### CAMBRIDGE

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1 Animal magic 1

### Minimum prior experience

counting to 20; doubling and halving; odd and even numbers

#### Resources

PCM 4, scissors, interlocking cubes in 2 colours for each pair; Blu-Tack, (PCM 3 for recording)

### Key vocabulary

number, zero, one, two, three . . . to twenty and beyond, odd, even, how many . . .?, double, half, halve

.....

# What's the problem?

Children count out a given numbers of cubes. They double and halve their cubes. They find other quantities of cubes that can be halved. They record results in a simple table.

Problem solving objectives

- Choose and use appropriate number operations and mental strategies to solve problems.
- Solve simple mathematical problems or puzzles; recognise and predict from simple patterns and relationships. Suggest extensions by asking 'What if . . .?' or 'What could I try next?'
- Explain methods and reasoning orally.

### Differentiation

Children find solutions with different quantities of cubes.

More able: 20 to 30 cubes.

Average: 10 to 20 cubes.

Less able: 6 to 12 cubes.

# Introducing the problem

Explain that on PCM 4 there are some animal faces and tails, but no bodies or legs. Tell children that their task is to make the bodies and legs from interlocking cubes. Demonstrate this with some cubes, one colour for the legs and another for the body, and a cut-out face and tail. Say: *Let's make a body for this animal. Let's make the body 8 cubes long. Now I want* 

each leg to be half as long as the body. How long will each leg be? Yes, 4 cubes long.

With help from children, who put together 2 towers of 4 cubes, and 1 of 8 cubes, assemble the animal. Use Blu-Tack to fix the head and tail on to the body. The finished animal will look something like this:



Explain that you would like children to build some animals. Ask them to begin, as follows:

More able: Body of 20 cubes; legs half that length.

Average: Body of 10 cubes; legs half that length.

Less able: Body of 6 cubes; legs half that length.

Explain that they make the body one colour and the legs another.

Ask children to write down, in a simple table, how many cubes they used for the body, and how many for each leg. Write the beginning of the table for the Less able group on the board. Children could use PCM 3 to help their recording.

Body	Legs
6	3

24

> Tell children that when they have made 1 animal, you would like them to make another animal with a different number of cubes, with each leg half the length of the body. Tell each group the range of quantity of cubes they can use for the body. Challenge them all to see how many different animals they can make. Some children may want to keep all their animals out. Explain that the last one they make, before the plenary, will be kept for a little while on display.

# **Teacher focus for activity**

All children: As children work, ask questions such as:

- Which numbers do you think you could try for the body?
- What is special about these numbers?

More able: Encourage children to predict which quantities of cubes they will need for the body. Ask children to explain what is special about the body numbers. They should recognise that these will all be even numbers. Encourage children to adopt a systematic approach to the problem, so that, in their table, they have the even numbers 20 to 30 in order for the body, and the corresponding numbers for the legs in order.

Average and Less able: If children are unsure about which quantities to use for the body, talk about the relationship between the legs and body: that each leg is half the body or that the body is double the leg. Ask children to tell you what is special about numbers that can be halved: they are even numbers.

### **Optional adult input**

Work with the Less able group. If children find it difficult to make the next animal after their starting point, ask: *What do you notice about the numbers for this animal with a body of 6 and legs of 3*? Children may try numbers at random, but will realise that odd numbers for the body are not possible.

## **Plenary**

- 1 Begin with the results for the Less able group. Invite pairs of children to show the animals they have made and to say how many cubes were needed for each body and a leg. On the board write the solutions in a table (see Solutions). Ask:
  - What can you tell me about the number of cubes needed for each body?
  - Are all the leg numbers even?
  - Are all the leg numbers odd?
  - Who can explain this?

- **2** Repeat this for the Average and More able results, again writing these onto the board. Now ask:
  - What would the next number for the body be after 30?
  - And how many cubes for a leg?
  - And the next . . . and the next . . .?
  - What pattern of numbers do we have for the body? (even numbers)
  - What pattern do we have for the leg? (counting numbers)

See Useful mathematical information, page 84 for further discussion of doubling and halving.

**3** Discuss how children wrote their results. Talk about how helpful it is to sort out the results so that they are in number order. Explain that this helps with spotting patterns.

Invite children to place the last animal they made on to a display table for all to admire.

### **Solutions**

6 to 12 cubes

Cubes for the body	Cubes for a leg
6	3
8	4
10	5
12	6

### 10 to 20 cubes

Cubes for the body	Cubes for a leg
10	5
12	6
14	7
16	8
18	9
20	10

#### 20 to 30 cubes

Cubes for the body	Cubes for a leg
20	10
22	11
24	12
26	13
28	14
30	15

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2 Cube animals

#### **Minimum prior experience**

counting objects up to 20; using vocabulary of length

#### Resources

interlocking cubes, paper for recording

#### Key vocabulary

number, zero, one, two, three . . . to twenty and beyond, how many . . .?, odd, even, total

.....

