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\cdot_b	backwards stochastic integral	276
\cong	Hilbert-space isomorphism	217
\otimes	Hilbert-space tensor product	218
$\langle \cdot, \cdot \rangle_{T, \rho}$	inner product in $\mathcal{H}_2(T, E)$	217
\circ	Itô's circle (Stratonovitch integral)	270
\circ_b	backwards Stratonovitch integral	276
\diamond	Marcus canonical integral	272
\diamond_b	backwards Marcus canonical integral	277
α	index of stability	34
$\Gamma(\alpha)$	gamma function $\int_0^\infty x^{\alpha-1} e^{-x} dx$	53
δ	mesh of a partition	110
η	Lévy symbol or characteristic exponent	31
η_X	Lévy symbol of a Lévy process X	45
δ_x	Dirac measure at $x \in \mathbb{R}^n$	24
$\Delta X(t) = X(t) - X(t-)$	jump process	98
η_Z	Lévy symbol of the subordinated process Z	58
μ	intensity measure	101
$\mu_1 * \mu_2$	convolution of probability measures	21
ν	Lévy measure	29
$\xi_t, t \in \mathbb{R}$	solution flow to an ODE	359
$\rho(A)$	resolvent set of generator A	158
$\rho_{s,t}$	transition density	146
$\sigma(T)$	spectrum of an operator T	208
$(\tau_a, a \in \mathbb{R}^d)$	translation group of \mathbb{R}^d	160
ϕ	local unit	182
$\Phi = (\Phi_{s,t}, 0 \leq s \leq t < \infty)$	solution flow to an SDE	384
χ_A	indicator function of the set A	6

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ψ	Laplace exponent	53
$\Psi = (\Psi_{s,t}, 0 \leq s \leq t < \infty)$	solution flow to a modified SDE	392
(Ω, \mathcal{F}, P)	probability space	3
A	infinitesimal generator of a semigroup	155
A^X	infinitesimal generator of a Lévy process	169
A^Z	infinitesimal generator of a subordinated Lévy process	169
(b, A, ν)	characteristics of an infinitely divisible distribution	45
(b, λ)	characteristics of a subordinator	52
\hat{B}	$\{x \in \mathbb{R}^d; x < 1\}$	29
$B = (B(t), t \geq 0)$	standard Brownian motion	46
$B_A(t)$	Brownian motion with covariance A	49
$\mathcal{B}(S)$	Borel σ -algebra of a Borel set $S \subseteq \mathbb{R}^d$	2
$B_b(S)$	bounded Borel measurable functions from S to \mathbb{R}	6
$\mathcal{C}(I)$	cylinder functions over $I = [0, T]$	296
$C_c(S)$	continuous functions with compact support on S	6
$C_0(S)$	continuous functions from S to \mathbb{R} that vanish at ∞	6
$C^n(\mathbb{R}^d)$	n -times differentiable functions from \mathbb{R}^d to \mathbb{R}	7
$\text{Cov}(X, Y)$	covariance of X and Y	7
dY	stochastic differential of a semimartingale Y	234
D	diagonal, $\{(x, x); x \in \mathbb{R}^d\}$	182
D_A	domain of a generator A	155
$D_\phi F$	directional derivative of Wiener functional F in direction ϕ	298
DF	gradient of a Wiener functional F	299
D_t	Malliavin derivative	318
δ	divergence (Skorohod integral)	323
\mathbb{E}	expectation	7
$\mathbb{E}_{\mathcal{G}}$	conditional expectation mapping	10
\mathbb{E}_s	$\mathbb{E}(\cdot \mathcal{F}_s)$ conditional expectation given \mathcal{F}_s	91
$\mathbb{E}(X; A)$	$\mathbb{E}(X \chi_A)$	7
$\mathbb{E}(X \mathcal{G})$	conditional expectation of X given \mathcal{G}	10

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\mathcal{E}	closed form, Dirichlet form	190
\mathcal{E}_Y	stochastic (Doléans-Dade) exponential of Y	279
\hat{f}	Fourier transform of f	163
$f^+(x)$	$\max\{f(x), 0\}$	5
$f^-(x)$	$-\min\{f(x), 0\}$	5
f_X	probability density function (pdf) of a random variable X	10
\mathcal{F}	σ -algebra	2
$(\mathcal{F}_t, t \geq 0)$	filtration	83
$(\mathcal{F}_t^X, t \geq 0)$	natural filtration of the process X	83
\mathcal{F}_∞	$\bigvee_{t \geq 0} \mathcal{F}_t$	83
\mathcal{F}_{t+}	$\bigcap_{\epsilon > 0} \mathcal{F}_{t+\epsilon}$	84
G_T	graph of the linear operator T	204
$(\mathcal{G}_t, t \geq 0)$	augmented filtration	84
$\mathcal{G}^X, t \geq 0$	augmented natural filtration of the process X	84
H	Hurst index	51
$\mathcal{H}_{\mathbb{C}}$	$L^2(\Omega, \mathcal{F}_T, P; \mathbb{C})$	300
\mathcal{H}_η	non-isotropic Sobolev space	176
$\mathbb{H}(I)$	Cameron–Martin space over $I = [0, T]$	296
$\mathcal{H}_2(T, E)$	Hilbert space of square-integrable, predictable mappings on $[0, T] \times E \times \Omega$	217
$\mathcal{H}_2^-(s, E)$	Hilbert space of square-integrable, backwards predictable mappings on $[0, s] \times E \times \Omega$	275
I	identity matrix	26
I	identity operator	144
$IG(\delta, \gamma)$	inverse Gaussian random variable	54
$I_T(F)$	Itô stochastic integral of F	223
$\hat{I}_T(F)$	extended Itô stochastic integral of F	227
$I_n(f_n)$	multiple Wiener–Lévy integral	307
$I_n^{(B)}(f_n)$	multiple Wiener integral	307
$I_n^{(N)}(f_n)$	multiple Poisson integral	307
$J_n(f_n)$	iterated Wiener–Lévy integral	312
$J_n^{(B)}(f_n)$	iterated Wiener integral	313
$J_n^{(N)}(f_n)$	iterated Poisson integral	313
K_ν	Bessel function of the third kind	342
l_X	lifetime of a sub-Markov process X	152
$L(B)$	space of bounded linear operators in a Banach space B	153
$L^p(S, \mathcal{F}, \mu; \mathbb{R}^d)$	L^p -space of equivalence classes of mappings from S to \mathbb{R}^d	8

