

Directed Algebraic Topology

This is the first authored book to be dedicated to the new field of directed algebraic topology that arose in the 1990s, in homotopy theory and in the theory of concurrent processes. Its general aim can be stated as ‘modelling non-reversible phenomena’ and its domain should be distinguished from that of classical algebraic topology by the principle that directed spaces have privileged directions and directed paths therein need not be reversible. Its homotopical tools (corresponding in the classical case to ordinary homotopies, fundamental group and fundamental groupoid) should be similarly ‘non-reversible’: directed homotopies, fundamental monoid and fundamental category. Homotopy constructions occur here in a directed version, which gives rise to new ‘shapes’, like directed cones and directed spheres. Applications deal with domains where privileged directions appear, including rewrite systems, traffic networks and biological systems. The most developed examples can be found in the area of concurrency.

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Directed Algebraic Topology
Models of Non-Reversible Worlds

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To
Maria Teresa
and
Marina

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