# What's the problem?

Children make a given shape with cubes, then follow rules given by their teacher to change the shape and the number of cubes used. They will predict the total numbers of cubes as they increase the size, including the length, of the shape.

### Problem solving objectives

- Choose and use appropriate number operations and mental strategies to solve problems.
- Solve simple mathematical problems or puzzles; recognise and predict from simple patterns and relationships. Suggest extensions by asking 'What if . . .?' or 'What could I try next?'
- Explain methods and reasoning orally.

### Differentiation

Children use different numbers of cubes.

More able: Up to 40.

Average: Up to 30.

Less able: Up to 20.

### Introducing the problem

Explain that you would like children to work in pairs to make a cube animal. Ask children to follow your instructions to make the animal. Make one at the same time, so that children can check that they have done what is asked. Say:

Put 3 cubes together to make a length. This is the animal's body.

Now put 2 cubes together. Put these at one end of the body, under the last cube, to make a leg. Do this again, at the other end of the body.

Put 2 cubes together. Put these, pointing up, above one of the legs to make the neck.

Now put one cube on to the side of the top cube of the neck to make the head.

Check that each pair has successfully completed an animal.



How many cubes did you use to make this animal? Agree that 10 cubes were used.

Now explain the problem. The animal keeps growing. I will tell each group how he grows. Make the new animal, and record how many cubes you have used each time.

**More able:** Each leg and the neck get taller by 2 cubes each time.

Average: Each leg and the neck get longer by 1 cube each time.

Less able: Each leg gets longer by 1 cube each time.

# **Teacher focus for activity**

All children: Ask questions as children work, such as:

- How many more cubes does this animal need than that one?
- How many cubes do you think you will need for the next animal?
- Why do you think that?

More able and Average: Once children have made 2 more animals, challenge them to predict what the next number of cubes will be. They can check their prediction by counting after they have made the next size of animal.

Less able: Ask children to tell you what they notice about the numbers they generate. They should notice that each animal contains 2 more cubes than the previous one. Ask them to say how many cubes they will need for the next animal, then to check by making and counting.

### **Optional adult input**

Work with the More able group. Ask children to predict how many more cubes they will need for each animal before they make it. Encourage them to make their predictions until the animal uses up all of their cubes, then to check by making the animal.

### **Plenary**

- 1 Invite each group to explain what they had to make, and to explain how many the animal grew by each time. Begin with the Less able group. Ask children to take turns to write on the board how many cubes they needed (see **Solutions**). Ask:
  - Are the totals odd or even numbers?
  - Can you explain why that is?

Discuss how these are the even numbers, and that they are 'every other number'.

- **2** Invite all children to predict what the next total after 20 would be, and the next ... (22, 24, 26...)
- **3** Invite the Average group to explain how many their animal grew by each time. Ask a child to write their results on the board. Ask:
  - What sort of numbers are your totals? (Yes, both odd and even.)
  - Why are there both odd and even numbers?
- **4** Ask children to predict what the next total after 28 would be, odd or even, and to say what the number is, then the next . . . : 31, 34, 37 . . .
- **5** Now ask the More able group to show some of their animals, and to explain how many cubes

their animal increased by each time. Invite a child to write the totals on the board. Ask:

- How many more cubes are there each time?
- Will the total always be even?
- Why is that?
- **6** Invite children to predict what the next total will be, and the next: 40, 46, 52 . . .

Ask various children to explain how they calculated their prediction. Where the addition fact was not known, children may have counted on in 1s from the larger number.

### **Solutions**

More able: The animal grows by 6 cubes each time: 10, 16, 22, 28, 34, 40 . . . The number of cubes is always even.

Average: The animal grows by 3 cubes each time: 10, 13, 16, 19, 22, 25, 28 . . . The number of cubes alternates even, odd, even, odd . . .

Less able: The animal grows by 2 cubes each time: 10, 12, 14, 16, 18 . . . There is always an even number of cubes.

See Useful mathematical information, page 84 for further discussion on finding patterns and making predictions.



# **3** Twos and threes

### Minimum prior experience

counting in 2s and 3s

### Resources

PCM 5, A3 enlargement of PCM 5, counters in 2 colours, paper for recording, Blu-Tack

### Key vocabulary

number, zero, one, two, three . . . to twenty and beyond, count in ones, twos . . ., odd, even, every other, pattern

.....

