg's coding system, analyzed my interviews, and found that Kohlberg's codes reduced the complex moral issues in the villagers' lives to reasoning represented by the first two of Kohlberg's six stages. I was pleased that my findings converged with other cross-cultural

studies from this era of research on moral development, but I was troubled as well, because my findings did not reflect the nuanced world of the Eskimo village, dense in moral complexities.

Upon reflection, I felt that my work failed to represent critical dynamics of culture-cognition relations in the moral domain, and I wondered if the same critique could be made of cognitive developmental research. The approach's emphasis on universals pushed the diversity and complexity of historical and cultural processes into a distantly backgrounded "context," and a question emerged that would later become the core of my Oksapmin research. How does one preserve the explanatory power of a developmentalist orientation to the progressive differentiation of forms of thought while creating techniques that yield greater insight into culture-cognition relations? The necessary focus on the individual's elaboration of cognitive structures in developmental treatments left culture as not much more than an appendage. But these issues were set aside after I entered graduate school in cognitive developmental psychology at U.C. Berkeley with Jonas Langer (1970–1975) and then held an 18-month postdoctoral fellowship in the cognitive neurosciences with Peter Wolff at Harvard University Medical School and Howard Gardner at the Boston Veterans Administration Hospital (1975–1977).

Graduate Study and Early Career

Throughout my graduate training and early post-Ph.D. years, I engaged with a thick reading of Piaget's and related developmental theories. Through a Piagetian lens, I examined, with graduate student colleagues and under the wise guidance of Jonas Langer, questions of the origins of human knowledge and corroborative evidence for the emergence of universal stages in logico-mathematical and physical understandings (Piaget, 1952). While I also read with great interest the work of Heinz Werner and Lev Vygotsky, their writings did not directly inform my dissertation research.

My dissertation engaged the long-standing discussion about relations between representation and understanding in cognitive development. In establishing a context for my work, I noted that Piaget, in his treatment of the development of numerical understandings, made offhand arguments about developmental relations between cultural systems of representing numbers, such as counting, and fundamental numerical understandings like number conservation. In my dissertation I endeavored to systematically explore counting-conservation relations, showing that counter to Piaget's

argument, children used counting to mediate numerical evaluations prior to their understanding conservation. I developed arguments that showed how one could work with core elements of a constructivist framework like Piaget's to produce a more adequate treatment of counting-conservation relations, even though my findings were at odds with Piaget's claim.

My dissertation led me to an NIMH postdoctoral fellowship at Children's Hospital Medical Center in Boston and the Boston Veterans Administration Hospital (VA) in the developing field of cognitive neuroscience. My focus was on atypical populations – children with varied forms of learning disabilities related to number and adults who had suffered focal brain lesions. With these populations, I sought to explore dissociations between representation and understanding in the domain of numerical thought. The postdoctoral period opened my eyes to questions of physiological mechanisms and the organization of cognitive functioning. At the VA I regularly attended rounds and engaged in a number of case studies involving people who had suffered loss of cognitive functioning. At Children's Hospital I worked in a learning disabilities clinic and conducted related work on children with focal difficulties with number. As I worked to find ways to help children and patients to circumvent cognitive impairments, I became attuned to the character of atypicalities. It was during this period in Boston that I received a letter from David Lancy, at the time Principal Research Officer of the Ministry of Education in Papua New Guinea. David wrote to invite me to submit a grant proposal to study the development of numerical cognition in indigenous groups. He had learned of my dissertation research and was looking for recent Ph.D.s who might be up for an adventure into other worlds. For me, the glove fit well: I was excited about the possibility of extending my interest in comparative approaches to communities far removed from me as well as the opportunity to participate in another cultural world. I wrote a small grant proposal, which was funded. In the interim, I received my first academic appointment, at the Graduate Center of the City University of New York in a Ph.D. program in Educational Psychology.

I delayed the Papua New Guinea trip for a year, and during that academic year in New York, I met two doctoral students in anthropology, Tom Moylan and Virginia Guilford, who were bound for Papua New Guinea to do dissertation research in linguistic anthropology. We discussed our Papua New Guinea research plans, and several months after Tom was in the field, I received a letter from him. Tom wrote of his early experiences with the Oksapmin, a remote group, and their number system, and invited me to come for a stay and study. I worked out an arrangement with David

Lancy at the Ministry of Education to engineer the Oksapmin area as a target for my own work. After a lot of help from my friends and family, I took off for New Guinea at the end of my first year at the Graduate Center.

Oksapmin 1978

My first entry into the remote community was unnerving. After a couple of failed landing approaches trying to find holes in rapidly moving clouds, the pilot put down his small Cessna on a notoriously hard-to-land-on dirt strip adjacent to a Baptist mission station. The airstrip was about 12 kilometers from Kweptanap, the village where Tom and Virginia resided. After a quick greeting and questioning, I took up the resident missionary's offer for a ride on the back of his motored dirt bike as far as the trail permitted to Tom and Virginia's Kweptanap location. Of course, we were a spectacle to onlookers. What I saw on the bumpy and muddy trail appeared as a fantasy world. The terrain was alien, and the people were so different from me. We passed bare-breasted women in grass skirts carrying string bags of sweet potatoes as well as fierce-looking men with bows and arrows, their pierced noses displaying boars' tusks and beetles' horns, dressed in cane belts and penis sheaths. When the dirt bike could go no farther, the missionary pointed to a densely forested trail. I set off worried that only a few hours of daylight remained. After an hour's slog through deep clay mud, I encountered some boys hunting with their bows. Excited to find me, they helped guide me (patiently, at a pace that I could follow) to a mountain bluff, where they called out with a mountain yodel to signal our impending arrival at Kweptanap. I doubted that Tom expected me to take him up on his invitation, but there I was.

