FIELD THEORY OF NON-EQUILIBRIUM SYSTEMS

The physics of non-equilibrium many-body systems is one of the most rapidly expanding areas of theoretical physics. Traditionally used in the study of laser physics and superconducting kinetics, these techniques have more recently found applications in the study of dynamics of cold atomic gases, mesoscopic and nano-mechanical systems.

The book gives a self-contained presentation of the modern functional approach to non-equilibrium field-theoretical methods. They are applied to examples ranging from biophysics to the kinetics of superfluids and superconductors. Its step-by-step treatment gives particular emphasis to the pedagogical aspects, making it ideal as a reference for advanced graduate students and researchers in condensed matter physics.

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Preface

The quantum field theory (QFT) is a universal common language of the condensed matter community. As any live language it keeps evolving and changing. The change comes as a response to new problems and developments, trends from other branches of physics and from internal pressure to optimize its own vocabulary to make it more flexible and powerful. There are a number of excellent books which document this evolution and give snapshots of "modern" QFT in condensed matter theory for almost half a century. In the beginning QFT was developed in the second quantization operator language. It brought such monumental books as Kadanoff and Baym [1], Abrikosov, Gor'kov and Dzyaloshinski (AGD) [2], Fetter and Walecka [3] and Mahan [4]. The advent of renormalization group and Grassmann integrals stimulated development of functional methods of QFT. They were reflected in the next generation of books such as Itzykson and Zuber [5], Negele and Orland [6] and Fradkin [7]. The latest generation, e.g. Tsvelik [8], Altland and Simons [9], and Nagaosa [10], is not only fully based on functional methods, but also deeply incorporates ideas of symmetry based on universality, geometry and topology. (I do not mention here some excellent specialized texts devoted to applications of QFT in superconductivity, magnetism, phase transitions, mesoscopics, one-dimensional physics, etc.)

Following AGD authority almost all these books (with the notable exception of Kadanoff and Baym) employ the imaginary time Matsubara formalism [11] of finite temperature equilibrium QFT. The irony is that a much more powerful non-equilibrium QFT pioneered by Schwinger [12], Konstantinov and Perel' [13], Kadanoff and Baym [1] and Keldysh [14] was developed almost at the same time as the Matsubara technique. Being widely scattered across the periodic scientific literature, it has barely broken into the mainstream pedagogical texts. The very few books I am aware of are Kadanoff and Baym [1], Lifshitz and Pitaevskii [15], Smith and Jensen [16], Haug and Jauho [17] and Rammer [18] (there are also a number of useful reviews [19, 20, 21, 22, 23]). The reasons for such a disparity are hard to explain. In my personal opinion they are two-fold. (i) There may be a perception that all subtle and interesting effects take place only in equilibrium; non-equilibrium systems are too "violent" to be treated by QFT. Instead, the kinetic equation approach is the best one can hope for; the latter may be obtained with the Golden Rule and thus does not need QFT. (ii) The formalism is too involved, too complicated and too non-intuitive to be a part of the "common" knowledge.

As far as the first reason is concerned, it was realized decades ago that even the "simple" kinetic equation may not be actually so simple. Time and again it was shown that the kinetic equation for superfluids, superconductors, fermion–boson mixtures, disordered normal metals, etc. can't be deduced from the Golden Rule and has to be derived using the non-equilibrium QFT methods. Yet, most of the traditional experimental systems, such as liquid helium, bulk magnets, superconductors or disordered normal metals can hardly be driven substantially away from equilibrium. This created a comforting impression that studying the equilibrium plus linear response properties is largely sufficient to describe experiment. The last two decades have changed this perception dramatically. First, mesoscopic normal metals and superconductors have demonstrated that non-equilibrium conditions may be achieved in controlled and reproducible ways and a number of unusual specifically non-equilibrium phenomena do emerge. Then came cold atomic gases in magnetic and optical traps. These systems are rarely truly at equilibrium, yet they exhibit a rich phenomenology, which calls for a theoretical description. Lately the flourishing fields of nano-mechanics and nano-magnetics emerged, which deal with stochastic mechanical and magnetic systems driven far away from equilibrium. All these developments call for the systematic non-equilibrium theory to be a part of the "standard package" of a theoretical physicist.