# What's the problem?

Children find which numbers occur in both counts of 2s and 3s.

### Problem solving objectives

- Choose and use appropriate number operations and mental strategies to solve problems.
- Solve simple mathematical problems or puzzles; recognise and predict from simple patterns and relationships. Suggest extensions by asking 'What if . . .?' or 'What could I try next?'
- Investigate a general statement about familiar numbers or shapes by finding examples that satisfy it.
- Explain methods and reasoning orally.

### Differentiation

All children use PCM 5.

More able: Limit the count to 30.

Average: Limit the count initially to 24, then extend if appropriate.

**Less able:** Limit the count initially to 12, then extend if appropriate.

# Introducing the problem

Pin the A3 enlargement of PCM 5 to the board. Explain the problem: Two children played jumping along the grid in the playground. They wanted to know which numbers they would visit if one of them jumped in 2s and the other in 3s. Your problem is to find all the numbers you would visit, counting in 2s and in 3s, then find the numbers that are visited by both 2s and 3s.

Explain that each group will have a different stopping point:

More able: Count to 30.

Average: Count to 24.

Less able: Count to 12.

The counts for the Average and Less able groups can be extended when they have a solution to their given count.

Suggest that children use 1 colour of counter for counting in 2s, and a different colour for counting in 3s. Check that they understand where to start: 2, 4, ... and 3, 6, ... by counting from 0, and pointing to the first 2 numbers in each counting pattern.

Remind children that they should start counting from the position of the child on PCM 5, i.e. **off** the playground grid, not from 1, so that this is like a zero starting point.

Ask children to work with a partner and to begin by deciding how they will record their work on paper.

# **Teacher focus for activity**

All children: As children work, ask questions such as:

- Which numbers are in the counting in 2s / counting in 3s pattern?
- Which numbers are in both the 2s and the 3s pattern?
- What is special about these numbers? (They are all even numbers.)

**More able**: Encourage children to use their oral counting skills to help them to place the counters correctly.

Average and Less able: If children are unsure about using their oral counting skills, discuss how they can find the pattern of 2s by marking 'every other' number position, and for 3s they may count 'one, two, three' each time to find the next position. Once they have marked these numbers with counters, ask them to say both number patterns. Check that they have an efficient recording system, such as writing down the numbers in each pattern, then making a list of those in both counting patterns.

See **Useful mathematical information**, page 84 for further discussion of these patterns.

### **Optional adult input**

Work with the Average group. Encourage them to count aloud in 2s, from 0, to about 12, in order to identify the numbers to be covered with counters. They can continue with this themselves to 24 or 30. Repeat for 3s. Use the questions above to stimulate discussion about what the children notice about the numbers they have covered.

### **Plenary**

- Invite children from the Less able group to say the numbers in the pattern of 2s that they found. Cover these numbers on an enlarged copy of PCM 5 with 1 colour of counter, fixed with Blu-Tack. Invite children from the Average group to continue the count, then those from the More able group to finish it. Ask the whole class to say the counting pattern of 2s together and ask: *What do you notice about these numbers?* Children should note that:
  - these are 'every other' numbers;
  - all these numbers are even;
  - the numbers form a pattern on the grid, of every other column.
- **2** Repeat this for counting in 3s. Use a different colour of counter to mark the pattern of 3s. Children should note the following:
  - the pattern of 3s alternates odd and even: 3, 6, 9...;
  - the numbers form a pattern on the grid of every third column.
- 3 Now ask:
  - Which numbers are in both the pattern of 2s and the pattern of 3s?

• What can you tell me about these numbers?

Children should note that:

- these numbers are all even;
- they are all in 1 column, shared by the 2s and the 3s patterns.
- 4 Demonstrate that the numbers have formed these neat columns because each row has 6 numbers in it. So each row allows 3 jumps of 2 and 2 jumps of 3.
- **5** Now invite everyone to say each counting pattern, and extend it beyond 30. Ask:
  - What comes after 30? And next?
  - What will the next number be in both the 2s and the 3s pattern?
  - Can anyone work out what the next number would be?
- **6** Discuss how children recorded their work. Give praise for systematic recording of the numbers.

### **Development**

For those children who are confident with reading and writing numbers to 100, repeat this with counting patterns of 2s and 5s.

### **Solutions**

This chart shows the patterns of 2s, 3s and 6s to 30. The 6s are the numbers included in both the 2s and the 3s, and are always even numbers.