The 1978 studies represented my first effort to engage the tension between a developmental analysis of universals in numerical cognition and culture-specific cognitive developments. My resolution was to focus on a unique and important historical development in Oksapmin communities, the 27-body-part counting system, and Oksapmin children's developing understanding and use of it. With the orienting help of Tom and Virginia, I investigated developmental shifts in one-to-one correspondence operations as evidence of the ways that children turn their culture's counting system into forms for counting and doing simple arithmetic operations (Studies 8-1 and 8-2, described in Part III). While my findings preserved a Piagetian orientation, they represented my determination to investigate developmental processes that were tied to culture-specific systems. This work opened up new lines of inquiry on my subsequent visits.

Oksapmin 1980

Returning to the area with Tom Moylan and Maryl Gearhart (who later became my partner in life) in 1980, I conducted developmental studies with a clearer and better-articulated link to people's daily activities and their use of the 27-body-part system to solve emerging problems.[1] In comparison to my prior work, my approach was decidedly more functional though it retained its Piagetian-developmental orientation; I wanted to capture the way that people were adapting and extending the body-part system to solve problems in daily activities. In a departure from my earlier concentration on the assessment of cognitive structures as articulated by Piaget, I now focused on cognition in practice, in the form of local people solving local problems. In this shift in investigative posture, which at the time I hardly appreciated as a shift, I had been reading works in the sociocultural tradition. These included Luria's seminal studies in the postrevolutionary Soviet Union (Luria, 1976) and the work of Michael Cole (and his collaborators) on cross-cultural methods for investigating the representations of knowledge of non-Western peoples (Cole, Gay, Glick, & Sharp, 1971; Cole & Scribner, 1974; Scribner & Cole, 1981). I also revisited the developmental perspectives of Werner and Kaplan in their treatment of symbol formation (Werner & Kaplan, 1963). These efforts led to developmental studies on the Oksapmin's use of the body system to measure string bags (Study 8-3), to compute arithmetic problems in bush schools (Studies 9-1 and 9-2), and to solve emergent problems in economic transactions in trade stores (Study 4-1).

Oksapmin 2001

Twenty-one years later, I returned to the Oksapmin, this time with my 19-year-old son Josh and two graduate students, Indigo Esmonde and Cliff McIntosh. My analytic concerns had deepened during 20 years of research on collective activities and cognitive development in diverse settings, with projects that included the study of candy selling and straw weaving in Brazil, number games in the homes of Brooklyn preschoolers, educational games for elementary students in Los Angeles, and classroom practices and mathematics education. Across these projects, I had developed a commitment

[1] Tom pursued his own research on the Oksapmin language and consulted with me on studies of number. Maryl collected field observations for a Ministry of Education curriculum project directed by Randall Souviney.

to exploring the interplay between individuals' developing understandings and the recurring problems that they cognized and solved in collective activities. For example, in 1985, in my work in Brazil with young candy sellers, my concern was to explore the structure of the children's practice of purchasing from wholesale stores, marking up the candy for retail sale, selling it to customers, and then searching for a better buy at wholesale stores. In this complex practice, the numerical problems that emerged for the young entrepreneurs (some of whom had virtually no schooling) arose from an interplay between their own understandings and the conditions of their practice, which was steeped in norms, values, and social conventions. My research showed that sellers, through their engagement with selling, had developed specialized forms of numerical representations and understandings deeply interwoven with their practice, and that their use of these forms was both powerful and insightful (and quite different from those of their nonselling peers).

When I first considered returning to Oksapmin, I conceptualized the analytic focus in simple terms: I would ask, had people's approaches to quantification changed since my early visits? And if so, how? With these questions, I would extend my prior work on understanding cognitive development in collective practices to incorporate a historical perspective. But once in the field, I found that my initial "simple" questions quickly morphed into an engagement with foundational issues: ways of understanding social history as a central theoretical construct in the conceptualization of cognitive processes as well as ways of understanding cognitive processes in the historical development of mathematical ideas. Oksapmin in 2001 became an arena to flesh out and pursue new ideas as I framed new questions and empirical projects.

In the thinking that came to usefully organize my work in 2001, people's present-day cognitive activities were reproducing as well as altering the collective practices in which they were participants, including forms of numerical representations they were using and the functions that these forms were serving as they pursued their own goals. At the same time, the complex dynamics of historical processes made manifest in collective activities framed people's goal-directed actions. This dialectical relationship between individual and social history that emerged in people's engagement with collective activities became an important frame for me in 2001. Bringing strands of my earlier 1978 and 1980 Oksapmin work forward, I focused on two arenas for analysis in 2001. The first arena was people's engagement with collective practices of economic exchange (Studies 4-2, 4-3, 4-4, 5-1, 5-2, 6-1, 6-2, and 6-3), and the second was students' and

teachers' participation with schooling (Studies 10-1, 10-2, 11-1, and 11-2). Though the shift in my own thinking about knowledge and its development seemed continuous and straightforward, the reader may detect a difference in analytic posture between the studies conducted between 1978 and 2001. To make the volume more coherent, I have organized it by making use of my current conceptual commitments related to understanding dynamic relations between individual, collective activity, and social history in the cultural development of mathematical ideas.

For readers interested in viewing video of scenes from Oksapmin communities in 1978, 1980, and 2001, please consult the following Web site: http://culturecognition.com/ or the YouTube channel: http://www.youtube.com/user/CultureAndNumber1. The videos include displays of the Oksapmin 27-body-part counting system and innovative uses of the system in arithmetical problem solving. The channel also includes video on collective practices, such as economic exchange and schooling; subsistence activities, such as hunting and gardening; and playing of (musical) percussion instruments.