

Index of notation

- $\langle a | b \rangle$ Cartan coefficient, 17
 γG reflection coset, 228
 \perp orthogonal, 9
 $[V, g]$ image of $1 - g$, 9
 $[x, y]$ commutator, 13
 $a(d)$ number of degrees divisible by d , 203
 $\mathcal{A}(G)$ reflecting hyperplanes of G , 176
 $A[[t]]$ formal power series, 54
 $\text{Av}(P)$ averaging operator, 42
 $\mathcal{B}_n^{(3)}$ mixed (3, 6)-system, 148
 $\mathcal{B}_n^{(k)}$ line system, 104
 $b(d)$ number of codegrees divisible by d , 205
 $C_G(X)$ centraliser, 13
 $C(G, M)$ sum of the M -exponents of G , 195
 C_n cyclic group, 13, 73
 $\mathcal{D}_n^{(k)}$ line system, 104
 $[f, P]$ non-degenerate pairing, 181
 $D(g)$ Dickson's invariant, 252
 $d(g, \zeta)$ dimension of $V(g, \zeta)$, 203
 $\dim A$, 211
 $\Delta = \Delta_V$ discriminant polynomial, 231
 D_m binary dihedral group, 73
 $\delta_{r,i}$ Demazure operator, 188
 \mathcal{D}_S algebra of differential operators, 179
 d -regular element, 217
 $\mathcal{E}_6, \mathcal{E}_7, \mathcal{E}_8$ 3-systems, 106–107
 $\text{End}_G(V)$ endomorphism ring, 14
 $F = SJ^+$ ideal generated by invariants, 42
 \mathcal{F}_4 4-system, 109
 $f_X(t)$ fake degree, 63
 $\text{Fix } g$ fixed points of g , 9
 $f_M(t)$ fake degree, 192
 G' , $[G, G]$ derived group, 13
 G_A pointwise stabiliser, 12
 $G \circ H$ central product, 13
 $G(d)$ reflection subquotient, 215
 $GL(V)$ general linear group, 8
 $G(m, p, n)$ imprimitive reflection group, 25
 $\Gamma_a, \Gamma_b, \Gamma_c, \Delta, \Lambda$ Goethals–Seidel decomposition, 111
 $H \leq G$ subgroup, 12
 $H \trianglelefteq G$ normal subgroup, 12
 \mathbb{H} quaternions, 67
 $\mathcal{H}_3, \mathcal{H}_4$ 5-systems, 110
 \mathcal{H} space of G -harmonic polynomials, 183
 $\text{Hom}_G(V, W)$ linear transformations, 14
 $H \wr G$ wreath product, 24
 \mathcal{I} binary icosahedral group, 73
 $\mathcal{I}(A)$ ideal of A , 208
 $J = S^G$ algebra of invariants, 41
 $\mathcal{J}_3^{(4)}$ 4-system, 108
 $\mathcal{J}_3^{(5)}$ 5-system, 110
 $\text{Jac}(\omega_G)$ Jacobian matrix, 172
 $\mathcal{K}_5, \mathcal{K}_6$ 3-systems, 107–108
 $\mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4$ ternary 6-systems, 148
 $\Lambda(V)$ exterior algebra, 57
 λ root of $\lambda^2 + \lambda + 2 = 0$, 108, 153
 L_H , 46
 $L(q), R(q)$ left and right multiplication, 68
 \mathcal{M}_3 mixed (3, 6)-system, 149
 Δ_M M -discriminant, 231
 $\varepsilon_i(M)$ M -factors, 230
 M_G hyperplane complement, 255