1	2	N	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30



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4 Teddy order

### Minimum prior experience

ordinal number; position of objects

#### Resources

PCM 6, alternatively use small teddies in 3 different colours, A3 enlargement of PCM 6 with the teddy tiles cut out, scissors, glue, paper, Blu-Tack

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### Key vocabulary

order, before, after, between, first, second, third

# What's the problem?

Children find different ways to order 3 objects. They learn to set out different combinations and arrangements.

Problem solving objectives

- Solve simple mathematical problems or puzzles; recognise and predict from simple patterns and relationships. Suggest extensions by asking 'What if . . . ?' or 'What could I try next?'
- Explain methods and reasoning orally.

### Differentiation

This activity is differentiated by outcome. Expect the children to find different numbers of solutions.

More able: All 6 solutions.

Average: At least 4 solutions.

Less able: At least 3 solutions.

## Introducing the problem

Model the problem using the enlarged teddy tiles. Show children 2 different tiles and ask them to say what is the same and what is different about the teddies. Say: *I can put these 2 teddies down in this order*. Attach the teddies to the board in a horizontal line. Show the same 2 types of teddy tiles as before: *How could I arrange these*? Invite a child to show the arrangement and pin that to the board, under the previous arrangement.

Ask children to say how the 2 arrangements are different.



Explain: Your problem is to use *3* different teddies each time and to put them in an order in a straight line. I wonder how many different ways you can find to do this? Challenge the children with the number of different ways suggested in **Differentiation**.

Ask children to work in pairs and to begin by cutting apart the teddy tiles. You could suggest ways of making the cutting more efficient. Suggest that they line up their different arrangements on the table, with one arrangement under another, and so on. When children have checked that each one is different, ask them to carefully glue their solutions on to a sheet of paper, keeping the teddy order the same as on the table in front of them.

## **Teacher focus for activity**

All children: Check that children choose the 3 different teddy tiles for each arrangement.

As children work, ask questions such as:

- How is this arrangement different from that one?
- Can you spot a pattern?
- How does that help you?

**More able:** Suggest that children try to find a pattern in the arrangement of the teddies.

Average and Less able: Children may not be systematic about this activity at this stage. However, do ask them to try to find a pattern in the arrangement of the teddies. When they have completed at least 3 or 4 solutions, challenge them to find all 6.

### **Optional adult input**

Work with the Less able group. Use the enlarged teddy tiles and work together initially. Children can model the solutions with their own teddy tiles and 'say' the pattern by describing each teddy in the arrangement. When children are confident with what they are doing, suggest that each pair now searches for some more solutions.

# **Plenary**

1 Invite children from the Less able group to begin to feed back. Children can use the enlarged teddy tiles to replicate each order, by sticking the tiles to the board. Continue, asking children from the Average and then the More able group, until all 6 solutions are on the board. Children can check which ones they have on their recording sheets as the solutions are pinned up.

Compare the different orders. Ask children to:

- Point out 2 that start with the same teddy.
- Find me 2 that have the same teddy in 3rd place.
- Find me 2 that have the same teddy in between the 1st and 3rd teddy.
- **2** Ask questions such as:
  - How many answers are there?
  - Is each one different?
  - How is this one different from this one?
  - What do you notice about the answers?
- **3** Encourage children to describe what they notice. They may say, for example:
  - 'Each teddy begins 2 different orders like, ABC and ACB.'
  - 'This one (CBA) is this one (ABC) backwards.'
- **4** Talk about how for orders such as this, one order can be the reverse of another, such as ABC and CBA. Invite children to find other reverses. There will be 3 pairs altogether:
  - ABC and CBA;
  - ACB and BCA;
  - BAC and CAB.

- 5 Now ask 3 children to stand at the front of the class. Invite the other children to help put the children into different orders, just as they have done with the teddy tiles. This will give them the opportunity to use the thinking skills practised in this activity. Emphasise what has been learnt, e.g. once one combination has been found see if children can find the reverse. Ask questions of order again:
  - Who is between . . . and . . .?
  - Who is first/middle/third/last?

As children come up with different orders, draw them on the board, or write the different names, so that children can see the 6 combinations.

See Useful mathematical information, page 85 for further discussion on order.

### **Development**

Challenge children to repeat this activity, this time with 4 different items to order, e.g. coloured counters. This is more complex and will produce more solutions (see **Useful mathematical information**, page 85 for possible outcomes).

### **Solutions**

These are the 6 possible combinations:



**5** Clock numbers

### Minimum prior experience

odd and even numbers; ordering numbers

#### Resources

PCM 7, A3 enlargement of PCM 7, counters in red, blue, green and yellow, coloured pencil crayons in red, blue, green and yellow, clock stamp and ink pad or PCM 8, Blu-Tack

### Key vocabulary

zero, one, two, three . . ., odd, even, more, less, compare, order, between

# What's the problem?

Children place counters on to the numbers on the clock, according to their properties. They consider value, shape, and whether numbers are odd or even.

