

Cambridge University Press

978-0-521-76036-2 - Directed Algebraic Topology: Models of Non-Reversible Worlds

Marco Grandis

Table of Contents

[More information](#)

Contents

<i>Introduction</i>	<i>page</i>	1
1	Aims and applications	1
2	Some examples	2
3	Directed spaces and other directed structures	3
4	Formal foundations for directed algebraic topology	5
5	Interactions with category theory	6
6	Interactions with non-commutative geometry	7
7	From directed to weighted algebraic topology	7
8	Terminology and notation	8
9	Acknowledgements	9
Part I First-order directed homotopy and homology		11
1	Directed structures and first-order homotopy properties	13
1.1	From classical homotopy to the directed case	14
1.2	The basic structure of the directed cylinder and cocylinder	28
1.3	First-order homotopy theory by the cylinder functor, I	40
1.4	Topological spaces with distinguished paths	50
1.5	The basic homotopy structure of d-spaces	61
1.6	Cubical sets	65
1.7	First-order homotopy theory by the cylinder functor, II	79
1.8	First-order homotopy theory by the path functor	89
1.9	Other topological settings	97
2	Directed homology and non-commutative geometry	105
2.1	Directed homology of cubical sets	106
2.2	Properties of the directed homology of cubical sets	114

Cambridge University Press

978-0-521-76036-2 - Directed Algebraic Topology: Models of Non-Reversible Worlds

Marco Grandis

Table of Contents

[More information](#)

viii

Contents

2.3	Pointed homotopy and homology of cubical sets	120
2.4	Group actions on cubical sets	127
2.5	Interactions with non-commutative geometry	130
2.6	Directed homology theories	140
3	Modelling the fundamental category	145
3.1	Higher properties of homotopies of d-spaces	146
3.2	The fundamental category of a d-space	153
3.3	Future and past equivalences of categories	165
3.4	Bilateral directed equivalences of categories	177
3.5	Injective and projective models of categories	185
3.6	Minimal models of a category	193
3.7	Future invariant properties	199
3.8	Spectra and pf-equivalence of categories	206
3.9	A gallery of spectra and models	214
	Part II Higher directed homotopy theory	227
4	Settings for higher order homotopy	229
4.1	Preserving homotopies and transposition	230
4.2	A strong setting for directed homotopy	239
4.3	Examples, I	250
4.4	Examples, II. Chain complexes	254
4.5	Double homotopies and the fundamental category	262
4.6	Higher properties of h-pushouts and cofibrations	269
4.7	Higher properties of cones and Puppe sequences	277
4.8	The cone monad	283
4.9	The reversible case	290
5	Categories of functors and algebras, relative settings	296
5.1	Directed homotopy of diagrams and sheaves	297
5.2	Directed homotopy in slice categories	301
5.3	Algebras for a monad and the path functor	309
5.4	Applications to d-spaces and small categories	319
5.5	The path functor of differential graded algebras	327
5.6	Higher structure and cylinder of dg-algebras	334
5.7	Cochain algebras as internal semigroups	342
5.8	Relative settings based on forgetful functors	345
6	Elements of weighted algebraic topology	351
6.1	Generalised metric spaces	352
6.2	Elementary and extended homotopies	362

Cambridge University Press

978-0-521-76036-2 - Directed Algebraic Topology: Models of Non-Reversible Worlds

Marco Grandis

Table of Contents

[More information](#)

<i>Contents</i>		ix
6.3	The fundamental weighted category	366
6.4	Minimal models	373
6.5	Spaces with weighted paths	376
6.6	Linear and metrisable w-spaces	387
6.7	Weighted non-commutative tori	391
6.8	Tentative formal settings for the weighted case	394
<i>Appendix A Some points of category theory</i>		397
A1	Basic notions	397
A2	Limits and colimits	405
A3	Adjoint functors	407
A4	Monoidal categories, monads, additive categories	410
A5	Two-dimensional categories and mates	414
<i>References</i>		418
<i>Glossary of symbols</i>		424
<i>Index</i>		427