$L(x, t)$	local time at x in $[0, t]$	70
$M = (M(t), t \geq 0)$	local martingale	69
\mathcal{M}	martingale space	90
$M(\cdot)$	random measure	103
$\langle M, N \rangle$	Meyer angle bracket	94
$[M, N]$	quadratic variation of M and N	245
$\mathcal{M}_1(\mathbb{R}^d)$	set of all Borel probability measures on \mathbb{R}^d	21
$N = (N(t), t \geq 0)$	Poisson process	49
$\tilde{N} = (\tilde{N}(t), t \geq 0)$	compensated Poisson process	49
$N(t, A)$	$\#\{0 \leq s \leq t; \Delta X(s) \in A\}$	99
$\tilde{N}(t, A)$	compensated Poisson random measure	105
p	Poisson point process	105
$p_{s,t}(x, A)$	transition probabilities	145
$(p_t, t \geq 0)$	convolution semigroup of probability measures	63
p_{t_1, t_2, \dots, t_n}	finite-dimensional distributions of a stochastic process	19
$p_t(x, A)$	homogeneous transition probabilities	149
P_X	probability law (distribution) of a random variable X	5
\mathcal{P}	partition	110
\mathcal{P}	predictable σ -algebra	216
\mathcal{P}^-	backwards predictable σ -algebra	275
$\mathcal{P}_2(T, E)$	predictable processes whose squares are a.s. integrable on $[0, T] \times E$	275
$\mathcal{P}_2^-(s, E)$	backwards predictable processes whose squares are a.s. integrable on $[s, T] \times E$	275
$P(\cdot)$	projection-valued measure	177
$P(A \mathcal{G})$	conditional probability of the set A given \mathcal{G}	11
$P_{Y \mathcal{G}}$	conditional distribution of a random variable Y , given \mathcal{G}	11
q_t	probability law of a Lévy process at time t	145
\mathcal{R}_α	regularly varying function of degree $\alpha \in \mathbb{R}$	72
\mathcal{R}_0	slowly varying function	72
$R_\lambda(A)$	resolvent of generator A at the point λ	158
(S, \mathcal{F}, μ)	measure space	2
$S(\mathbb{R}^d)$	Schwartz space	163
$S(T, E)$	simple processes on $[0, T]$	218
$S^-(s, E)$	backwards simple processes on $[s, T]$	275
$(S(t), t \geq 0)$	stock price process	346
$(\tilde{S}(t), t \geq 0)$	discounted stock price process	346

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Σ	Malliavin covariance matrix	415
T	stopping time	91
T_A	first hitting time to a set A	91
\bar{T}	closure of a closable operator T	204
T^c	dual operator to T	206
T^*	adjoint operator to T	206
$T = (T(t), t \geq 0)$	subordinator	52
$(T_{s,t}, 0 \leq s \leq t < \infty)$	Markov evolution	144
$(T_t, t \geq 0)$	semigroup of linear operators	153
$(T_t^X, t \geq 0)$	semigroup associated with a Lévy process X	169
$(T_t^Z, t \geq 0)$	semigroup associated with a subordinated Lévy process Z	169
$V(t)$	value of a portfolio at time t	328
$\text{var}_{\mathcal{P}}(g)$	variation of a function g over a partition \mathcal{P}	110
V_g	(total) variation of g	110
$\text{Var}(X)$	variance of X	7
$\mathcal{W}_0(I)$	Wiener space over $I = [0, T]$	295
$X \sim N(m, A)$	X is Gaussian with mean m and covariance A	26
$X \sim \pi(c)$	X is Poisson with intensity c	27
$X \sim \pi(c, \mu_Z)$	X is compound Poisson with Poisson intensity c and Lévy measure $c\mu(\cdot)$	28
$X \sim S\alpha S$	X is symmetric α -stable	36
$X(t-)$	left limit of X at t	98

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