ORIGAMI, ELEUSIS, AND THE SOMA CUBE

For 25 of his 90 years, Martin Gardner wrote "Mathematical Games and Recreations," a monthly column for *Scientific American* magazine. These columns have inspired hundreds of thousands of readers to delve more deeply into the large world of mathematics. He has also made significant contributions to magic, philosophy, debunking pseudoscience, and children's literature. He has produced more than 60 books, including many best sellers, most of which are still in print. His *Annotated Alice* has sold more than a million copies. He continues to write a regular column for the *Skeptical Inquirer* magazine. (The photograph is of the author in 1959.)

THE NEW MARTIN GARDNER MATHEMATICAL LIBRARY

From 1957 through 1986 Martin Gardner wrote the "Mathematical Games" columns for *Scientific American* that are the basis for these books. *Scientific American* editor Dennis Flanagan noted that this column contributed substantially to the success of the magazine. The exchanges between Martin Gardner and his readers gave life to these columns and books. These exchanges have continued and the impact of the columns and books has grown. These new editions give Martin Gardner the chance to bring readers up to date on newer twists on old puzzles and games, on new explanations and proofs, and on links to recent developments and discoveries. Illustrations have been added and existing ones improved, and the bibliographies have been greatly expanded throughout.

1. Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi: Martin Gardner's First Book of Mathematical Puzzles and Games
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Origami, Eleusis, and the Soma Cube

MARTIN GARDNER'S MATHEMATICAL DIVERSIONS

Martin Gardner

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Introduction

Since the appearance of the first *Scientific American Book of Mathematical Puzzles & Diversions*, in 1959, popular interest in recreational mathematics has continued to increase. Many new puzzle books have been printed, old puzzle books have been reprinted, kits of recreational math materials are on the market, a new topological game (see Chapter 7) has caught the fancy of the country's youngsters, and an excellent little magazine called *Recreational Mathematics* has been started by Joseph Madachy, a research chemist in Idaho Falls. Chessmen – those intellectual status symbols – are jumping all over the place, from TV commercials and magazine advertisements to Al Horowitz's lively chess corner in *The Saturday Review* and the knight on Paladin's holster and have-gun-will-travel card.

This pleasant trend is not confined to the United States. A classic four-volume French work, *Récréations Mathématiques,* by Eduouard Lucas, has been reissued in France in paperback. Thomas H. O'Beirne, a Glasgow mathematician, is writing a splendid puzzle column in a British science journal. A handsome 575-page collection of puzzles, assembled by mathematics teacher Boris Kordemski, is selling in Russian and Ukrainian editions. It is all, of course, part of a worldwide boom in math – in turn a reflection of the increasing demand for skilled mathematicians to meet the incredible needs of the new triple age of the atom, spaceship, and computer.

Computers are not replacing mathematicians; they are breeding them. It may take a computer less than 20 seconds to solve a thorny problem, but it may have taken a group of mathematicians many months to program the problem. In addition, scientific research is becoming more and more dependent on the

mathematician for important breakthroughs in theory. The relativity revolution, remember, was the work of a man who had no experience in the laboratory. At the moment, atomic scientists are thoroughly befuddled by the preposterous properties of some 30 different fundamental particles, "a vast jumble of odd dimensionless numbers," as J. Robert Oppenheimer has described them, "none of them understandable or derivable, all with an insulting lack of obvious meaning." One of these days a great creative mathematician, sitting alone and scribbling on a piece of paper, or shaving, or taking his family on a picnic, will experience a flash of insight. The particles will spin into their appointed places, rank on rank, in a beautiful pattern of unalterable law. At least, that is what the particle physicists *hope* will happen. Of course the great puzzle solver will draw on laboratory data, but the chances are that he will be, like Einstein, primarily a mathematician.

Not only in the physical sciences is mathematics battering down locked doors. The biological sciences, psychology, and the social sciences are beginning to reel under the invasion of mathematicians armed with strange new statistical techniques for designing experiments, analyzing data, and predicting probable results. It may still be true that if the president of the United States asks three economic advisers to study an important question, they will report back with four different opinions, but it is no longer absurd to imagine a distant day when economic disagreements can be settled by mathematics in a way that is not subject to the usual dismal disputes. In the cold light of modern economic theory, the conflict between socialism and capitalism is rapidly becoming, as Arthur Koestler has put it, as naïve and sterile as the wars in Lilliput over the two ways to break an egg. (I speak only of the economic debate; the conflict between democracy and totalitarianism has nothing to do with mathematics.)

But those are weighty matters, and this is only a book of amusements. If it has any serious purpose at all, it is to stimulate popular interest in mathematics. Such stimulation is surely desirable, if for no other reason than to help the layman understand what the scientists are up to. And they are up to plenty.

I would like to express again my gratitude to the publisher, editors, and staff of *Scientific American*, the magazine in which these

chapters first appeared; to my wife for assistance in many ways; and to the hundreds of friendly readers who continue to correct my errors and suggest new material. I would like also to thank, for her expert help in preparing the manuscript, Nina Bourne of Simon and Schuster.

MARTIN GARDNER