

## Notation Index

- $:=, A := B$  means  $A$  is defined by  $B$ , xii  
 $=:, A =: B$  means  $B$  is defined by  $A$ , xii  
 $\asymp$ , of the same order, 288  
 $\boxtimes$ , set of Cartesian products, 193  
 $\Rightarrow$ , converges in law, 136  
 $\otimes$ , product  $\sigma$ -algebra, 152  
 $\cap$ , class of intersections, 175  
 $\cap_{j=1}^k$ , class of intersections, 286  
 $\sqcup$ , class of unions, 193  
 $\sqcup_{j=1}^k$ , class of unions, 286  
 $\|\cdot\|_\alpha$ , differentiability norm, 287  
 $\|\cdot\|_{BL}$ , bounded Lipschitz norm, 154  
 $\|\cdot\|_C$ , 179  
 $\|\cdot\|_{\mathcal{F}}$ , 137  
 $\|\cdot\|_L$ , Lipschitz seminorm, 154  
 $\|\cdot\|'$ , dual norm, 67  
 $[\cdot, \cdot], [f, g] := \{h : f \leq h \leq g\}$ , 269  
 $(\cdot, \cdot)_{0,p}$ , covariance, 62  
 $\alpha_n := n^{1/2}(F_n - F)$ , 3  
 $AEC(P, \tau)$ , asymptotic equicontinuity condition, 159  
 $\beta(\cdot, \cdot)$ , bounded Lipschitz distance, 157  
 $B(k, n, p)$ , binomial probability, 13  
 $C(F) := I(F) \cup R(F)$ , 297  
 $C^1$ , polar of  $C$ , 87  
 card, cardinality, 175  
 $\text{cov}_p$ , covariance, 62  
 $\mathcal{C}(\alpha, K, d)$ , subgraphs of  $\alpha$ -smooth  $f$ 's, 288  
 $D(\varepsilon, A, d)$ , packing number, 8  
 $\Delta$ , symmetric difference, 137  
 $\Delta\Delta$ , set of symmetric differences, 184  
 $\Delta^C$ , number of induced subsets, 175  
 ${}_\delta A$ ,  $\delta$ -interior of  $A$ , 298  
 $\delta_x$ , point mass at  $x$ , 1  
 $D_F^{(p)}(\delta, \mathcal{F})$ , 239  
 $D_F^{(p)}(\varepsilon, \mathcal{F}, Q)$ , 205  
 $D^{(p)}(\varepsilon, \mathcal{F})$ , 205  
 $D^{(p)}(\varepsilon, \mathcal{F}, Q)$ , 205  
 $D^p$ , partial derivative, 287  
 dens, combinatorial density, 178  
 diam, diameter, 8  
 $d_P, d_P(A, B) := P(A\Delta B)$ , 200  
 $d_{p,Q}, L^p(Q)$  distance, 205  
 $d_{\text{sup}}$ , supremum distance, 269  
 $E(k, n, p)$ , binomial probability, 13  
 $E^*$ , upper expectation, 138  
 $e^\mu, \mu$  a signed measure, 332  
 ess.inf, essential infimum, 138  
 ess.sup, essential supremum, 66  
 $\Phi$ , normal distribution function, 98  
 $\phi$ , normal density, 98  
 $F$ , distribution function, 1  
 $F_n$ , empirical distribution function, 1  
 $F_{\mathcal{F}}$ , envelope, 205  
 $f_*$ , 141  
 $f \circ g$ , composition, 136  
 $f^*$ , measurable cover function, 138  
 $\mathcal{F}_{\alpha,K}(F)$ ,  $\alpha$ -differentiable functions, 287  
 $\mathcal{F}'(\delta, d)$ , 373  
 $G_P$ , Gaussian limit process, 61  
 $\mathcal{G}_{\alpha,K,d}$ , differentiable functions, 288  
 $H(\varepsilon, A, d) = \log N(\varepsilon, A, d)$ , 8  
 $\bar{H}_s(\mathcal{F}, M)$ , hull, 169  
 $h(\cdot, \cdot)$ , Hausdorff metric, 284

- $I(F)$ , inside  $F$  boundary, 297  
 $I = [0, 1]$ , unit interval, 57  
 i.i.d., indep. identically dist., 1  
  
 $L(\cdot)$ , isonormal process, 64  
 $L(A)^*$ ,  $\text{ess. sup}_A L$ , 66  
 $|L(A)|^*$ ,  $\text{ess. sup}_A |L|$ , 66  
 $\mathcal{L}^2(\mu)$ , set of square-integrable functions, 64  
 $L^2$ , equivalence-classes for  $\mathcal{L}^2$ , 64  
 $\mathcal{L}$ , law = distribution, 62  
 $\mathcal{L}^0$ , 137  
 $\mathcal{L}_0^2$ , 134  
 $\mathcal{L}\mathcal{L}_d$ , lower layers in  $\mathbb{R}^d$ , 300  
 $\mathcal{L}\mathcal{L}_{d,1}$ , lower layers in unit cube, 300  
  
 $m^{\mathcal{C}}(n)$ , 175  
  
 $v_n$ , empirical process, 133  
 $v_n^B$ , bootstrap empirical process, 324  
 $N\mathcal{C}_{\leq k}$ ,  $N$  choose  $\leq k$ , 176  
 $N(\varepsilon, A, S, d)$ , ball covering number, 8  
 $N(\varepsilon, C, d)$ , covering number, 8  
 $N_I(\varepsilon, C, P)$ , 270  
 $N_{[]}^{(q)}(\varepsilon, \mathcal{F}, P)$ , 269  
 $nn(g) = \{x : g(x) > 0\}$ , 179  
  
 $\pi_0$ , centering projection, 133  
 $P_n$ , empirical measure, 1  
 $P_n^B$ , bootstrap empirical measure, 323  
  
 $P^*$ , outer probability, 137  
 $\text{pos}(G) = \{\text{pos}(g) : g \in G\}$ , 179  
 $\text{pos}(g) = \{x : g(x) > 0\}$ , 179  
 $[p] = p_1 + \cdots + p_d$ , 287  
  
 $\rho(\cdot, \cdot)$ , Prokhorov distance, 157  
 $\rho_P$ ,  $\text{cov}_P$  distance, 134  
 $R(F)$ ,  $\text{range}(F)$ , 297  
 $\overline{\mathbb{R}} = [-\infty, \infty]$ , 136  
 $\mathbb{R}^X$ , all functions  $X \mapsto \mathbb{R}$ , 180  
  
 $f^*$ , upper integral, 136  
 $f_*$ , lower integral, 141  
 $S^1$ , unit circle, 179  
 $\overline{\text{sc}o}$ , symmetric closed convex hull, 91  
 $\text{sc}o$ , symmetric convex hull, 159  
 $\mathcal{S}(\mathbb{R}^n)$ , L. Schwartz space, 80  
  
 $U[0, 1]$ , uniform law on  $[0, 1]$ , 168  
  
 $V(C) = S(C) + 1$ , VC index, 175  
 $\text{var}(X) = E((X - EX)^2)$ , 9  
  
 $W_P$ , isonormal process on  $L^2(P)$ , 65  
  
 $[x]$ , 57  
 $x^p = x_1^{p(1)} x_2^{p(2)} \cdots$ , 287  
 $x_t$ , Brownian motion, 3  
  
 $y_t$ , Brownian bridge, 3

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