As for the technical complexity and lack of the "esthetic" appeal, there is some truth to it, especially when the story is told in the old operator formalism (this is exactly how most currently existing books approach the subject). I can see how one can be overwhelmed by a number of different Green functions, rules to follow and by the length of calculations. Fortunately, the structure of the theory becomes much more transparent when it is presented in the functional formalism. Instead of keeping track of matrix Green functions and tensor vertices, one has to follow the scalar action, which is a functional of just two fields. Yes, one still has to double the number of degrees of freedom. However, being taken in appropriate linear combinations (Keldysh rotation), they acquire a transparent physical meaning. Then the causality principle emerges as a simple and natural way to navigate through the calculations. The main goal of this book is to present a thorough and self-contained exposition of the non-equilibrium QFT *entirely in the functional formalism.*

I tried to pay special attention to specific peculiarities of non-equilibrium (i.e. closed time contour) QFT, which do not show up in the imaginary time or $T = 0$

formalism. The presentation starts from the simplest possible systems and develops all minute technical details, exposes pitfalls and explains the internal structure for such "trivial" situations. Then the systems are gradually taken to be more and more complex. I still tried my best to emphasize peculiarities of non-equilibrium calculations in comparison with the probably more familiar equilibrium ones. Although the book is meant to be entirely self-contained, some common subjects between equilibrium and non-equilibrium techniques (e.g. diagrammatic expansion, the Dyson equation, the renormalization group) are introduced in a rather compact way. In such cases I mostly focused on differences between the two approaches, possibly at the expense of the common themes. Therefore some prior familiarity with the imaginary time QFT is probably beneficial (although not compulsory) for a reader.

What are the benefits of learning non-equilibrium QFT? (i) It naturally provides a way to go beyond the linear response and derive consistent kinetic theory (e.g. quasiparticle kinetics coupled to the dynamics of the order parameter). As was mentioned above, there is a rapidly growing list of fields where such an approach is unavoidable. (ii) Even for linear response problems it allows one to circumvent the analytical continuation procedure to real time, which may be quite cumbersome. (iii) In its functional form it provides a natural and seamless connection to the huge field of classical stochastic systems, their universality classes and phase transitions. In fact, roughly a third of this book is devoted to such classical problems. For this reason the word "quantum" does not appear in the title. Yet from the point of view of the formalism, non-equilibrium QFT and the theory of classical stochastic systems are virtually undistinguishable. (iv) Some subjects of great current interest (e.g. full counting statistics, or fluctuation relations) can not even be approached without the formalism presented here. (v) Non-equilibrium QFT (again in its functional form) appears to be extremely effective in dealing with systems with quenched disorder. Even if purely equilibrium, or linear response properties are in question, the closed time contour QFT is much more natural and efficient than the imaginary time one. All these items are the subjects of the present book. I hope you'll find it useful.

The book is directed to advanced graduate students, post-docs and faculty who want to enrich their understanding of non-equilibrium physics. It may be used as a guide for an upper division graduate class on QFT methods in condensed matter physics. Having in mind such a mature and busy audience, I opted to omit exercises. On the other hand, there are plenty of calculations within the book which require "filling in the blanks" and may be suggested as exercises for graduate students. There is practically no discussion of relevant experimental results in the book. This is done intentionally, since the scope is rather broad and inclusion of experiments could easily increase the volume by a factor. Finally, the bibliography list is not meant to be exhaustive or complete. In most cases the references are given to the original works, where the presented results were obtained, and to their immediate extensions and the review articles. I sincerely apologize to many authors whose works I was not able to cover.

Finally, this is an opportunity to express my deep appreciation to all of my coauthors and colleagues, from whom I learned a great deal about the subjects of this book. During the work on the book I was partially supported by the National Science Foundation grant DMR-0804266. Last but not least, I am indebted to my family, whose love and support made this book possible.