- \mathcal{N}_4 4-system, 109
 $N_G(X)$ normaliser, 13
 N/C reflection subquotient, 214
 $N(q)$ quaternion norm, 67
 \mathcal{O} binary octahedral group, 73
 \mathcal{O}_4 4-system, 109
 ω cube root of unity, 10, 103
 $\widehat{\Omega}(V, Q), \widehat{\Omega}_{2m}^\varepsilon(\mathbb{F}_q), \widehat{\Omega}_n(\mathbb{F}_q)$ kernel of the spinor norm, 253
 $O_n(\mathbb{F}_q), O_{2m}^+(\mathbb{F}_q), O_{2m}^-(\mathbb{F}_q)$ finite orthogonal groups, 251
 $O_p(G)$ largest normal p -subgroup, 13, 156
 $[f, P]$ pairing of $S(V) \times S(V^*)$, 181
 $\Phi(P)$ Frattini subgroup, 13
 φ_d -Sylow theory, 269
 Π skew invariant, 172
 Π_M M -skew polynomial, 194
 $P_M^G(t)$ Poincaré polynomial, 55
 $P_{(S \otimes_{\Lambda M^*})G}(t, u)$ Poincaré series, 201
 \mathcal{Q} quaternion group, 73
 $\mathbb{Q}(G)$ field of definition, 19
 $q_i(M)$ M -exponent, 192
 $q \cdot r$ inner product of quaternions, 69
 r_a reflection of order two, 101
 $R_{T_w}^G(\theta_w)$ Deligne–Lusztig character, 267
 S^3 unit sphere, 69
 S/F coinvariant algebra, 51
 $Sp(V)$ symplectic group, 250
 $(S \otimes M)^G$ module of M -covariants, 192
 $SU_n(\mathbb{C})$ special unitary group, 8
 $S(V)$ symmetric algebra, 40, 56
 $S(V^*)$ coordinate ring, 40
 $Sym(n)$ symmetric group, 11
 $S_\zeta(G)$ poset of eigenspaces, 270
 \mathcal{T} binary tetrahedral group, 73
 t_a reflection of order three, 147
 τ golden section: $\tau^2 = \tau + 1$, 73, 91
 $\text{Tr}(q)$ quaternion trace, 67
 $T^r(V)$ tensor power, 39
 $T(V)$ tensor algebra, 39
 $U(V), U_n(\mathbb{C})$ unitary group, 8
 $U_n(\mathbb{F}_{q^2})$ finite unitary group, 250
 $V(d)$ union of eigenspaces, 212
 $V(g, \zeta)$ ζ -eigenspace of g , 203
 V^σ Galois twist, 198
 $\mathcal{V}(T)$ variety of T , 208
 $W(A_m)$, 29
 $W(\mathcal{A}_n) = \text{Sym}(n+1)$, structure of, 157
 $W(C)$ Weyl group of a Cartan matrix, 18
 $W(\mathcal{E}_6) = G_{35}$, structure of, 167
 $W(\mathcal{E}_7) = G_{36}$, structure of, 167
 $W(\mathcal{E}_8) = G_{37}$, structure of, 167
 $W(\mathcal{F}_4) = G_{28}$, structure of, 165
 $W(\mathcal{H}_3) = G_{23}$, structure of, 159
 $W(\mathcal{H}_4) = G_{30}$, structure of, 158
 $W(\mathcal{J}_3^{(4)}) = G_{24}$, structure of, 160
 $W(\mathcal{J}_3^{(5)}) = G_{27}$, structure of, 160
 $W(\mathcal{K}_5) = G_{33}$, structure of, 166
 $W(\mathcal{K}_6) = G_{34}$, structure of, 166
 $W(\mathcal{L})$ Weyl group of a line system, 102
 $W(\mathcal{L}_3) = G_{25}$, structure of, 161
 $W(\mathcal{L}_4) = G_{32}$, structure of, 162
 $W(\mathcal{M}_3) = G_{26}$, structure of, 161
 $W(\mathcal{N}_4) = G_{29}$, structure of, 165
 $W(\mathcal{O}_4) = G_{31}$, structure of, 165
 X/G orbits of G on X , 12
 $Z(G)$ centre, 13
 $\mathbb{Z}(G)$ ring of definition, 20

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