SOLITON EQUATIONS AND THEIR ALGEBRO-GEOMETRIC SOLUTIONS

Volume II: (1 + 1)-Dimensional Discrete Models

As a partner to Volume I: (1 + 1)-*Dimensional Continuous Models*, this monograph provides a self-contained introduction to algebro-geometric solutions of completely integrable, nonlinear, partial differential-difference equations, also known as soliton equations.

The systems studied in this volume include the Toda lattice hierarchy, the Kac–van Moerbeke hierarchy, and the Ablowitz–Ladik hierarchy. An extensive treatment of the class of algebro-geometric solutions in the stationary as well as time-dependent contexts is provided. The theory presented includes trace formulas, algebro-geometric initial value problems, Baker–Akhiezer functions, and theta function representations of all relevant quantities involved.

The book uses basic techniques from the theory of difference equations and spectral analysis, some elements of algebraic geometry and, especially, the theory of compact Riemann surfaces. The presentation is constructive and rigorous, with ample background material provided in various appendices. Detailed notes for each chapter, together with an exhaustive bibliography, enhance understanding of the main results.

Reviews of Volume I:

'. . . this is a book that I would recommend to any student of mine, for clarity and completeness of exposition. . . Any expert as well would enjoy the book and learn something stimulating from the sidenotes that point to alternative developments. We look forward to Volumes II and III!'
Mathematical Reviews

'The book is very well organized and carefully written. It could be particularly useful for analysts wanting to learn new methods coming from algebraic geometry.'
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Volume II: (1 + 1)-Dimensional Discrete Models

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Helge Holden har mottatt støtte fra Det faglitterære fond

To

Gloria
Christian, Mads, Frederik, and Daniel
Elli, Peter, and Franziska
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Acknowledgments

It's been a hard day's night,
and I've been working like a dog.
It's been a hard day's night,
I should be sleeping like a log.

J. Lennon/P. McCartney[1]

This monograph is the second volume focusing on a certain class of solutions, namely the algebro-geometric solutions of hierarchies of soliton equations. While we studied nonlinear partial differential equations in one space and one time dimension in the first volume, with the Korteweg–de Vries (KdV) and AKNS hierarchies as the prime examples, we now discuss differential-difference equations, where the time variable is continuous, while the one-dimensional spatial variable is discretized in this second volume. The key examples treated here in great detail are the Toda and Ablowitz–Ladik lattice hierarchies.

As in the case of the previous volume, we have tried to make the presentation as detailed, explicit, and precise as possible. The text is aimed to be self-contained for graduate students with sufficient training in analysis. Ample background material is provided in the appendices. The notation is consistent with that of Volume I, whenever possible (but the present Volume II is independent of Volume I).

To a large extent this enterprise is the result of joint work with several colleagues and friends, in particular, Wolfgang Bulla and Jeff Geronimo.

The writing, and in particular the typesetting of a technical manuscript is no easy task. As was the case for Volume I we have had the great fortune to be assisted by Harald Hanche-Olsen whenever we got stuck, and we appreciate his unselfish assistance.

Parts of the manuscript have been read by Emma Previato and Maxim Zinchenko. We gratefully acknowledge their constructive comments. We are particularly indebted to Emma Previato for the comprehensive list of misprints we received and

[1] *A Hard Day's Night* (1964).

for her enthusiasm about this project in general. Her efforts required a considerable time commitment and we truly appreciate her help. We are also very grateful to Engui Fan for supplying us with a large number of corrections for Volume I.

The web-page with URL

`www.math.ntnu.no/~holden/solitons`

contains an updated list of misprints and comments for Volume I and will include the same for this volume. Please send pertinent comments to the authors.

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