#### Problem solving objectives

- Choose and use appropriate number operations and mental strategies to solve problems.
- Solve simple mathematical problems or puzzles; recognise and predict from simple patterns and relationships. Suggest extensions by asking 'What if . . .?' or 'What could I try next?'
- Investigate a general statement about familiar numbers or shapes by finding examples that satisfy it.
- Explain methods and reasoning orally.

### Differentiation

The activity is differentiated by the resources provided and methods used.

More able: Mental strategies.

Average: Mental strategies and counting using the clock face.

Less able: Counting using the clock face.

## Introducing the problem

Pin to the board the enlarged clock face (PCM 7). Tell children that this is a clock face. Write in the numbers in the circles with children saying which go where. Explain that the problem today involves using



different properties of the numbers. Read together the properties on the sheet:

- a blue counter can go on an odd number;
- a red counter can go on an even number;
- a green counter can go on a number larger than 9;
- a yellow counter can go on a number with only straight lines.

Summarise these on the board to remind children, e.g. draw a blue circle and write 'even' next to it, with other symbols and key words for children to refer to.

Explain that you would like children to work in pairs to solve the problem. Say: *Each number needs a counter on it. The counters must go on to the clock so that they fit the instructions.* Suggest that, for instance, children could put a blue counter on 11 because it is odd, or they could put a yellow one on because it is a number with straight lines or a green one because it is more than 9. Each pair will need about 6 counters of each colour.

Children need to draw the numbers on the clock at given marks. This gives them an opportunity to think about their properties before they start with the counters.

> Ask children to record what they do. They should decide with their partner how they will record and they can use coloured crayons if they wish.

# **Teacher focus for activity**

All children: As children work, ask questions such as:

- Why did you choose to put that counter on . . .?
- What other counter could go there? Why?
- What else could you try?

More able: Challenge children to find many different solutions to this problem. Ask them to explain why they have placed a particular counter. Discuss with them how they will record, so that their recording is an accurate record of what they have done. Children may, for example, draw a clock face and colour in the numbers to represent their counters, or write the numerals in a line, marking each one with its colour.

Average: Children may find it helpful to count around the clock face, using it as a form of number track in order, for example, to identify odd or even numbers. Discuss how children intend to record their work. They may find it helpful to stamp clock faces on to a sheet, for example, and mark in the colours on to the numbers on the clocks to represent the counters. They could also use PCM 8, and colour in next to the relevant numbers.

Less able: Children will find it helpful to count around the clock face to identify the required number properties. Provide a clock stamp and ink pad or PCM 8 and children can use this to help them to record, as for the Average ability group.

### **Optional adult input**

Work with the Less able group. Children may need help in counting, e.g. to find odd and even numbers, as the clock face shows the numbers in a different visual orientation from how they are presented on a number track. Discuss different ways of covering each number.

# Plenary

1 Draw 3 clock faces on the board. Ask 1 pair from each ability group to come out and apply counters for each number property required. They can do this by fixing counters with Blu-Tack on to the clock faces. Ensure children put counters *next to* the numbers and not on them.

- **2** Encourage children to explain how their arrangement of counters matches the criteria. Ask:
  - Which are the odd/even numbers?
  - Which numbers are larger than 9?
  - Which numbers have straight lines?
- **3** Ask children to explain the strategies they used to find the numbers to be covered. The Less able group can demonstrate counting around the clock, to find even numbers. The More able group can show how they used mental strategies, such as counting patterns of odd or even numbers, using a mental number track image to think which numbers are larger than 9...
- **4** Ask children to sit at their tables with their PCM and some counters. Explain that you will ask them to cover some numbers. Say:
  - Only put 1 counter on each number.
  - Put blue counters on numbers less than 6 and more than 2. (3, 4, 5)
  - Put red counters on numbers between 4 and 8. (5, 6, 7)
  - Put yellow counters on numbers with curves.
    (2, 3, 5, 6, 8, 9, 10, 12)
  - Which numbers are not covered? (1, 11) Why is that?

Congratulate the children on their hard work. Their recording could become part of a display on properties of numbers.

### **Solutions**

Numbers can be covered in different ways.

	1	2	3	4	5	6	7	8	9	10	11	12
Odd numbers	V		~		~		~		~		~	
Even numbers		V		~		V		~		~		~
Numbers												
larger than 9												
Numbers with												
only straight	~			~			~				~	
lines												

See Useful mathematical information, page 85 for further discussion of properties of numbers.