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Differential Tensor Algebras and their Module Categories

R. BAUTISTA, L. SALMERÓN AND R. ZUAZUA

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Contents

Preface

This monograph is concerned with the notions of ditalgebras (an acronym for "differential tensor algebras") and the study of their categories of modules. It involves reduction techniques which have proved to be very useful in the development of the theory of representation of finite-dimensional algebras. Our aim has been to present in a systematic, elementary and self-contained as possible way some of the main results obtained with these methods. They were originally introduced by the Kiev School in representation theory of algebras, in an attempt to formalize and generalize matrix problems methods.

The presentation given here has many common features with the original one of A. V. Roiter and M. Kleiner [46], in terms of differential graded categories, as well as with the formulation given by Y. Drozd [28] (and further developed by W. Crawley-Boevey [19] and [20]), in terms of bocses. It is clear that some applications of these techniques, notably in the study of coverings in representation theory of algebras, will require the categorical formulation of the theory, as suggested in [30]. However, for the sake of simplicity, we preferred to work here in the more concrete ring theoretical context of ditalgebras. We assume from the reader some familiarity with the basics of representation theory of algebras and homological algebra (including the basics of the theory of additive categories with exact structures), which can be obtained from the first sections of [29], [47] and [3] (respectively, [32] and [27]).

In the representation theory of finite-dimensional algebras, the notions of finite, tame and wild representation type play a central role. An algebra is of finite representation type if it has finitely many pairwise non-isomorphic indecomposable modules. It is of wild representation type, or simply wild, if it contains the problem of finding a normal form for pairs of square matrices over a field under simultaneous conjugation by a non-singular matrix. Finally, it is of tame representation type, or simply tame, if the pairwise non-isomorphic indecomposable modules in each dimension can be described by a finite number

of one-parameter families. For precise definitions, see Sections 22 and 27. This monograph includes a fresh point of view of well-known facts on tame and wild ditalgebras, on tame and wild algebras, and on their modules. But there are also some new results and some new proofs.

We will review, for instance, Drozd's Tame and Wild Theorem, stating that a finite-dimensional algebra over an algebraically closed field is either tame or wild, but not both. We review also Crawley's Theorem on the existence of generic modules for tame finite-dimensional algebras over an algebraically closed field, and his Structure Theorem for its Auslander–Reiten quiver.

Our approach presents a formal alternative to the use of bocses with underlying additive categories and pull-back reduction constructions. This is replaced by the use of some special dual basis and what we call "reduction by an admissible module". This approach permits to perform explicit calculations with a reasonable effort. As an illustration of this, Section 24 includes a more conceptual proof of the fact that critical bocses are wild than the original proof of Drozd (see [19]) or than some of its subsequent simplifications (see [49]), where an explicit bimodule which produces wildness is exhibited.

The presentation given here of the reduction by an admissible module is more general than the one given in [6]. We believe that this approach has some promising potential since it provides a systematic treatment of a wide variety of reductions.

Let us comment on one interesting new result proved in Section 31. Let K denote a field extension of our ground field k. As usual, if Λ is some k-algebra, Λ-Mod denotes the category of left Λ-modules. We consider the induced K-algebra $\Lambda^K = \Lambda \otimes_k K$. Recall that the *endolength* of a Λ-module M is by definition the length of the right $\mathrm{End}_\Lambda(M)^{op}$-module M. A *generic* Λ-module is an indecomposable Λ-module with finite endolength and not finitely generated over Λ.

We will prove that if Λ is a finite-dimensional algebra over an algebraically closed field k and the induced algebra Λ^K is not wild, then every generic Λ^K-module is *rationally induced* from a generic Λ-module. More precisely, any generic Λ^K-module is of the form $G \otimes_{k(t)} K(t)$, where G is some generic Λ-module equipped with a natural structure of Λ-$k(t)$-bimodule. This is related to the study in [37], where it is shown that the extension of a field to its algebraic closure preserves generic tameness.

Now assume that k is algebraically closed and let $K = k(t)$, the rational function field of k. It has been proved in [11] that Λ^K is of finite representation type if and only if every indecomposable Λ^K-module is induced from a Λ-module. In Section 31, we show that Λ^K is not wild if and only if every generic Λ^K-module is rationally induced from a generic Λ-module. Our proof is derived

from the compatibility of the scalar extension with reduction operations and a "scalar extended version" of Crawley's article [20] on the existence and description of generic modules for tame algebras Λ over an algebraically closed field k.

We have included a series of exercises in order to illustrate and enrich the content of these notes. As usual, some of them contain parts of various research works. We have added reference paragraphs at the end of some sections, where we tried to provide fair recognition of previous work on the subject from which these notes developed.

R. Bautista, L. Salmerón and R. Zuazua